High-Resolution DOA Estimation Method Based on Covariance Matrix Reconstruction Under Array Element Failures

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Abstract

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Full Text

High-Resolution DOA Estimation Method Based on Covariance Matrix Reconstruction in Presence of Element Failure

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Abstract

Traditional subspace-based methods rely on the array correlation matrix, which becomes rank-deficient when array elements fail, causing these methods to fail.

To address this problem, this paper proposes a method that starts from the covariance fitting criterion, uses the Toeplitz structure of the covariance matrix as a constraint, and recovers the covariance matrix based on the principle of low-rank matrix reconstruction. Subsequently, subspace-based methods are employed for target direction-of-arrival (DOA) estimation. Under element failure conditions, this method can effectively reconstruct the array covariance matrix, recover the degrees of freedom of failed elements, and solve the problem of high-precision target bearing estimation with failed elements. Numerical simulations demonstrate that under element failure conditions, the proposed method can restore the performance of a damaged array to that of a fully functional array, particularly showing superior performance in multi-target scenarios.

Keywords: array element failure; degree of freedom recovery; covariance matrix reconstruction; low-rank matrix reconstruction
Classification: TN911.7

1 Introduction

Array signal processing is a key technology in radar, sonar, deep space communications, and other fields. Most literature assumes perfect array elements for target detection, recognition, and localization. However, in practical applications, element failure is often inevitable due to harsh natural environments, human interference, element aging, and physical damage. Underwater conditions are particularly complex and severe, with high pressure and corrosive environments increasing the probability of array element damage. Element failure destroys the geometric symmetry and uniformity of uniform linear arrays, causing distortion in array output signals, sharp changes in array response, and beam pattern distortion, potentially leading to complete array failure in severe cases. Element failure degrades detection performance and increases false alarm rates. For target bearing estimation, element failure alters the sample covariance matrix, rendering traditional subspace-based DOA estimation methods ineffective. For applications with complex maintenance, high real-time requirements, or expensive costs, restoring array performance under element failure is a critical research problem.

Researchers have extensively studied the impact of element failure on array performance, sidelobe level elevation, and effective recovery methods. Regarding performance analysis, Zhu Dezhi et al. analyzed the effects of failed elements at different positions in uniform linear arrays on main lobe amplitude, uniform weighting beam patterns, tapered beam patterns, and monopulse angle measurement [1][2]. Li Zheng et al. analyzed the impact of element failure on circular array patterns, concluding that the effect is relatively uniform regardless of failure position [3]. For rectangular planar arrays, Gao Fei et al. discussed the effects of different element failure scenarios on array patterns, clutter characteristics, and STAP performance, analyzing impacts from array aperture, clutter degrees of freedom, system degrees of freedom, and coherent integration values [4]. These studies examined performance impacts in uniform linear, circular, and rectangular arrays but did not provide signal processing methods for element failure scenarios.

To address sidelobe elevation caused by element failure, Cui Lin et al. proposed using time delay estimation and linear prediction to predict failed element outputs, suppressing sidelobe levels and improving array directivity [5]. Xu Zhaoyang et al. proposed forcing the weights of failed elements to zero and re-optimizing the weights of remaining functional elements according to the desired beam pattern to suppress sidelobe levels [6]. For effective recovery of failed elements, Yang Yang et al., using grid arrays as examples, analyzed the impact of element failure on sound field reconstruction and proposed compensation methods that either ignore failed elements and treat the damaged array as a new grid or irregular array, or reorganize data for reconstruction using the new array geometry [7]. Zhang Yuanbiao et al. proposed using linear prediction to reconstruct echo signals from failed elements before synthetic aperture imaging to solve image distortion problems [8]. Xu Zhaoyang et al. utilized the characteristic that adjacent elements receive signals from the same source with only a fixed phase shift, synthesizing outputs from other functional elements to recover failed element outputs and suppress sidelobe growth [9].

