# Type-III two Higgs doublet model plus a pseudoscalar confronted with h → μτ , muon g − 2 and dark matter (Postprint)

**Authors:** Xuewen Liu, Ligong Bian, Xue-Qian Li Jing Shu

**Date:** 2017-08-03T09:04:38+00:00

## Abstract

在本研究中，我们向 III 型两希格斯二重态模型（2HDM）引入一个额外的单态赝标量，以期解决现代粒子宇宙学中的一系列问题。在轻赝标量存在的情况下，CMS 实验测量到的 h → μτ 超出以及（g − 2)μ 反常可在某些参数空间内同时得到解释，而这些参数空间也能容纳 LHC 上获得的关于味破坏过程 $\tau \to \mu\gamma$ 和希格斯衰变的数据。在相同的参数空间内，DM 遗迹丰度亦能得到很好解释。此外，近期观测到的银河系中心伽马射线超出（GCE）被认为可通过暗物质（DM）对湮灭来实现，本研究也探讨了该湮灭过程由赝标量介导的情形。

## Full Text

### Type-III Two Higgs Doublet Model Plus a Pseudoscalar Confronted with $h \to \mu\tau$, Muon $g-2$ and Dark Matter

**Xuewen Liu,**[1] **Ligong Bian,**[2,3] **Xue-Qian Li,**[1] **and Jing Shu**[3]

[1]School of Physics, Nankai University, Tianjin 300071, China
[2]Department of Physics, Chongqing University, Chongqing 401331, China
[3]State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China

**Abstract.** In this work, we introduce an extra singlet pseudoscalar into the Type-III two Higgs doublet model (2HDM) to address a series of problems in modern particle cosmology. With a light pseudoscalar, the $h \to \mu\tau$ excess measured at CMS and the $(g-2)_\mu$ anomaly can be simultaneously explained within certain parameter spaces that also satisfy constraints from flavor-violating processes $\tau \to \mu\gamma$ and Higgs decay data from the LHC. Within the same parameter spaces, the DM relic abundance is well accommodated. Moreover, the recently observed Galactic Center gamma-ray excess (GCE) is proposed to arise through dark matter (DM) pair annihilations, and in this work we also address the scenario where the annihilation is mediated by the pseudoscalar.

## Introduction

Even within the framework of a minimally extended Standard Model (SM) with non-zero neutrino masses, leptonic flavor violation (LFV) processes are negligible due to the smallness of neutrino masses that have been experimentally confirmed. Therefore, direct searches for LFV processes provide an ideal probe for new physics beyond the SM, and any observational anomaly may hint at its existence. Besides B-factories, the LHC—with its high energy and luminosity—is definitively the machinery for such exploration. A search for LFV has been performed by the CMS collaboration via two channels $h \to \mu\tau_e$ and $h \to \mu\tau_h$, reporting a $2.4\sigma$ excess with a branching fraction $\mathrm{BR}(h \to \mu\tau) = (0.84^{+0.39}_{-0.37})\%$ \cite{CMS_{excess}}. If one interprets this excess as an anomaly, there must be mechanisms beyond the SM responsible for it.

The Type-III two Higgs doublet model (2HDM) is one such framework, because the model contains flavor-violating Yukawa interactions that may contribute to LFV at tree level. The model has been explored extensively [?, ?, ?, ?, ?, ?, ?, ?, ?] to study this phenomenological observation. Furthermore, the Yukawa interaction contributes to the muon $g-2$ via one-loop diagrams and thus provides a possibility to explain the $(g-2)_\mu$ discrepancy [?, ?, ?]. Meanwhile, the flavor-changing Yukawa interaction would induce substantial contributions to the radiative decay $\tau \to \mu\gamma$, so the flavor-changing Yukawa interaction might be rigorously constrained by available experimental data [?, ?, ?].

One of the main characteristics of the 21st century is that cosmology has already become a precision science, and corresponding observations must be combined with precise measurements and new discoveries at Earth-based facilities to test existing theories. The identity of dark matter (DM) and the interactions that determine the behavior of DM particles are key points, and searching for them is the most challenging task for both experimentalists and theorists in high-energy physics and cosmology.

Recently, Fermi Large Area Telescope data show an excess of gamma rays at energies of a few GeV coming from the Galactic Center (GCE) [?, ?, ?, ?, ?, ?, ?, ?, ?]. To explain this observation, it has been suggested that annihilation of DM particles weighing $30-70$ GeV into $b\bar{b}$ is responsible for the GCE [?, ?, ?]. Even though other proposals exist to explain the excess, such as a population of millisecond pulsars (MSP) [?, ?] that might be responsible for the GCE, it is not easy to explain both the energy spectrum and spatial distribution of the GCE [?, ?]. Thus in this work we discard astrophysical source explanations [?, ?, ?, ?] and focus on the DM scenario.

Dwarf galaxies are considered the cleanest sources for detecting gamma rays produced by DM annihilations, so data on gamma rays observed from Reticulum II [?, ?, ?] may imply abundant DM in our galaxy. To be certain, we need to compute the DM annihilation cross section in a particle physics model. Meanwhile, other cosmological phenomena must also be considered—namely, the DM annihilation cross section required by the new data should be of the same order as that determined by thermal DM relic abundance.

However, the original Type-III 2HDM does not provide a natural explanation for the DM particle annihilation cross section. Thus we need to extend the model to also accommodate DM annihilation. It is noteworthy that a pseudoscalar could mediate the annihilation of dark matter (DM) pairs, while due to the small momentum transfer in the t-channel, the interaction between DM particles coming from outer space and nuclei in detectors is not affected by the existence of the new pseudoscalar, so DM particles may evade direct searches at not-very-sensitive detectors.

This idea has been implemented in various models [**?**, **?**, **?**, **?**, **?**, **?**, **?**, **?**, **?**, **?**, **?**, **?**, **?**, **?**, **?**]. It further motivates us to consider the DM explanation for the GCE and introduce a Dirac fermion field serving as the DM candidate. In this work, we introduce a pseudoscalar $a_0$ into the Type-III 2HDM. The pseudoscalar does not directly couple to SM particles, but it slightly mixes with the CP-odd Higgs that exists in the original Type-III 2HDM, thus it would effectively couple to the SM via this mixing. Therefore, the pseudoscalar can mediate an effective interaction between the DM $\chi\bar{\chi}$ and SM fermions $b\bar{b}$.

Moreover, its introduction may bring two additional advantages:

1. An extra pseudoscalar would open a new decay channel for the Higgs and thus affect the $h \to \mu\tau$ excess.
2. There could be mixing of the newly introduced pseudoscalar with the CP-odd scalar $A_0$ in the original Type-III 2HDM, which induces a new contribution to the value of $(g-2)_\mu$. With increased mixing, the contribution of the new pseudoscalar would cancel that of the heavy Higgs, thus in this extended model the theoretical prediction for $(g-2)_\mu$ can be decreased to a tolerable level.

In this work the pseudoscalar also plays a role in explaining the $h \to \mu\tau$ excess and the discrepancy between theoretical prediction and data for $(g-2)_\mu$.

The paper is organized as follows: after this introduction, we discuss in Section II the new scenario where a light pseudoscalar is introduced to extend the Type-III 2HDM. In Section III, we investigate relevant topics including the $h \to \mu\tau$ excess observed at CMS, the muon $(g-2)_\mu$ anomaly, the Galactic Center gamma-ray excess (GCE), and constraints from the $\tau \to \mu\gamma$ process and LHC Higgs data. Numerical results are presented in Section IV, and the final section is devoted to our conclusions and discussion.

