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Abstract

Compared with traditional single-point magnetic field detection, coordinated multi-point magnetic field detection can simultaneously obtain magnetic field measurements at each observation point, eliminating temporal variations in the detected magnetic field and enabling better calculation of spatial current density. Based on the computational method for calculating spatial current density from multi-point magnetic field inversion, this paper conducts numerical simulation analysis to investigate the influence of satellite formation number, satellite formation configuration, satellite positioning deviation, satellite attitude measurement error, magnetic field measurement error, external magnetic field strength, and external current density on current inversion error. Simulation results demonstrate that a 5-satellite formation outperforms a 4-satellite formation. Under the 5-satellite formation condition, satellite attitude measurement error and external magnetic field strength constitute the primary sources of inversion error, while satellite formation configuration also represents a significant source of inversion error. According to the simulation results, when the satellite attitude error is 0.001° and the formation scale is approximately 100 km, the relative error of current density inversion in the equatorial region is approximately 24%.

Full Text

Multi-Point Magnetic Field Cooperative Detection for Ionospheric Current Density Inversion

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Abstract

Compared with traditional single-point magnetic field measurements, multi-point cooperative magnetic field detection can simultaneously obtain magnetic field data from multiple measurement points, eliminating temporal variations in the measured magnetic field and enabling better calculation of spatial current density. Based on the computational method for deriving spatial current density from multi-point magnetic field measurements, this paper conducts numerical simulation analysis to examine how various factors affect current density inversion error, including the number of satellites, satellite formation configuration, satellite positioning deviation, satellite attitude measurement error, magnetic field measurement error, external magnetic field intensity, and external current density. Simulation results demonstrate that a five-satellite formation outperforms a four-satellite formation. Under five-satellite formation conditions, satellite attitude measurement error and external magnetic field intensity constitute the primary sources of inversion error, while satellite formation configuration also represents a significant error source. According to simulation results, when satellite attitude error is 0.001° and formation scale is approximately 100 km, the relative error in current density inversion in the equatorial region is about 24%.

Keywords: multi-point observation, magnetic field measurement, ionospheric current density

1. Introduction

Space magnetic field detection has long served as one of the primary means for space science research, with a history spanning nearly 50 years. Based on space magnetic field measurements, researchers can establish magnetic field models, investigate magnetic field variation responses, develop Earth' s main magnetic field models, study magnetic field changes during disturbance processes and responses to upstream space events, and conduct related research on space plasma [1,2,3].

Traditional space magnetic field detection primarily employs single-satellite in-situ measurements. As the satellite moves, the magnetic field measurements couple temporal variations with spatial distributions of the magnetic field. When calculating current density using measured magnetic fields at various points along the satellite path, one must neglect temporal magnetic field variations because current density depends only on the spatial distribution of the magnetic field [4]. Multi-point cooperative magnetic field detection through satellite formations can eliminate these temporal variations.

With advances in satellite formation technology, multi-point cooperative detection has become an important approach for magnetic field measurements, most notably exemplified by the Cluster mission [5]. Launched in 2000, Cluster consists of four satellites carrying 11 scientific instruments including magnetometers. Through its tetrahedral configuration, Cluster has conducted observations of plasma boundaries, three-dimensional structures, and current densities. Cluster has enabled numerous important scientific discoveries and inaugurated a new era of scientific research using distributed satellite networks. China subsequently implemented the Double Star program, which coordinated with Cluster to conduct the first six-point stereo detection in human history, performing multi-temporal and multi-scale investigations of Earth' s magnetosphere [6]. Distributed satellite network observations reflect the average current density within the spatial scale of the satellite formation. Cluster' s inter-satellite distance ranges from approximately 600 to 20,000 kilometers, while the Double Star program consisted of one polar-orbiting satellite and one equatorial satellite. Their combination cannot effectively observe relatively small-scale physical phenomena, particularly near polar regions where magnetic field structures vary significantly and current distributions are complex, often containing small-scale physical phenomena [7]. Another example is THEMIS, launched in 2007, which comprises five satellites in a large-scale collinear network with apogees of 10Re, 20Re, and 30Re, aligning in a straight line in the magnetotail every four days. This large-scale network facilitates studies of global physical processes and magnetotail physics, enabling investigation of the temporal sequence of magnetotail processes [8], but is similarly unsuitable for studying fine structures and current density distributions.