Yerriswamy et al. [10] proposed recovering spatiotemporally missing data through low-rank matrix completion under the assumption of random spatiotemporal distribution of element data loss, then directly applying matrix pencil methods for DOA estimation in the data domain. However, in practice, the random distribution assumption often cannot be satisfied simultaneously, as element damage may cause complete data loss at certain spatial positions, rendering the method ineffective. Yang Dong et al. [11] considered scenarios where an entire array element is completely damaged or unsampled in spatial sampling, proposing to rearrange data from a single time snapshot using the equal-ratio characteristic of uniform linear array data to satisfy random distribution conditions, thereby equivalently transforming the problem into a low-rank matrix completion problem. Although this method addresses non-random spatiotemporal data distribution due to element damage or downsampling, it has two significant drawbacks. First, the rearranged matrix completion involves dimensionality reduction while maintaining the low-rank condition, which greatly reduces array degrees of freedom. Second, performing data filling for each snapshot individually requires substantial computational resources, making it impractical for real applications. Effective signal recovery and high-resolution bearing estimation and imaging under element failure have become important research topics in array signal processing [12].

This paper addresses element failure in uniform linear arrays. Assuming random element damage distribution and that no more than half of the elements fail, we propose a method starting from the covariance fitting criterion, using the Toeplitz structure of the covariance matrix as a constraint [13], and exploiting the low-rank property to recover the covariance matrix, followed by subspace-based methods for target DOA estimation. Under element failure conditions, this method can effectively recover the covariance matrix, restore degrees of freedom of failed elements, and solve the high-precision DOA estimation problem.

2.1 Undamaged Uniform Linear Array Model

Consider a uniform linear array (ULA) with M elements. Assume K uncorrelated narrowband target signals $s_k(t)$, $k = 1, 2, \dots, K$, illuminate the array from far-field directions $\varphi_1, \varphi_2, \dots, \varphi_K$. The array received signal vector can be expressed as:

$$\mathbf{x}(t) = \mathbf{A}(\varphi)\mathbf{s}(t) + \mathbf{n}(t)$$

where $\mathbf{A}(\varphi) = [\mathbf{a}(\varphi_1), \mathbf{a}(\varphi_2), \dots, \mathbf{a}(\varphi_K)]$ is the array manifold, $\mathbf{a}(\varphi_k) = [1, e^{-j\pi\sin(\varphi_k)}, \dots, e^{-j(M-1)\pi\sin(\varphi_k)}]^T$ is the steering vector for direction φ_k (with normalization $d = \lambda/2$ where λ is the wavelength), $\mathbf{s}(t) = [s_1(t), s_2(t), \dots, s_K(t)]^T$ is the source signal vector, and $\mathbf{n}(t)$ is complex Gaussian noise with mean zero and variance σ_n^2, independent of the signals. The source signals $s_k(t)$ are independent Gaussian random variables with variance σ_k^2. The covariance matrix of $\mathbf{x}(t)$ is:

$$\mathbf{R} = E\{\mathbf{x}(t)\mathbf{x}^H(t)\} = \mathbf{A}(\varphi)\mathbf{R}_s\mathbf{A}^H(\varphi) + \sigma_n^2\mathbf{I}$$

where $\mathbf{R}_s = \text{diag}(\sigma_1^2, \sigma_2^2, \dots, \sigma_K^2)$ is the source covariance matrix and $\mathbf{I}$ is the identity matrix. In practice, the covariance matrix is unknown and can only be estimated precisely with infinite snapshots. Typically, it is obtained through the sample covariance matrix:

$$\hat{\mathbf{R}} = \frac{1}{N}\sum_{t=1}^{N}\mathbf{x}(t)\mathbf{x}^H(t)$$

For undamaged arrays, subspace methods such as MUSIC and ESPRIT can be used for DOA estimation by searching for peaks in the pseudo-spectrum. However, element damage causes rank deficiency and phase ambiguity, rendering traditional subspace methods ineffective. This paper proposes to recover the covariance matrix using the covariance fitting criterion with Toeplitz structure constraint and low-rank characteristics, then apply subspace methods for target DOA estimation.