## The Model

In this work, a Dirac fermion $(\chi)$ of mass $m_\chi$ serves as the DM candidate and a gauge singlet pseudoscalar $a_0$ is introduced to extend the Type-III two Higgs doublet model, where $a_0$ mediates the coupling between the dark matter and SM particles. The interaction Lagrangian reads

$$\mathcal{L}_{\rm dark} = -y_\chi a_0 \bar{\chi} i \gamma_5 \chi.$$

The pseudoscalar $a_0$ mixes with the pseudoscalar in the original 2HDM and then couples to SM particles through the potential, given as [?]

$$V = V_{\rm 2HDM} + V_{\rm portal},$$

$$V_{\rm portal} = -iBa_{0H}1^\dagger H_2 + \text{h.c.},$$

where $B$ is a parameter of mass dimension, and the Higgs potential is [?]

$$V_{\rm 2HDM} = \mu_1 H_1^\dagger H_1 + \mu_2 H_2^\dagger H_2 + (\mu_3 H_1^\dagger H_2 + \text{h.c.}) + \frac{\lambda_1}{2}(H_1^\dagger H_1)^2 + \frac{\lambda_2}{2}(H_2^\dagger H_2)^2 + \lambda_3 (H_1^\dagger H_1)(H_2^\dagger H_2) + \lambda_4 (H_1^\dagger H_2)(H_2^\dagger H$$

We can explicitly rewrite $H_1$ and $H_2$ in the Higgs basis as

$$H_1 = \begin{pmatrix} G^+ \\ \frac{v+\phi_1+iG^0}{\sqrt{2}} \end{pmatrix}, \quad H_2 = \begin{pmatrix} H^+ \\ \frac{\phi_2+iA_0}{\sqrt{2}} \end{pmatrix},$$

where $G^+$ and $G^0$ are the Nambu-Goldstone bosons and $H^+$ and $A_0$ are a charged Higgs boson and a CP-odd Higgs boson, respectively. Without loss of generality, let us concentrate on the CP-conserving case, where $a_0$ does not develop a vacuum expectation value (VEV) and all $\lambda_i$ and $\mu_3$ are set to be real.

Then the potential is minimized to $\langle H_1 \rangle = v/\sqrt{2}$, $\langle H_2 \rangle = 0$, with $v = 246$ GeV.

In the basis of $(\phi_1, \phi_2)$, the mass matrix for the CP-even Higgs is $M_h^2$ whose elements are

$$h_{11} = 2\lambda_1 v^2, \quad h_{22} = m_H^2 + \lambda_5 v^2, \quad h_{12} = h_{21} = \lambda_6 v^2.$$

Diagonalizing the matrix, one obtains the physical CP-even states $h$ and $H$ $(m_h \le m_H)$ as

$$\begin{pmatrix} h \\ H \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix},$$

with eigen-masses

$$m_{h,H}^2 = \frac{h_{11}+h_{22}}{2} \mp \frac{1}{2}\sqrt{(h_{11}-h_{22})^2 + 4(M_h^2)^2},$$

and mixing angle

$$\tan 2\alpha = \frac{2h_{12}}{h_{11} - h_{22}}.$$

We consider the eigenstates $h$ and $H$ as the SM-like and heavy Higgs bosons, respectively.

The CP-odd Higgs $A_0$ mixes with $a_0$ due to the potential $V_{\text{portal}}$ (Eq. 3), and the mass matrix in the $(A_0, a_0)$ basis is

$$\begin{pmatrix} m_A^2 & Bv \\ Bv & m_a^2 \end{pmatrix},$$

where $m_A^2 = m_H^2 + \lambda_4 v^2/2 - \lambda_5 v^2$. Thus, the relation between $A_0, a_0$ and mass eigenstates $A$ and $a$ is

$$\begin{pmatrix} A_0 \\ a_0 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} A \\ a \end{pmatrix},$$

with mixing angle

$$\tan 2\theta = \frac{2Bv}{m_A^2 - m_a^2},$$

and masses

$$m_{a,A}^2 = \frac{m_A^2 + m_a^2}{2} \mp \frac{1}{2}\sqrt{(m_A^2 - m_a^2)^2 + 4B^{2v^2}}.$$

The parameter $B$ in terms of $m_{a,A}$ and $\theta$ can be expressed as

$$B = \frac{(m_A^2 - m_a^2)\sin 2\theta}{2v}.$$

The effective coupling of the CP-even Higgs bosons to the SM $W$ is $igm_W C_{\phi WW} g_{\mu\nu}$ with $C_{hWW} = \sin\alpha$, $C_{HWW} = \cos\alpha$, and the CP-odd Higgs bosons $A(a)$ do not couple to $W$, i.e., $C_{A(a)WW} = 0$. The $V_{\text{portal}}$ is recast in terms of mass eigenstates and mixing angle as

$$V_{\text{portal}} = -\frac{(m_A^2 - m_a^2)}{2v}\left[\sin^4\theta\, aA + \sin^2\theta\cos^2\theta(A^2 - a^2)\right] \times (\sin\alpha\, h + \cos\alpha\, H).$$

The Yukawa interactions in the extended Type-III 2HDM are

$$\mathcal{L}_{\text{Yukawa}} = -\bar{Q}_L V_{\text{CKM}} H_1 y_i^u u_R - \bar{Q}_L \tilde{H}_1 y_i^d d_R - \bar{L}_L H_1 y_i^e e_R - \bar{Q}_L V_{\text{CKM}} H_2 \rho_{ij}^u u_R - \bar{Q}_L \tilde{H}_2 \rho_{ij}^d d_R - \bar{L}_L H_2 \rho_{ij}^e e_R - \bar{L}_L \tilde{H}_2 \rho$$

where $Q = (u_L, V_{\text{CKM}} d_L)^T$, $L = (V_{\text{MNS}} \nu_L, e_L)^T$ and $\tilde{H}_i$ stands for $i\sigma_2 H_i^*$. $V_{\text{CKM}}$ ($V_{\text{MNS}}$) is the Cabibbo-Kobayashi-Maskawa (Maki-Nakagawa-Sakata) matrix. The general 3-by-3 complex matrices $\rho_{ij}^f$ induce Higgs-mediated Flavor Changing Neutral Current (FCNC). In the mass eigen-basis of the Higgs bosons, the Yukawa interactions are recast as

$$\mathcal{L}_{\text{Yukawa}} = -y_{\phi ij} \bar{f}_{Li} \phi f_{Rj} - \bar{\nu}_{Li} (V_{\text{MNS}}^\dagger \rho^e)_{ij} H^+ e_{Rj} - \bar{u}_i (V_{\text{CKM}} \rho^{d\dagger} P_R - \rho^{u\dagger} V_{\text{CKM}} P_L)_{ij} H^+ d_j + \text{h.c.},$$

with $\phi = h, H, A, a$, $f = u, d, e, \nu$, and

$$y_{hij} = \frac{m_{fi}}{v} \cos\alpha \delta_{ij} + \frac{\rho_{ij}^f}{\sqrt{2}} \sin\alpha, \quad y_{Hij} = -\frac{m_{fi}}{v} \sin\alpha \delta_{ij} + \frac{\rho_{ij}^f}{\sqrt{2}} \cos\alpha,$$

$$y_{Aij} = \frac{\rho_{ij}^f}{\sqrt{2}} \cos\theta, \quad y_{aij} = \frac{\rho_{ij}^f}{\sqrt{2}} \sin\theta, \quad (f = u),$$

$$y_{Aij} = i\frac{\rho_{ij}^f}{\sqrt{2}} \cos\theta, \quad y_{aij} = i\frac{\rho_{ij}^f}{\sqrt{2}} \sin\theta, \quad (f = d, e),$$

where the couplings $y_{Aij}$ and $y_{aij}$ exist in the new Feynman rules and are accompanied by $\gamma_5$. For investigating the $(g-2)_\mu$ excess, we do not need to invoke the so-called Cheng-Sher ansatz for $\rho_{ij}^f$ since the corresponding parameter space is highly restricted [**?**, **?**]. The smallness of the mixing parameter $\cos\alpha$ is favored by current LHC Higgs coupling measurements, and we will study this issue later. In this scenario, the coupling of the SM-like Higgs to fermions $y_{hf\bar{f}}$ approaches the SM value, thus flavor-violating processes mediated by the SM-like Higgs boson are almost completely suppressed.

