Power Transfer Coefficients Between Two 3D Semi-Infinite Solids in EFEA Formulation Postprint

Authors: Chen, XL, Zhang, WW

Date: 2017-04-06T00:00:00+00:00

Abstract

The energy finite element analysis (EFEA) is more effective for analyzing high frequency vibration problem compared to the traditional finite element method (FEM). When applying the EFEA to complicated structures, it is necessary to obtain power transfer

Full Text

Power Transfer Coefficients Between Two 3D Semi-Infinite Solids in EFEA Formulation

Xiliang Chen[1], and Wenwu Zhang[1],
[1]Ningbo Institute of Materials Technology & Engineering, Chinese Academy of Sciences, China
chenxl@nimte.ac.cn, zhangwenwu@nimte.ac.cn

Keywords: EFEA, Power Transfer Coefficients, 3D Solid, Wave

Abstract. Energy finite element analysis (EFEA) is more effective than the traditional finite element method (FEM) for analyzing high-frequency vibration problems. When applying EFEA to complicated structures, it is necessary to obtain power transfer coefficients at structural joints. This paper proposes theoretical formulations for computing power transfer coefficients between two 3D semi-infinite solids and demonstrates the validity of the developed formulations through two examples with different incident waves: dilatational waves and distortional waves.

Introduction

For high-frequency vibration problems in complicated structures, the traditional finite element method (FEM) requires extremely dense meshing because element size must be smaller than the vibration wavelength. Consequently, FEM demands excessive computational time for complex structures, and sometimes fails to achieve accurate results or even obtain solutions. To overcome these inherent limitations, energy finite element analysis (EFEA) has been developed [1]. In EFEA, the primary variable—energy density—is discontinuous at locations of discontinuities within a single member or at junctions where different members connect. Therefore, a special approach based on power flow continuity across joints is employed to formulate the global system equations at these junctions [2]. This continuity is expressed in terms of power transfer coefficients. Thus, when applying EFEA to such structures, obtaining power transfer coefficients at structural joints becomes essential.

Power transfer coefficients for plate junctions or beam-plate connections have been previously developed [3]. This paper proposes theoretical formulations for computing power transfer coefficients between two 3D semi-infinite solids and validates the developed theoretical framework.

Theoretical Formulations

For simplicity in deriving the equations, we define the x-z plane as the wave propagation plane, with the y-axis perpendicular to the x-z plane and oriented outward. In 3D solids, two types of waves can propagate: dilatational and distortional waves. Define Φ as a scalar quantity called the dilatation, which corresponds to the dilatational motion of particles. Define = × u as a vector potential corresponding to the rotational motion of particles, where u is the displacement vector.

From the partial differential equations of motion for free vibrations in a homogeneous isotropic elastic 3D solid, the following second-order partial differential equation (PDE) for the dilatation is obtained: (cid:1) is satisfied for vector quantity as a vector quantity representing the displacement is displacement (cid:1) (cid:1) vj wk (cid:1) (cid:1) (cid:1) (cid:1) $\Phi =$ where is wave speed of dilatational wave.

Similarly, the following PDE for the distortional wave is obtained: (cid:1) (cid:1) where is wave speed of distortional wave.

We consider either one dilatational plane wave (Fig. 1a [Figure 1: see original paper]) with heading to the interface Li or one distortional plane wave (Fig. 1b) with heading to the interface Si incident in the upper 3D semi-infinite elastic isotropic solid. When the wave encounters the interface between two 3D semi-infinite elastic isotropic solids, part of the wave transmits into the lower 3D semi-infinite solid while part reflects back into the upper 3D semi-infinite solid. The transmitted and reflected waves may include both dilatational and distortional waves regardless of whether the incident wave is dilatational or distortional.

We assume the scalar and vector quantities for incident, transmitted, and reflected waves as follows: (a) For the incident wave, (b) For the transmitted wave, 2Φ=Φ − (cid:1) (cid:1) (cid:1) 1 (c) For the reflected wave, We assume that all components of wave numbers in the x direction are identical. The totality of waves in the upper 3D solid is 1 Φ+Φ=Φ + . The totality of waves in the lower 3D solid is There are eight unknown complex amplitudes. To solve for these eight unknowns, we employ six boundary conditions (displacement continuity and stress continuity) at the interface, where superscript "+" denotes terms belonging to the upper solid while superscript "-" denotes terms belonging to the lower solid.

The eight unknown complex amplitudes can be solved using eight equations: the relationships between strains and displacements, and the relationships between stresses and strains. Once the complex amplitudes are obtained, particle displacements can be calculated using the following equation:) and two distortional wave requirements [4]: (cid:1) (cid:1)),,,((cid:1)),,,((cid:1)),,,(

The power transfer coefficients can be calculated as the ratio of power transmitted by the generated wave to the total incident wave power on the interface for a specific wave type with frequency and heading to the interface [3]: where the indices p, r, i, j represent the incident wave type, the transmitted or reflected wave type, the medium carrying the incident wave, and the medium carrying the transmitted wave, respectively (j=i represents reflected wave). We use "L" to represent dilatational wave type and "S" for distortional wave type.

Since the total power incident on the interface equals the total power carried away by the generated waves, energy conservation requires that 1= .

Validation of Theoretical Formulations

The developed theory for calculating power transfer coefficients is validated through several examples. In these examples, Poisson' s ratio is 1 . The ratio of density in the upper solid to the lower solid is . The ratio of group speed of dilatational wave in the upper solid to that in the lower solid is C C. The heading of the incident wave is .

The results are listed in Table 1 for dilatational incident waves and in Table 2 for distortional incident waves. In Tables 1 and 2, are reflected power coefficients, where the first subscript denotes incident wave type and the second subscript denotes transmitted or reflected wave type. "L"represents dilatational wave type and "S"represents distortional wave type. SS are transmitted power coefficients, and LS , SL , LL , SL , LS , LL ,

Table 1 Power Transfer Coefficients (Dilatational Incident Wave) Present Ref [5] (cid:2) (cid:2) Present Ref [5] (cid:2) Present Ref [5]

Table 2 Power Transfer Coefficients (Distortional Incident Wave) Present Ref [5] (cid:2) (cid:2) Present Ref [5] Present (cid:2) Ref [5]

Conclusions

The developed theory and program can simulate wave propagation in 3D semi-infinite elastic media very well. When a dilatational or distortional wave passes through the interface between two 3D semi-infinite media, both transmitted and reflected waves are generated (or portions thereof). The generated waves consist of both dilatational and distortional components (or portions thereof). The power carried away by the transmitted and reflected waves equals the total power carried by the incident wave, as expected.

References

[1] W.G. Zhang, A.M. Wang, N. Vlahopoulos, An alternative energy finite element formulation based on incoherent orthogonal waves and its validation for marine structures, Finite Elements in Analysis and Design Vol. 38 1095-1113 (2002).

[2] X.Y. Yan, Energy finite element analysis developments for high frequency vibration analysis of composite structures, PhD dissertation of Dept. of Naval Architecture and Marine Engineering, The University of Michigan (2008).

[3] R.S. Langley and K.H. Heron, Elastic wave transmission through plate/beam junctions, Journal of Sound and Vibration Vol.143(2) 241-253(1990).

[4] P. Borejko, Reflection and transmission coefficients for three-dimensional plane waves in elastic media, Wave Motion Vol.24 371-393(1996).

[5] M. Muskat and M.W. Meres, Reflection and transmission coefficients for plane waves in elastic media, Geophysics Vol. 115-148 (1940).