2.2 Damaged Uniform Linear Array Model

We make the following assumptions: 1. The spatial positions of failed elements are known. 2. Element failure means complete malfunction with no receivable amplitude or phase information. If the i-th element fails, the corresponding entry in the steering vector $\mathbf{a}(\varphi)$ is replaced by zero. 3. The spatial positions of damaged elements are randomly distributed, and the number of failed elements does not exceed half of the total.

A damaged ULA can be viewed as spatial subsampling of an undamaged ULA, where the damaged array output is a subset of the original uniform array output. Let Ψ denote the set of positions of failed elements, where $\Psi \subset \{1, 2, \dots, M\}$. Assume the set indices are sorted in ascending order. Let $L = M - |\Psi|$ be the number of remaining functional elements. The steering vector of the damaged array can be expressed as $\mathbf{a}_{\text{im}}(\varphi) = \Gamma\mathbf{a}(\varphi)$, where $\Gamma \in \{0, 1\}^{L\times M}$ is a selection matrix. Each row j of Γ has a single entry of 1 at position Ψ_j and zeros elsewhere. The array output becomes:

$$\mathbf{x}_{\text{im}}(t) = \Gamma\mathbf{A}(\varphi)\mathbf{s}(t) + \Gamma\mathbf{n}(t) = \mathbf{A}_{\text{im}}(\varphi)\mathbf{s}(t) + \mathbf{n}_{\text{im}}(t)$$

The corresponding covariance matrix is:

$$\mathbf{R}_{\text{im}} = E\{\mathbf{x}_{\text{im}}(t)\mathbf{x}^H_{\text{im}}(t)\} = \mathbf{A}_{\text{im}}(\varphi)\mathbf{R}_s\mathbf{A}^H_{\text{im}}(\varphi) + \sigma_n^2\mathbf{I}_L$$

In practice, the sample covariance matrix is used:

$$\hat{\mathbf{R}}_{\text{im}} = \frac{1}{N}\sum_{t=1}^{N}\mathbf{x}_{\text{im}}(t)\mathbf{x}^H_{\text{im}}(t)$$

3 Covariance Matrix Reconstruction Based on Low-Rank Matrix Constraints

For the undamaged array model, let $\mathbf{B} = \mathbf{R} - \sigma_n^2\mathbf{I}$. It is easy to show that $\text{rank}(\mathbf{B}) = K \leq M - 1$ and $\mathbf{B} \succeq \mathbf{0}$. $\mathbf{B}$ is a Hermitian Toeplitz matrix that can be expressed as $\mathbf{B} = \mathcal{T}(\mathbf{u})$, where $\mathbf{u} \in \mathbb{C}^M$ and $\|\mathbf{u}\|_0 \leq M$. The Toeplitz structure is:

$$\mathcal{T}(\mathbf{u}) = \begin{bmatrix} u_1 & u_2 & \cdots & u_M \\ u_2^* & u_1 & \cdots & u_{M-1} \\ \vdots & \vdots & \ddots & \vdots \\ u_M^* & u_{M-1}^* & \cdots & u_1 \end{bmatrix}$$

Subtracting the covariance matrix from the sample covariance matrix yields:

$$\hat{\mathbf{R}} - \mathcal{T}(\mathbf{u}) - \sigma_n^2\mathbf{I} = \mathbf{E}$$

where $\mathbf{E}$ is the measurement error. Due to finite snapshots, this error cannot be zero and contains cross-correlation terms between signals and between signal and noise, which diminish as the number of snapshots increases.

Let $\mathcal{D} \subset \{1, 2, \dots, M\}$ denote the set of element positions. When $\mathcal{D}$ is defined on a ULA, it forms a redundant array, and the damaged array can be viewed as a co-array of a non-uniform linear array. To ensure redundancy of the damaged array, we assume random element failure positions with no more than half the elements failed. This assumption is reasonable in practice and is sufficient but not necessary—even if the co-array is not redundant, covariance reconstruction can still be guaranteed [14].