## Several Relevant Topics Specifically Addressed

### A. Constraints on the Parameter Space of the Aforementioned Model

At first, we explore possible constraints coming from B physics and electroweak precision tests, and find that the model is more advantageous than the Type-II 2HDM as it may evade those constraints in the situation $m_{H^+} \sim m_A$ because the $\tan\beta$ enhancement effect does not exist. The $T$ parameter in our model is obtained as [**?**, **?**]

$$\Delta T = \frac{1}{16\pi^2 s_W^2 m_W^2} \times \left[ F(m_{H^+}^2, m_A^2) \cos^2\theta + \sin^2\alpha F(m_{H^+}^2, m_H^2) - \cos^2\theta F(m_A^2, m_H^2) \right] ,$$

where

$$F(x,y) = \frac{x+y}{2} - \frac{xy}{x-y} \ln \frac{x}{y}.$$

**1. Constraints from B Physics** A light $a$ can mediate the initial and final states of the decay $B_s \to \mu^+\mu^-$ in addition to the SM contribution, hence imposing a stringent constraint on the model. For $m_a \ll m_Z$, the correction due to $a$ exchange at the s-channel was calculated [?] and the results are

$$\mathrm{BR}(B_s \to \mu^+\mu^-) \approx \mathrm{BR}(B_s \to \mu^+\mu^-)_{\mathrm{SM}} \left[ 1 + \frac{v^2 m_{B_s}}{4 m_\mu (m_a^2 - m_{B_s}^2)} \frac{\sin^2\theta \rho_{bb} \rho_{\mu\mu}}{(1+Y(x_t))^2} f(x_t, y_t, r) \right]^2 ,$$

for $\rho_{tb}^u = 0$, and

$$\mathrm{BR}(B_s \to \mu^+\mu^-) \approx \mathrm{BR}(B_s \to \mu^+\mu^-)_{\mathrm{SM}} \left[ 1 + \frac{v^2 m_{B_s}}{2 m_\mu (m_a^2 - m_{B_s}^2)} \frac{\sin^2\theta \rho_{bb} \rho_{\mu\mu}}{(1+Y(x_t))^2} f(x_t, y_t, r) \right]^2 ,$$

for $\rho_{tb}^u = \rho_{bb} s_\theta$, with $x_t = m_t^2/m_W^2$, $y_t = m_{H^\pm}^2/m_t^2$, $r = m_t^2/m_{H^\pm}^2$, and the $f$ and $Y$ functions can be found in Eq. A.3 of the Appendix. The average of the LHCb and CMS measurements on this mode is $\mathrm{BR}(B_s \to \mu^+\mu^-) = (2.9 \pm 0.7) \times 10^{-9}$ [?, ?, ?]. This can be compared against the SM prediction, which is taken to be $(3.65 \pm 0.23) \times 10^{-9}$ [?, ?]. We note that the $\tan\beta$ enhancement effect in the calculation of $B_s \to \mu^+\mu^-$ with the Type-II 2HDM [?, ?, ?] does not appear in our model, thus the constraint from $B_s$ leptonic decay is relaxed.

The $a/A$ and $h/H$ mixing is highly constrained by current LHC data to be around $\sin\alpha \sim 1$ and $\cos\theta \sim 1$. Along with the parameter choice $m_{H^+} \sim m_A$, the $T$ parameter is suppressed, as seen in Eq. (19), therefore it does not actually affect applications of this model as indicated in [?].

## B. Relevant Processes Under Investigation

In the following sections, we investigate several relevant processes in this extended Type-III 2HDM model. All puzzles regarding the $h \to \mu\tau$ excess, muon $(g-2)_\mu$ anomaly, constraints from $\tau \to \mu\gamma$, dark matter relic abundance, and GCE explanations that were not solved in previous Type-III 2HDM studies will be addressed.

**1. $h \to \mu\tau$ Excess** The existence of flavor-violating Yukawa couplings in the extended Type-III 2HDM may possibly explain the $h \to \mu\tau$ excess observed by the CMS collaboration. Let us compute the branching ratio of $h \to \mu\tau$ in terms of our model; the result is shown as

$$\mathrm{BR}(h \to \tau\mu) = \frac{(|y_{h\tau\mu}|^2 + |y_{h\mu\tau}|^2)}{16\pi\Gamma_h},$$

where $\Gamma_h$ is the total decay width of the SM-like Higgs boson. To meet the observed excess, the flavor mixing should be of a magnitude

$$\bar{\rho}_{\mu\tau} \equiv \sqrt{|\rho_{\mu\tau}|^2 + |\rho_{\tau\mu}|^2} \gtrsim 0.0018\frac{\sqrt{\cos\alpha}}{|c_{\beta\alpha}|}.$$

In the Type-III 2HDM, the constraint on the $T$ parameter imposed by electroweak precision tests may be the most stringent. Following the method proposed by the authors of Ref.~[**?**, **?**], for $\Gamma_h \sim 4.2$ MeV and $c_{\beta\alpha} = 0.01$, as long as $\bar{\rho}_{\mu\tau} \sim \mathcal{O}(0.1)$ is reached, the new model is able to accommodate the $h \to \mu\tau$ excess.

**2. Muon $(g-2)_\mu$ Anomaly and $\tau \to \mu\gamma$** With the lepton flavor-violating Lagrangian, previous studies indicated that new physics may contribute to the anomalous magnetic moment of the muon and the radiative process $\tau \to \mu\gamma$ via a chirality-flipping dipole operator [**?**]

$$\mathcal{L} = \frac{em_l}{\Lambda^2}\bar{l}_j[i\sigma_{\mu\nu}q^\nu(A^L_{ij}P_L + A^R_{ij}P_R)l_j] + \text{h.c.},$$

where $i, j$ denote the flavors of the external leptons, $F_{\alpha\beta}$ is the electromagnetic strength tensor, and $\sigma_{\alpha\beta} = \frac{i}{2}[\gamma_\alpha, \gamma_\beta]$. The diagonal component contributes to the anomalous magnetic moment of the muon, whereas the off-diagonal component corresponds to a dipole transition from $\tau$ to $\mu$.

It is noted that the coefficient $C_{ij}$ is derived in various new physics models and has different values that help determine the corresponding parameter space. In this work, we derive this coefficient in the extended Type-III 2HDM. The flavor-violating Yukawa couplings and the newly introduced pseudoscalar induce additional contributions to $(g-2)_\mu$ and $\tau \to \mu\gamma$ via one-loop and two-loop Barr-Zee diagrams [**?**], as shown in the two panels of Figure 1. We include these extra contributions in the numerical computations.

**1. The Muon $g-2$ Anomaly** In our case, the model-dependent coefficient $C_{ij}/\Lambda^2$ could be expressed as [**?**]

$$\frac{C_{\mu\mu}}{\Lambda^2} = \frac{N_c A_{\mu\mu} e m_\mu}{16\pi^2 m_\tau^2}.$$

The contributions from the two CP-odd Higgs bosons give rise to

$$\delta a_\mu^{\text{1-loop}} \simeq -\frac{\alpha_{\text{em}} m_\mu}{4\pi^3 m_f^2} \sum_f y_{\phi\mu\mu} y_{\phi f \bar{f}} f_\phi(r_f),$$

where $\phi = A, a$, $r_f = m_f^2/m_\phi^2$, and $y_{\phi f\bar{f}}$ is defined in Eq.~(16). The function $f_\phi$ can be found in Eq. A.1 of the attached Appendix. The $W$ and Goldstone loops would not contribute to the Barr-Zee diagrams because $C_{A(a)WW} = 0$.

The other contributions from the CP-even Higgs bosons are induced by the 2-loop Barr-Zee diagrams where an intermediate photon and a $W$-boson are involved [**?**, **?**].

**2. The $\tau \to \mu\gamma$ Process** The branching ratio of $\tau \to \mu\gamma$ is calculated as

$$\text{BR}(\tau \to \mu\gamma) = \text{BR}(\tau \to \mu\nu\bar{\nu}) \times \frac{48\pi^3 \alpha_{\text{em}}}{G_F^2 m_\tau^4} \left(|A_L|^2 + |A_R|^2\right).$$

Due to $|y_{h\tau\mu}| = |y_{h\mu\tau}|$ in the model, we have $|A_L| = |A_R| \equiv |A|$. The two CP-odd Higgs bosons contribute to the form factor $A$ through 1-loop and 2-loop Barr-Zee diagrams as shown in Fig. 1, and the new form factor is obtained as

$$A = A_1 + A_{t,b},$$

where

$$A_1 = \sum_\phi \frac{y_{\phi\mu\tau} y_{\phi\tau\tau}}{16\pi^2 m_\tau} f_\phi(r_\tau),$$

$$A_{t,b} = \sum_{f=t,b} \frac{y_{\phi\mu\tau} y_{\phi f\bar{f}} \alpha_{\text{em}} Q_f^2 N_c}{8\pi^2 m_f} f_\phi(r_f).$$

**3. DM Annihilation and GCE** For $m_a \ll m_A$, the Dirac DM fermions annihilate into $b\bar{b}$ primarily through exchanging $a$ at s-channel. The annihilation cross section for the relative velocity $v_r$ is given as

$$\langle \sigma v_r \rangle \simeq \frac{y_\chi^2 \rho_{bb}^2 s^2}{8\pi (s - m_a^2)^2 + m_a^2 \Gamma_a^2},$$

where $s$ is the center-of-mass energy of the annihilating DM fermions, and $m_a$ and $\Gamma_a$ are the mass and decay width of the mediator boson $a$, respectively. In the non-relativistic approximation $s \sim 4m_\chi^2 + m_\chi^2 v_r^2$, thus Eq. 33 can be rewritten as

$$\langle \sigma v_r \rangle \simeq \frac{y_\chi^2 \rho_{bb}^2}{256\pi m_\chi^2} \frac{(\delta + v_r^2/4)^2 + \gamma^2}{(\delta^2 + \gamma^2)^2},$$

where $\gamma \equiv m_a \Gamma_a / 4m_\chi^2$ and $\delta$ are two dimensionless parameters, and the kinematic factor $\delta$ is defined as $\delta = 1 - m_a^2/(4m_\chi^2)$. As long as $\delta$ is not too small, the DM annihilation could occur in a region far away from resonance, and then the cross section is almost independent of velocity. In this case, the GCE and correct thermal DM relic density could be accommodated simultaneously provided that the parameter $y_\chi^2 \rho_{bb}^2 \sin^2 2\theta$ is adjusted to an appropriate value. For small $\delta$, the resonance effect would enhance the DM annihilation cross section. In that case, adjusting the parameter $y_\chi^2 \rho_{bb}^2 \sin^2 2\theta/(\delta^2 + \gamma^2)$ can give a reasonable explanation for the GCE observation. When $\delta > 0$, Eq. 34 indicates that the magnitude of $\langle \sigma v_r \rangle$ decreases as temperature increases, and the process $\bar{\chi}\chi \to a \to b\bar{b}$ does not sufficiently reduce the DM abundance at the freeze-out epoch; therefore, some other annihilation processes that affect the DM relic abundance must exist in the Lee-Weinberg evolution equation.

## A Synthesis of All the Ingredients

In this section, we perform a numerical analysis to investigate the CMS $h \to \mu\tau$ excess, muon $(g-2)_\mu$ anomaly and the $\tau \to \mu\gamma$ constraint, as well as DM relic abundance and GCE in the extended Type-III 2HDM. The model has been implemented in the program FeynRules [?], and the model file for CalcHEP [?] has been employed in the packages micrOMEGAs 4.1.8 [?] to calculate the relic density and annihilation cross section of DM.

The Higgs masses in the model are set as: $m_H = 150$ GeV, $m_{H^\pm} = m_A = 300$ GeV. Here the value of $m_{H^\pm}$ is allowed by flavor physics constraints [?], and $m_A = m_{H^\pm}$ is suggested by the $T$ parameter constraint as indicated in Section III A 2. The value of $m_H$ employed in our computations is consistent with vacuum stability requirements [?].

The magnitude of the two CP-even Higgs mixing angle $\alpha$ is severely constrained by recent Higgs data [?], and its closeness to $\pi/2$ will be explored later. The invisible and undetected decays of the SM-like Higgs boson are accounted as decays beyond the SM (BSM) [?], denoted as $\Gamma(h \to aa) + \Gamma(h \to \mu\tau) = \Gamma_{\text{BSM}}(h)$. Since the BSM contribution to the decay branching ratio is bounded below 0.34 at 68% CL, the decays $h \to aa$ and $h \to \mu\tau$ in this model are constrained.

The best fit of the branching ratio of the undetected decays is $\leq 0.23$ at 68% CL, and this limit allows $\text{BR}(h \to \mu\tau) = (0.84^{+0.39}_{-0.37})\%$, implying that the constraint on the flavor-violating process is relaxed. As $m_a < m_h/2$, the $a/A$ mixing angle $\theta$ dominates the exotic decay rate of the SM-like Higgs $h \to aa$ and changes the total decay width of the SM-like Higgs, by which the predicted value of $\text{BR}(h \to \mu\tau)$ in this model is affected.

The magnitude of $\theta$ is required to be of order $\mathcal{O}(0.01-0.1)$ by the present Higgs signal fit, which is also welcome for estimates of the DM relic abundance and GCE interpretation since this value affects the magnitude of $\langle\sigma v_r\rangle$ (see Eq. 34).

The parameter dependence of the coupling $\rho_{bb}$ (denoted as $\rho_{bb}$ for simplicity) needs careful analysis for the following reasons: the coupling $\rho_{bb}$ is responsible for one of the dominant decay modes of the SM-like Higgs boson $h \to b\bar{b}$, thus any change of $\rho_{bb}$ would affect the theoretical prediction for $\mathrm{BR}(h \to \mu\tau)$. Meanwhile, the DM annihilation $\chi\chi \to b\bar{b}$ is supposed to be the dominant process, and its cross section is proportional to the square of $\rho_{bb}$, as given in Eq. 34. $\rho_{bb}$ is set at the same order as $\rho_{\mu\tau}$, i.e., $\mathcal{O}(0.1)$ as aforementioned in Sec. III B 1. The contributions of the tau lepton and top quark dominate $\mathrm{BR}(\tau \to \mu\gamma)$, as shown in Eq. 31 and Eq. 32. Thus the two parameters $\rho_{\tau\tau}$ and $\rho_{tt}$ (denoted as $\rho_{\tau\tau}$ and $\rho_{tt}$ for simplicity) need to be explored; here we use the parameter range $\rho_{\tau\tau}, \rho_{tt} \sim \rho_{bb}$. Even though we do not adopt the Cheng-Sher ansatz for $\rho^f_{ij}$, we still choose a negative value for $\rho^f_{ij}$ as in the Cheng-Sher ansatz [**?**, **?**], except for $\rho_{\tau\tau}$ and $\rho_{tt}$ while considering the current experimental constraint from $\tau \to \mu\gamma$.

To interpret the GCE, the DM fermion mass and the coupling of the DM fermion to the mediator $(a, A)$ are set as $m_\chi = 30$ GeV and $y_\chi = 0.5$, and the range of $m_a$ is taken between 30 GeV and 95 GeV to account for resonance effects in the DM annihilation process.

To obtain the parameter spaces favored by the physical picture including all aforementioned constraints, we perform a complete numerical analysis for all possible parameter spaces: $\alpha - \rho_{\mu\tau}$, $\alpha - \theta$, $\theta - \rho_{\mu\tau}$, $\rho_{bb} - \rho_{\mu\tau}$, $\rho_{tt} - \rho_{\tau\tau}$, and $m_a - \theta$, with relevant parameters free within the ranges: $30\,\mathrm{GeV} \leq m_a \leq 95\,\mathrm{GeV}$, $0.025 < \theta < 0.1$, $1.475 < \alpha < 1.57$, $-0.115 < \rho_{\mu\tau} < 0$, $-0.3 < \rho_{bb} < 0$, $-0.28 < \rho_{tt} < 0.28$, $-0.05 < \rho_{\tau\tau} < 0.05$ based on the aforementioned arguments. For each specific parameter space listed above, several parameters need to be fixed as shown in the following:

- For parameter spaces of $\alpha - \theta$ ($\alpha - \rho_{\mu\tau}$): $\rho_{\mu\mu} = -0.01$, $\rho_{\tau\tau} = 0.012$, $\rho_{tt} = -0.2$, $\rho_{bb} = -0.2$, and $m_a = 50$ GeV (46 GeV), $\rho_{\mu\tau} = -0.102$ ($\theta = 0.06$);
- For parameter spaces of $\rho_{tt} - \rho_{\tau\tau}$ ($\rho_{bb} - \rho_{\mu\tau}$): $m_a = 50$ GeV, $\alpha = 1.546$, $\rho_{\mu\mu} = -0.01$, $\theta = 0.045$, $\rho_{bb} = -0.2$ and $\rho_{\mu\tau} = -0.102$ ($\rho_{\tau\tau} = 0.012$ and $\rho_{tt} = -0.2$);
- For the $\rho_{\mu\tau} - \theta$ ($m_a - \theta$) parameter spaces: $\alpha = 1.546$, $\rho_{\tau\tau} = 0.015$, $\rho_{tt} = -0.2$, $\rho_{bb} = -0.2$, and $\rho_{\mu\mu} = -0.01$ $(-0.02)$, $m_a = 50$ GeV ($\rho_{\mu\tau} = -0.102$).