For ULAs, we can reconstruct the covariance matrix using low-rank matrix reconstruction theory:

$$\begin{aligned} \min_{\mathbf{u}} \quad & \operatorname{rank}(\mathcal{T}(\mathbf{u})) \\ \text{s.t.} \quad & \mathcal{T}(\mathbf{u}) \succeq \mathbf{0} \\ & \|\mathbf{W}^{1/2}\operatorname{vec}(\hat{\mathbf{R}} - \mathcal{T}(\mathbf{u}) - \sigma_n^2\mathbf{I})\|_2 \leq \eta \end{aligned}$$

where $\mathbf{W}$ is a weighting matrix. The vectorized error $\operatorname{vec}(\mathbf{E})$ asymptotically follows a normal distribution $\mathcal{N}(\mathbf{0}, \mathbf{W})$, where $\mathbf{W} = \mathbf{R}^T \otimes \mathbf{R}$ and $\otimes$ denotes the Kronecker product. In practice, $\mathbf{W}$ can be estimated as $\hat{\mathbf{W}} = \hat{\mathbf{R}}^T \otimes \hat{\mathbf{R}}$. Consequently, $\|\hat{\mathbf{W}}^{-1/2}\operatorname{vec}(\mathbf{E})\|_2^2$ follows a chi-square distribution with $2N$ degrees of freedom. Introducing a parameter η, the covariance matrix reconstruction problem can be solved through the following rank minimization:

$$\begin{aligned} \min_{\mathbf{u}} \quad & \operatorname{rank}(\mathcal{T}(\mathbf{u})) \\ \text{s.t.} \quad & \mathcal{T}(\mathbf{u}) \succeq \mathbf{0} \\ & \|\hat{\mathbf{W}}^{-1/2}\operatorname{vec}(\hat{\mathbf{R}} - \mathcal{T}(\mathbf{u}) - \sigma_n^2\mathbf{I})\|_2 \leq \eta \end{aligned}$$

However, this is an NP-hard problem. Therefore, convex relaxation is applied by replacing the pseudo-rank norm with the nuclear norm or trace norm of positive semidefinite matrices. The optimization problem becomes:

$$\begin{aligned} \min_{\mathbf{u}} \quad & \operatorname{tr}(\mathcal{T}(\mathbf{u})) \\ \text{s.t.} \quad & \mathcal{T}(\mathbf{u}) \succeq \mathbf{0} \\ & \|\hat{\mathbf{W}}^{-1/2}\operatorname{vec}(\hat{\mathbf{R}} - \mathcal{T}(\mathbf{u}) - \sigma_n^2\mathbf{I})\|_2 \leq \eta \end{aligned}$$

Once the optimal solution $\hat{\mathbf{u}}$ is obtained, the reconstructed covariance matrix $\mathcal{T}(\hat{\mathbf{u}})$ can be used with root-MUSIC for target DOA estimation. Due to the Toeplitz structure of $\mathcal{T}(\hat{\mathbf{u}})$, target directions can also be efficiently estimated through Vandermonde decomposition.

When the array is damaged, the covariance reconstruction problem can be described as:

$$\begin{aligned} \min_{\mathbf{u}} \quad & \operatorname{tr}(\mathcal{T}(\mathbf{u})) \\ \text{s.t.} \quad & \mathcal{T}(\mathbf{u}) \succeq \mathbf{0} \\ & \|\hat{\mathbf{W}}_{\text{im}}^{-1/2}\text{vec}(\hat{\mathbf{R}}_{\text{im}} - \Gamma\mathcal{T}(\mathbf{u})\Gamma^T - \sigma_n^2\mathbf{I}_L)\|_2 \leq \eta' \end{aligned}$$

where η' follows an asymptotic chi-square distribution with $2L$ degrees of freedom. This method can potentially recover the covariance matrix of the damaged array. The complete algorithm flow is:

Covariance Matrix Reconstruction DOA Estimation Algorithm: 1. Compute the damaged array covariance matrix $\hat{\mathbf{R}}_{\text{im}}$; 2. Compute the weighting matrix $\mathbf{W}_{\text{im}}$ and covariance fitting error parameter η'; 3. Solve the trace minimization problem to reconstruct $\mathcal{T}(\mathbf{u})$; 4. Apply root-MUSIC to estimate target directions for the damaged array.