Conducting a numerical analysis using the above parameter setup, the relevant processes are depicted in Figure 2 (see the caption for details).

## 1. The $h \to \mu\tau$ Excess

To understand the CMS excess $\mathrm{BR}(h \to \mu\tau) = 0.84^{+0.39}_{-0.37}\%$, the relation between the coupling $\rho_{\mu\tau}$ and $\alpha$ given in Eq. 22 should be satisfied. The region colored magenta in the top-left plot of Fig. 2 [Figure 2: see original paper] is allowed to explain this excess. With a properly fixed $\alpha$, both plots in the middle panel show that a sizeable coupling $\rho_{\mu\tau} \sim 0.1$ is required.

The exotic decay mode $h \to aa$ with a large branching ratio can efficiently change the total Higgs decay width, thus the $a/A$ mixing angle $\theta$ and the mediator mass $m_a$ affect the prediction of $\mathrm{BR}(h \to \mu\tau)$, as plotted in the bottom-right panel. The entire magenta region in the bottom-right panel with $\mathrm{BR}(h \to \mu\tau) = 0.61$ implies that when $\theta$ is relatively small or in the case $2m_\chi < m_a$, the decay process $h \to aa$ cannot occur, so it would not affect the Higgs total width and becomes irrelevant to the $h \to \mu\tau$ process. Eq. 21 clearly interprets this situation.

It is worth noting that when this work was close to being finalized, the ATLAS collaboration published a new analysis on $h \to \mu\tau_h$, obtaining a slightly smaller excess compared to the CMS result, while its upper bound is consistent with the CMS result [**?**].

## 2. The Muon $g-2$ Anomaly

The contribution of two-loop Barr-Zee diagrams to $\delta a_\mu$ is negligible because of the smallness of $\cos\alpha$. The dominant contributions to $\delta a_\mu$ include the one-loop diagrams where CP-even Higgs $H$ and CP-odd Higgs $A$ are mediators; these two diagrams respectively provide negative and positive contributions.

The contribution of the one-loop diagram where CP-odd $a$ is the mediator to $\delta a_\mu$ is positive, becoming larger for smaller $m_a$ and/or larger $\theta$, therefore canceling the contribution of the diagram where $H$ is involved. This situation is depicted in the top- and bottom-right panels of Fig. 2 (by the red dashed curves).

To obtain a result consistent with the present experimental value ($\delta a_\mu/10^{-9} = 2.61 \pm 0.8$) [**?**], a sizeable $\rho_{\mu\tau}$ is required, which is also consistent with the CMS $h \to \mu\tau$ excess, and its dependence on $\rho_{\mu\tau}$ is demonstrated in the plots (top-left and middle panels). Within the allowed parameter space in $\rho_{\tau\tau} - \rho_{tt}$, a value of muon $g-2$: $\delta a_\mu = 2.80 \times 10^{-9}$ is reached.

## 3. Constraints from $\tau \to \mu\gamma$

The dominant contributions come from the one-loop diagrams ($A$-loop and $H$-loop diagrams) and the Barr-Zee diagram with a top quark involved, as shown in Eq. (30,31,32). The existence of opposite signs between the contributions of the CP-even Higgs (negative) and the CP-odd Higgs $A$ (positive) one-loop diagrams [**?**] leads to a cancellation effect; however, this does not occur as long as $m_A > m_H$. With a large coupling $\rho_{tt}$ and under the limit $\sin\alpha \sim 1$, the Barr-Zee diagram involving the top quark provides the dominant (positive) contribution to $\tau \to \mu\gamma$.

Additionally, as in the case of $\delta a_\mu$, when $m_a$ is smaller and $\theta$ is larger, the importance of the one-loop diagram where $a$ is involved in $\mathrm{BR}(\tau \to \mu\gamma)$ enhances, as shown in the bottom-right panel of Fig. 2. As well, a smaller $\alpha$ would also induce an enhancement of the contribution of the $h$-mediated Barr-Zee diagram with a $W$ loop. Thus, due to the contributions of this Barr-Zee diagram and the $a$ one-loop diagram, for a region with larger $\theta$ and smaller $\alpha$, our calculation would predict an even larger $\mathrm{BR}(\tau \to \mu\gamma)$ (see the top-right panel).

The theoretical prediction for $\mathrm{BR}(\tau \to \mu\gamma)$ with respect to $\rho_{\mu\tau}$ (plotted in the top-left and middle panels) shows behavior similar to that for $\delta a_\mu$.

It is noteworthy that since the form factor $A$ is related to the tau lepton and top quark through the relevant loops (see Eq. (31,32)), the current experimental bound $\mathrm{BR}(\tau \to \mu\gamma) < 4.4 \times 10^{-8}$ [**?**, **?**] constrains $\rho_{\tau\tau}$ and $\rho_{tt}$ strictly. The red region in the bottom-left panel of Fig. 2 is allowed. It is interesting to note that $\rho_{tt}$ and $\rho_{\tau\tau}$ should have opposite signs as favored by the data. This result agrees with that given by the authors of Ref.~[**?**]. One can see that the upper bound demands $\rho_{\tau\tau}$ to be small, about $|\rho_{\tau\tau}| < 0.04$.

## 4. DM Relic Density and GCE

Assuming that the observed GCE is caused by dark matter annihilation, the lower bound of the annihilation cross section $\langle \sigma v_r \rangle$ should be about $0.5 \times 10^{-26}\,\mathrm{cm}^3/\mathrm{s}$ as discussed in Ref.~[**?**]. The upper bound of $\langle \sigma v_r \rangle$ is determined to be $4.0 \times 10^{-26}\,\mathrm{cm}^3/\mathrm{s}$ at 95% CL [**?**]. Also, the Pass 8 data of Fermi-LAT [**?**] from dwarf spheroidal satellite galaxies set a new upper bound ($\sim 1.0 \times 10^{-26}\,\mathrm{cm}^3/\mathrm{s}$) on the dark matter annihilation cross section at a DM mass of 30 GeV. The areas in Fig. 2 that give an annihilation cross section of $\langle \sigma v_r \rangle = 0.5 - 4.0(1.0) \times 10^{-26}\,\mathrm{cm}^3/\mathrm{s}$ are depicted as the cyan (orange) regions. For the top-left (bottom-left) parameter space, the calculated values are $\langle \sigma v_r \rangle = 1.8(2.5) \times 10^{-26}\,\mathrm{cm}^3/\mathrm{s}$.

The blue curves and contours in Fig. 2 represent the correct dark matter relic density ($\Omega h^2 = 0.1197 \pm 0.0022$) [**?**]. We note that the range of $\langle \sigma v_r \rangle$ favored by the DM relic density and GCE, as expected, highly depends on $m_a$, $\theta$, and $\rho_{bb}$ (see Sec. III B 3). Due to the enhancement of the annihilation cross section when the mediator mass $m_a$ is close to $\sim 2m_\chi$, the dark matter relic density rules out a range in the parameter space (see the bottom-right panel). With a sizeable $\rho_{bb} \sim \mathcal{O}(0.1)$, the DM relic density and GCE could be accommodated simultaneously in a range of $\langle \sigma v_r \rangle = (0.5-4) \times 10^{-26}\,\mathrm{cm}^3/\mathrm{s}$. However, the results newly reported by the Fermi-LAT and DES collaborations constrain $\langle \sigma v_r \rangle$ to be smaller than $1.0 \times 10^{-26}\,\mathrm{cm}^3/\mathrm{s}$ for 30 GeV dark matter fermions, which is slightly smaller than the value required by the thermal relic abundance; thus, there should exist other additional DM annihilation channels to make up the correct DM relic density [**?**, **?**].

## 5. The CMS Constraints

Recently, the CMS collaboration combined comprehensive sets of production and decay measurements of the 125 GeV Higgs boson, including decay channels into $\gamma\gamma$, $ZZ^*$, $WW^*$, $\tau^+\tau^-$, $b\bar{b}$, and $\mu^+\mu^-$, and found no significant deviation from Standard Model predictions [?]. This synthesis should severely constrain the parameter spaces of all built models.

More specifically, the signal strengths for $\tau^+\tau^-$, $b\bar{b}$, $WW^*$, $ZZ^*$, and $\gamma\gamma$ channels are defined as

$$\mu_i = \frac{\sigma(h) \times \mathrm{BR}_i}{\sigma_{\mathrm{SM}}(h) \times \mathrm{BR}_i^{\mathrm{SM}}},$$

which can be obtained by fitting the CMS data within $1\sigma$ tolerance [?]. In the text, $\sigma(h)$ and $\sigma_{\mathrm{SM}}(h)$ ($\mathrm{BR}_i$ and $\mathrm{BR}_i^{\mathrm{SM}}$) correspond to the Higgs production cross section (decay branching fractions of the five decay modes) predicted by this model and the SM, respectively, as $\mathrm{BR}_i = \Gamma(h \to ii)/(\Gamma_{\mathrm{BSM}}(h) + \Gamma_{\mathrm{SM}}(h))$.

We find that the upper bound on $\mu_W$ and the lower bound on $\mu_\gamma$ set more rigorous constraints on the parameters of this model, and the grey areas in Fig. 2 are excluded. With the allowed parameter spaces, the ranges of the signal strengths $\mu_i$ are given in Table I .

The CMS Higgs decay signal strength strongly constrains the $h/H$, $a/A$ mixing angles $\alpha$ and $\theta$, concretely demanding both $\sin\alpha$ and $\cos\theta$ to be close to unity (see the middle-left and top panels). The constraint on $\alpha$ is found to be compatible with the parameter space obtained by accounting for the $h \to \mu\tau$ excess. $\rho_{bb}$ is also severely bounded since it could affect the primary decay mode of the SM-like Higgs significantly. As shown in the middle-right panel of Fig. 2, which is a contour diagram of $\rho_{bb} - \rho_{\mu\tau}$, only a small band with $\rho_{bb}$ between $-0.15$ and $-0.22$ is not ruled out. Meanwhile, from the contour diagram of $m_a - \theta$, one can see that as long as $\mathrm{BR}(h \to aa)$ is larger than 0.12, the grey regions in the figure are excluded by the CMS Higgs signal strengths.

For the not-yet-detected channel $h \to aa$, the pseudoscalar primarily decays into $b\bar{b}$, so there should be an additional contribution from the process $h \to aa \to 4b$ in the $h \to b\bar{b}$ searches [?], and the CMS experiment data [?, ?] would definitely constrain the coupling between $hb\bar{b}$ as long as $m_a < m_h/2$. The upper bound on the undetected decay of the Higgs will be further improved as the $b\bar{b}$ pair production is measured at 13 (14) TeV. Moreover, the $h \to 2b2\nu$ searches at the LHC [?] could give more rigorous constraints on the parameters $m_a$ and $\theta$.

## 6. Collider Observation of the Pseudoscalar

For the benchmark scenarios of this work, when $m_a > 2m_\chi$, the mass relation $m_A > m_h + m_a$ opens the channel $pp(gg) \to A \to h(h \to \gamma\gamma)a(a \to \chi\bar{\chi})$ for observation of the pseudoscalar $a$ at the LHC, which corresponds to the mono-Higgs searches in [?]. The backgrounds are dominated by the SM process $pp \to Z\gamma\gamma$ with $Z \to \nu\nu$, and the Higgs associated production process $pp \to Zh$ with $Z \to \nu\nu$. The collider analysis is performed by generating signal and background events with MadGraph5_{aMC}@NLO [?, ?] at 14 TeV, then passing them to Pythia 8.1 [?] for parton shower and hadronization, and finally conducting detector simulation with Delphes 3 [?]. After implementing the selection cuts $m_{\gamma\gamma} \in [120, 130]$ GeV, as well as $E_T^{\text{miss}}$, $P_T^{\gamma\gamma} > 76$ GeV following Ref.~[?], the number of signal and background events obtained are 37 and 48, respectively, giving a significance for observing $a$ of $S/\sqrt{S+B} \sim 4$ for the benchmark $m_a - \theta$ with $\theta = 0.08$, $m_a = 76$ GeV and an integrated luminosity $\mathcal{L} = 300\,\text{fb}^{-1}$. The pseudoscalar could also be probed with hard $b$-jets and large missing transverse energy [?] when $m_a > 2m_\chi$ and the pseudoscalar dominantly decays to dark matter. When the pseudoscalar decays mostly to $b$-quarks, collider searches for the pseudoscalar could follow Ref.~[?].

## Conclusions and Discussions

The anomalies discovered by LHC experiments and dark matter observations greatly excite human curiosity and inspire enthusiasm for searching for new physics beyond the Standard Model. However, so far the trend has not been very successful. Even though we know new physics must exist, we do not know its scale. Direct and indirect searches for dark matter, LHC experiments, long-baseline and short-baseline neutrino experiments, and numerous lower-energy experiments including BELLE, BES, and many others provide hints toward new physics beyond the Standard Model, while simultaneously setting increasingly rigorous constraints on models proposed to explain anomalies observed in astronomy and high-energy experiments. Some models survive the current measurements, while many have been ruled out.

In this work, we have extended the Type-III 2HDM by introducing a pseudoscalar $a$ that can mediate DM pair annihilation. In a recent paper, Han et al.~[?] also extended the 2HDM with an aligned Yukawa sector to explain the $(g-2)_\mu$ excess. In comparison, our scheme is somewhat different from theirs. We not only consider the $(g-2)_\mu$ excess but also many other constraints from both terrestrial experiments and cosmology.

In this framework, the LFV process $h \to \mu\tau$ observed at the LHC is addressed, and the possibility of explaining the muon $g-2$ anomaly, dark matter relic abundance, and GCE has been investigated. The role played by the newly introduced $a$ is studied in detail. It is found that there indeed exist certain parameter spaces that can accommodate the $h \to \mu\tau$ excess, muon $g-2$ discrepancy, and dark matter relic abundance.

With the flavor-violating coupling $\rho_{\mu\tau} \sim \mathcal{O}(0.1)$ and a tiny mixing between $h$ and $H$ around $\sin\alpha \sim 1$, the observed $h \to \mu\tau$ excess can be easily accommodated. The pseudoscalar $a$ opens an important undetected decay channel for the SM-like Higgs ($h \to aa$) as $m_a < m_h/2$, thus affecting the branching ratio of $h \to \tau\mu$. It also plays a role in explaining the discrepancy between theoretical prediction and data for $(g-2)_\mu$. A smaller $m_a$ and a slightly larger CP-odd $a/A$ mixing $\theta$ help interpret the muon $g-2$ anomaly. Increasing the CP-odd Higgs mixing angle and $|\rho_{\mu\tau}|$ increases BR($\tau \to \mu\gamma$), thus further constraining the parameter space of our model. The measurement of the branching ratio of $\tau \to \mu\gamma$ sets a stringent bound on the flavor-conserving Yukawa couplings $\rho_{tt}$ and $\rho_{\tau\tau}$, and moreover determines an opposite sign between these two couplings. There are parameter regions in our model allowed by current experimental data on $\tau \to \mu\gamma$ where both the measured $h \to \mu\tau$ excess and the muon $g-2$ anomaly can be explained.

To account for the dark matter relic density, the $a/A$ mixing angle should be of order $\mathcal{O}(0.01)$ and $\rho_{bb} \sim \mathcal{O}(0.1)$ is required; however, this parameter region does not coincide with that favored by the GCE observation if it is postulated that the GCE fully originates from $\chi\bar{\chi}$ DM pair annihilation. This inconsistency may imply that there are other sources contributing to the GCE besides the pure $\chi\bar{\chi}$ DM annihilation mechanism.