4 Simulation and Analysis

This section presents simulation results to verify the effectiveness of the proposed high-resolution estimation method based on covariance matrix reconstruction. In all simulations, the uniform linear array consists of 10 elements with half-wavelength spacing. Targets are far-field uncorrelated narrowband signals, and noise is zero-mean Gaussian white noise with unit variance. Four elements fail at random positions [2, 4, 6, 9] in the ULA. The failed element positions are shown in [Figure 1: see original paper], where   indicates functional elements and × indicates failed elements, satisfying the condition that the number of failed elements does not exceed half of the total.

Experiment 1: Performance Comparison Under Element Failure

[Figure 1: see original paper] shows the array configuration. [Figure 2: see original paper] compares the spatial spectra under different conditions for two targets at directions [-4°, 4°]. When failed element outputs are treated as zero, the MUSIC method cannot resolve the two closely spaced targets. However, using the reconstructed covariance with root-MUSIC achieves performance close to that of a fully functional array, demonstrating the effectiveness of the proposed method in recovering array degrees of freedom.

Experiment 2: Estimation Bias vs. Angular Separation

The input SNR is 10 dB. Two equal-power independent incoherent signals impinge on the 10-element ULA. Two hundred Monte Carlo trials are conducted to analyze the average DOA estimation bias. One target angle is fixed at -10°, while the other moves away with 0.5° steps. The SNR is fixed at 0 dB with 500 snapshots. Three scenarios are compared: undamaged array with MUSIC, damaged array with MUSIC, and damaged array with covariance matrix reconstruction followed by root-MUSIC. The MUSIC method scans the [-90°, 90°]

region with 1° intervals. The relationship between bias and angular separation is shown in [Figure 3: see original paper].

With four randomly failed elements, the proposed covariance matrix reconstruction combined with root-MUSIC achieves excellent estimation performance even at small angular separations, outperforming MUSIC with a complete array. This is because the Toeplitz structure constraint and the inherent superiority of root-MUSIC over MUSIC contribute to improved performance. With a damaged array, MUSIC can estimate targets at large angular separations because the array still contains elements spaced at half-wavelength, but its performance degrades for multiple targets. The proposed method' s estimation bias approaches zero at 6° separation, while the intact array requires 8° separation for bias to approach zero, confirming that the method successfully restores the performance of a fully functional array.

Experiment 3: Multi-Target DOA Estimation Performance

Root Mean Square Error (RMSE) is used as the performance metric, defined as:

$$\text{RMSE} = \sqrt{\frac{1}{MK}\sum_{m=1}^{M}\sum_{k=1}^{K}(\hat{\varphi}_{k,m} - \varphi_k)^2}$$

where M is the number of Monte Carlo trials, K is the number of incident signals, and φ_k and $\hat{\varphi}_{k,m}$ are the true and estimated directions of the k-th signal in the m-th trial, respectively.

Assume 5, 6, 7, and 8 far-field uncorrelated narrowband targets are uniformly distributed in [-60°, 60°]. [Figure 4: see original paper] and [Figure 5: see original paper] show RMSE versus snapshots and SNR, respectively. [Figure 4: see original paper] shows the RMSE versus snapshots at 0 dB SNR with 200 Monte Carlo trials. As the number of targets increases, estimation accuracy relatively decreases, but the method maintains high precision for 5, 6, and 7 targets. [Figure 5: see original paper] shows RMSE versus SNR with 500 snapshots and 200 Monte Carlo trials. At SNR below 0 dB, the method shows lower accuracy, but as SNR increases, it achieves good resolution precision for 5, 6, and 7 targets. With four failed elements, traditional MUSIC cannot resolve more targets than the number of remaining functional elements, while the proposed method successfully addresses this limitation.

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Author Contributions:
Tan Weijie: Proposed research ideas, designed research methodology, performed theoretical analysis, conducted simulations, drafted manuscript;
Feng Xi' an: Revised final manuscript.

Note: Figure translations are in progress. See original paper for figures.