The LFV process $h \to \mu\tau$ is studied in terms of our model, and our prediction for BR($h \to \mu\tau$) is qualitatively consistent with that observed by CMS and ATLAS at 8 TeV. However, to reach a decisive conclusion, more data are needed, and the LHC Run II at 14 TeV should help. A synthesis of data accumulated by the high-energy collider LHC, the future SPPC at 100 TeV, perhaps some lower-energy experiments, and new astronomical observations would make the whole picture clearer, enabling us to judge whether this model indeed works or needs further modification. For the case $m_a > 2m_\chi$, the mono-Higgs search provides one possible probe of the pseudoscalar.

## Appendix: Loop Functions

The functions $f_\phi$ and $h$ used for the calculation of $g-2$ and $\tau \to \mu\gamma$ are given by

$$f_{A,a}(r) = \int_0^1 dx \frac{x(1-x)}{x(1-x)-r}.$$

The functions $f(x,y,r)$ and Inami-Lim function $Y(x)$ used in Eq. (17) are

$$f(x,y,r) = \frac{r}{(x-1)(r-x)} + \frac{x\ln x}{(x-1)(x-y)} + \frac{y\ln y}{(y-1)(y-x)} + \frac{r\ln r}{(r-x)(r-y)},$$

$$Y(x) = \frac{x}{8}\left[\frac{x-4}{x-1} + \frac{3x\ln x}{(x-1)^2}\right].$$

## Acknowledgments

We are grateful to Peng-Fei Yin and Zuowei Liu for helpful discussions on the Galactic Center gamma-ray excess. This work is supported by the National Natural Science Foundation of China under Grants No. 11375128 and No. 11547305, and by Fundamental Research Funds for the Central Universities under Grant No. 0903005203404.

## References

[1] A. Vicente, arXiv:1411.2372 [hep-ph].
[2] V. Khachatryan et al. [CMS Collaboration], arXiv:1502.07400 [hep-ex].
[3] A. Crivellin, A. Kokulu and C. Greub, Phys. Rev. D 87, 094031 (2013) doi:10.1103/PhysRevD.87.094031 [arXiv:1303.5877 [hep-ph]].
[4] A. Crivellin, G. D'Ambrosio and J. Heeck, Phys. Rev. Lett. 114, 151801 (2015) [arXiv:1501.00993 [hep-ph]].
[5] A. Crivellin, J. Heeck and P. Stoffer, arXiv:1507.07567 [hep-ph].
[6] A. Crivellin, G. D'Ambrosio and J. Heeck, Phys. Rev. D 91, no. 7, 075006 (2015) [arXiv:1503.03477 [hep-ph]].
[7] I. Dorner, S. Fajfer, A. Greljo, J. F. Kamenik, N. Konik and I. Niandic, JHEP 1506, 108 (2015) [arXiv:1502.07784 [hep-ph]].
[8] A. Hektor, K. Kannike and L. Marzola, JCAP 1510, 025 (2015) doi:10.1088/1475-7516/2015/10/025 [arXiv:1507.05096 [hep-ph]].
[9] D. Aristizabal Sierra and A. Vicente, Phys. Rev. D 90, no. 11, 115004 (2014) [arXiv:1409.7690 [hep-ph]].
[10] W. Altmannshofer, S. Gori, A. L. Kagan, L. Silvestrini and J. Zupan, arXiv:1507.07927 [hep-ph].
[11] Y. Omura, E. Senaha and K. Tobe, JHEP 1505, 028 (2015) [arXiv:1502.07824 [hep-ph]].
[12] S. Davidson and G. J. Grenier, Phys. Rev. D 81, 095016 (2010) [arXiv:1001.0434 [hep-ph]].
[13] V. Vitale et al. [Fermi-LAT Collaboration], arXiv:0912.3828 [astro-ph.HE].
[14] L. Goodenough and D. Hooper, arXiv:0910.2998 [hep-ph].
[15] D. Hooper and L. Goodenough, "Dark Matter Annihilation in The Galactic Center As Seen by the Fermi Gamma Ray Space Telescope," Phys. Lett. B697 (2011) 412, arXiv:1010.2752 [hep-ph].
[16] D. Hooper and T. Linden, "On The Origin Of The Gamma Rays From The Galactic Center," Phys. Rev. D84 (2011) 123005, arXiv:1110.0006 [astro-ph.HE].
[17] K. N. Abazajian and M. Kaplinghat, "Detection of a Gamma-Ray Source in the Galactic Center Consistent with Extended Emission from Dark Matter Annihilation and Concentrated Astrophysical Emission," Phys. Rev. D86 (2012) 083511, arXiv:1207.6047 [astro-ph.HE].
[18] T. Daylan, D. P. Finkbeiner, D. Hooper, T. Linden, S. K. N. Portillo, et al., "The Characterization of the Gamma-Ray Signal from the Central Milky Way: A Compelling Case for Annihilating Dark Matter," arXiv:1402.6703

[astro-ph.HE].

[19] B. Zhou, Y.-F. Liang, X. Huang, X. Li, Y.-Z. Fan, et al., “GeV excess in the Milky Way: The Role of Diffuse Galactic gamma ray Emission template,” arXiv:1406.6948 [astro-ph.HE].

[20] F. Calore, I. Cholis, and C. Weniger, “Background model systematics for the Fermi GeV excess,” JCAP 1503 (2015) 038, arXiv:1409.0042 [astro-ph.CO].

[21] S. Murgia, “Observation of the high energy gamma-ray emission towards the Galactic center,” (2014), Talk given on Fifth Fermi Symposium, Nagoya, 20-24 October 2014.

[22] F. Calore, I. Cholis, C. McCabe and C. Weniger, Phys. Rev. D 91, 063003 (2015) [arXiv:1411.4647 [hep-ph]].

[23] P. Agrawal, B. Batell, P. J. Fox and R. Harnik, JCAP 1505, 011 (2015) [arXiv:1411.2592 [hep-ph]].

[24] J. Fan, S. M. Koushiappas and G. Landsberg, arXiv:1507.06993 [hep-ph].

[25] K. N. Abazajian, JCAP 1103, 010 (2011) [arXiv:1011.4275 [astro-ph.HE]].

[26] Q. Yuan and B. Zhang, JHEAp 3-4, 1 (2014) [arXiv:1404.2318 [astro-ph.HE]].

[27] D. Hooper, I. Cholis, T. Linden, J. Siegal-Gaskins and T. Slatyer, Phys. Rev. D 88, 083009 (2013) [arXiv:1305.0830 [astro-ph.HE]].

[28] I. Cholis, D. Hooper and T. Linden, arXiv:1407.5583 [astro-ph.HE].

[29] R. Bartels, S. Krishnamurthy and C. Weniger, arXiv:1506.05104 [astro-ph.HE].

[30] S. K. Lee, M. Lisanti, B. R. Safdi, T. R. Slatyer and W. Xue, arXiv:1506.05124 [astro-ph.HE].

[31] A. Geringer-Sameth, M. G. Walker, S. M. Koushiappas, S. E. Koposov, V. Belokurov, et al., “Evidence for Gamma-ray Emission from the Newly Discovered Dwarf Galaxy Reticulum 2,” arXiv:1503.02320 [astro-ph.HE].

[32] D. Hooper and T. Linden, “On The Gamma-Ray Emission From Reticulum II and Other Dwarf Galaxies,” arXiv:1503.06209 [astro-ph.HE].

[33] The Fermi-LAT Collaboration, The DES Collaboration, A. Drlica-Wagner, A. Albert, K. Bechtol, M. Wood, L. Strigari, M. Sanchez-Conde, L. Baldini, R. Essig, and et al., “Search for Gamma-Ray Emission from DES Dwarf Spheroidal Galaxy Candidates with Fermi-LAT Data,” arXiv:1503.02632 [astro-ph.HE].

[34] C. Boehm, M. J. Dolan, C. McCabe, M. Spannowsky and C. J. Wallace, JCAP 1405, 009 (2014) doi:10.1088/1475-7516/2014/05/009 [arXiv:1401.6458 [hep-ph]].

[35] J. Kozaczuk and T. A. W. Martin, JHEP 1504, 046 (2015) doi:10.1007/JHEP04(2015)046 [arXiv:1501.07275 [hep-ph]].

[36] O. Buchmueller, S. A. Malik, C. McCabe and B. Penning, Phys. Rev. Lett. 115, no. 18, 181802 (2015) doi:10.1103/PhysRevLett.115.181802 [arXiv:1505.07826 [hep-ph]].

[37] A. Berlin, S. Gori, T. Lin and L. T. Wang, Phys. Rev. D 92, no. 1, 015005 (2015) [arXiv:1502.06000 [hep-ph]].

[38] J. Cao, L. Shang, P. Wu, J. M. Yang and Y. Zhang, arXiv:1506.06471 [hep-ph].

[39] J. Cao, L. Shang, P. Wu, J. M. Yang and Y. Zhang, Phys. Rev. D 91, no.

5, 055005 (2015) [arXiv:1410.3239 [hep-ph]].
[40] J. Guo, J. Li, T. Li and A. G. Williams, Phys. Rev. D 91, no. 9, 095003 (2015) [arXiv:1409.7864 [hep-ph]].
[41] K. Ghorbani, JHEP 1501, 045 (2015) [arXiv:1408.4929 [hep-ph]].
[42] J. Huang, T. Liu, L. T. Wang and F. Yu, Phys. Rev. D 90, no. 11, 115006 (2014) [arXiv:1407.0038 [hep-ph]].
[43] A. Berlin, D. Hooper and S. D. McDermott, Phys. Rev. D 89, no. 11, 115022 (2014) [arXiv:1404.0022 [hep-ph]].
[44] J. M. Cline, G. Dupuis, Z. Liu and W. Xue, Phys. Rev. D 91, no. 11, 115010 (2015) [arXiv:1503.08213 [hep-ph]].
[45] P. Agrawal, B. Batell, D. Hooper and T. Lin, Phys. Rev. D 90, no. 6, 063512 (2014) [arXiv:1404.1373 [hep-ph]].
[46] A. Berlin, P. Gratia, D. Hooper and S. D. McDermott, Phys. Rev. D 90, no. 1, 015032 (2014) [arXiv:1405.5204 [hep-ph]].
[47] C. Arina, E. Del Nobile and P. Panci, Phys. Rev. Lett. 114, 011301 (2015) [arXiv:1406.5542 [hep-ph]].
[48] S. Ipek, D. McKeen and A. E. Nelson, Phys. Rev. D 90, no. 5, 055021 (2014) [arXiv:1404.3716 [hep-ph]].
[49] J. F. Gunion and H. E. Haber, Phys. Rev. D 67, 075019 (2003) doi:10.1103/PhysRevD.67.075019 [hep-ph/0207010].
[50] T. P. Cheng and M. Sher, Phys. Rev. D 35, 3484 (1987).
[51] W. Skiba and J. Kalinowski, Nucl. Phys. B 404, 3 (1993).
[52] S. Chatrchyan et al. [CMS Collaboration], Phys. Rev. Lett. 111, 101804 (2013) [arXiv:1307.5025 [hep-ex]].
[53] R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. 111, 101805 (2013) [arXiv:1307.5024 [hep-ex]].
[54] CMS and LHCb Collaborations, CMS-PAS-BPH-13-007, LHCb-CONF-2013-012, CERN-LHCb-CONF-2013-012.
[55] C. Bobeth, M. Gorbahn, T. Hermann, M. Misiak, E. Stamou and M. Steinhauser, Phys. Rev. Lett. 112, 101801 (2014) [arXiv:1311.0903 [hep-ph]].
[56] A. J. Buras, R. Fleischer, J. Girrbach and R. Knegjens, JHEP 1307, 77 (2013) [arXiv:1303.3820 [hep-ph]].
[57] H. E. Logan and U. Nierste, Nucl. Phys. B 586, 39 (2000) [hep-ph/0004139].
[58] W. Grimus, L. Lavoura, O. M. Ogreid and P. Osland, J. Phys. G 35, 075001 (2008) [arXiv:0711.4022 [hep-ph]].
[59] M. Baak et al. [Gfitter Group Collaboration], Eur. Phys. J. C 74, 3046 (2014) [arXiv:1407.3792 [hep-ph]].
[60] M. Raidal et al., Eur. Phys. J. C 57, 13 (2008) [arXiv:0801.1826 [hep-ph]].
[61] S. M. Barr and A. Zee, Phys. Rev. Lett. 65, 21 (1990) [Phys. Rev. Lett. 65, 2920 (1990)].
[62] D. Chang, W. S. Hou and W. Y. Keung, Phys. Rev. D 48, 217 (1993) [hep-ph/9302267].
[63] A. Alloul, N. D. Christensen, C. Degrande, C. Duhr and B. Fuks, Comput. Phys. Commun. 185, 2250 (2014) [arXiv:1310.1921 [hep-ph]].
[64] A. Belyaev, N. D. Christensen and A. Pukhov, Comput. Phys. Commun. 184, 1729 (2013) [arXiv:1207.6082 [hep-ph]].

[65] G. Belanger, F. Boudjema, A. Pukhov and A. Semenov, Comput. Phys. Commun. 192, 322 (2015) [arXiv:1407.6129 [hep-ph]].
[66] F. Mahmoudi and O. Stal, Phys. Rev. D 81, 035016 (2010) [arXiv:0907.1791 [hep-ph]].
[67] P. M. Ferreira and D. R. T. Jones, JHEP 0908, 069 (2009) [arXiv:0903.2856 [hep-ph]].
[68] V. Khachatryan et al. [CMS Collaboration], Eur. Phys. J. C 75, no. 5, 212 (2015) [arXiv:1412.8662 [hep-ex]].
[69] G. Aad et al. [ATLAS Collaboration], arXiv:1508.03372 [hep-ex].
[70] K. Hagiwara, R. Liao, A. D. Martin, D. Nomura and T. Teubner, J. Phys. G 38, 085003 (2011) [arXiv:1105.3149 [hep-ph]].
[71] B. Aubert et al. [BaBar Collaboration], Phys. Rev. Lett. 104, 021802 (2010) [arXiv:0908.2381 [hep-ex]].
[72] K. Hayasaka et al. [Belle Collaboration], Phys. Lett. B 666, 16 (2008) [arXiv:0705.0650 [hep-ex]].
[73] M. Ackermann et al. [Fermi-LAT Collaboration], arXiv:1503.02641 [astro-ph.HE].
[74] P. A. R. Ade et al. [Planck Collaboration], arXiv:1502.01589 [astro-ph.CO].
[75] C. Cheung, M. Papucci, D. Sanford, N. R. Shah and K. M. Zurek, Phys. Rev. D 90, 075011 (2014) [arXiv:1406.6372 [hep-ph]].
[76] X. J. Bi, L. Bian, W. Huang, J. Shu and P. F. Yin, Phys. Rev. D 92, no. 2, 023507 (2015) [arXiv:1503.03749 [hep-ph]].
[77] CMS Collaboration, CMS-PAS-HIG-14-009.
[78] D. Curtin et al., Phys. Rev. D 90, no. 7, 075004 (2014) [arXiv:1312.4992 [hep-ph]].
[79] S. Chatrchyan et al. [CMS Collaboration], Phys. Rev. D 89, no. 1, 012003 (2014) [arXiv:1310.3687 [hep-ex]].
[80] J. M. No, Phys. Rev. D 93, no. 3, 031701 (2016) [arXiv:1509.01110 [hep-ph]].
[81] J. Alwall et al., JHEP 1407, 079 (2014) [arXiv:1405.0301 [hep-ph]].
[82] J. Alwall, M. Herquet, F. Maltoni, O. Mattelaer and T. Stelzer, JHEP 1106, 128 (2011) [arXiv:1106.0522 [hep-ph]].
[83] J. Lilley, CERN-THESIS-2011-013.
[84] T. Sjostrand, S. Mrenna and P. Z. Skands, Comput. Phys. Commun. 178, 852 (2008) [arXiv:0710.3820 [hep-ph]].
[85] J. de Favereau et al. [DELPHES 3 Collaboration], JHEP 1402, 057 (2014) [arXiv:1307.6346 [hep-ex]].
[86] T. Lin, E. W. Kolb and L. T. Wang, Phys. Rev. D 88, no. 6, 063510 (2013) [arXiv:1303.6638 [hep-ph]].
[87] T. Han, S. K. Kang and J. Sayre, arXiv:1511.05162 [hep-ph].