Near polar regions, magnetic fields and plasmas vary intensely, with numerous upward and downward particle phenomena. Under typical conditions, polar region field-aligned current densities exceed 1 A/m^2, with current sheet thicknesses of several hundred kilometers [9]. Through multi-point cooperative detection that eliminates temporal magnetic field variations, high-precision current density distributions can be obtained based on the spatial distribution of magnetic fields. Numerous small-scale fine structures exist near polar regions, and investigating small-scale magnetic field structures and ionospheric current density distributions near polar regions is crucial for understanding magnetosphere-ionosphere coupling and ionospheric responses to upstream solar wind.

Currents in the ionosphere arise when electrons and ions move in different directions under various forces. The primary driving forces in the ionosphere include collisions between neutral winds and charged particles, gravity, and pressure gradients. Consequently, the main current types in the ionosphere are neutral wind currents, gravity drift currents, and pressure gradient currents (diamagnetic currents). Neutral wind currents concentrate primarily in the E layer (dynamo layer) at approximately 90–150 km altitude, decreasing rapidly with increasing height. Gravity drift currents and pressure gradient currents are smaller in the E layer, gradually increasing with altitude and reaching maximum values at approximately 400 km, becoming the dominant current forms in the F layer. Overall, ionospheric current density is relatively large at low latitudes during daytime, reaching several thousand nA/m^2 in the E layer and several tens of nA/m^2 at 400 km altitude [10,11], with specific intensities closely related to ionospheric and solar activity. This paper investigates the method for ionospheric current density inversion using multi-point magnetic field measurements and the key factors affecting current density inversion error.

2. Calculation Method

In space plasma, displacement current can typically be neglected, allowing spatial current density to be calculated using Ampère’ s law. Based on magnetic field observations from multiple satellites, the magnetic field gradient tensor at the geometric center of the measurement points can be computed, thereby obtaining magnetic field curl and current density [12,13].

The magnetic field gradient tensor is expressed as: MATH_0 . In a coordinate system with the geometric center of satellite positions as the origin, assuming the spatial positions of each satellite are MATH_1 . Expanding the magnetic field values measured by each satellite at the origin yields: MATH_2 , where the subscript c denotes the value at the origin, i, j, k represent three directions, MATH_3 denotes the partial derivative of the i-direction component of the magnetic field in the j direction, and MATH_4 represents the satellite index.

Multiplying both sides of the above equation by MATH_5 , where l represents three directions, and summing and averaging yields: MATH_6 , where MATH_7 is a column vector and MATH_8 represents the magnetic field values measured by the satellites. Letting MATH_9 , where MATH_10 is a column vector representing satellite positions and the superscript T denotes transpose. Defining the volume tensor of the satellite formation configuration as MATH_11 and omitting higher-order terms yields: MATH_12 , where MATH_13 represents the magnetic field gradient tensor at the geometric center and the omitted higher-order terms are of order MATH_14 , where D is the spatial scale of magnetic field variation and L is the scale of measurement point spacing. This demonstrates that when L « D, the above method can be used to determine magnetic field curl.

3. Numerical Simulation

During simulation, the satellite formation orbit, background magnetic field, and background current density distribution are predetermined to simulate magnetic field measurement data, which is then used to invert spatial current density and investigate factors affecting inversion error.

The satellite formation employs an elliptical polar orbit with perigee at 500 km and apogee at 1500 km, with inter-satellite distances of 10–100 km, covering all local times through 13.5 orbital periods. In the geographic coordinate system (X-axis in Earth’ s equatorial plane through the zero-degree meridian, Z-axis parallel to the rotation axis, Y-axis determined by the right-hand rule), the flight trajectory of the formation center is shown in Figure 1 [Figure 1: see original paper].

Figure 1. Schematic diagram of the flight trajectory of the satellite formation center in the geographic coordinate system

The satellite formation adopts a non-flattened design: based on a tetrahedral configuration of four satellites, one additional satellite flies in parallel as a companion. Due to orbital and formation configuration evolution, the tetrahedral formation can degenerate into a planar configuration in certain regions. The companion satellite maintains a favorable three-dimensional configuration for all five satellites throughout all regions.

The background magnetic field comprises the geomagnetic field and ionospheric magnetic field. The geomagnetic field uses the IGRF geomagnetic model [14], while the ionospheric magnetic field is calculated using a coupled magnetic field-current model proposed by U. Engels [15]. This model simultaneously provides analytical expressions for spatial magnetic fields and currents, and the magnetic field distribution satisfies the Biot-Savart law calculated from the current distribution in the model, making it highly suitable for verification of inversion calculations. This magnetic field model in spherical coordinates can be written as: MATH_15 , where MATH_16 represents Schmidt-form associated Legendre functions, Re is Earth' s radius, MATH_17 is colatitude, and MATH_18 is longitude. Since the geomagnetic field is irrotational, all background currents originate from the ionosphere. The current model in the above model in spherical coordinates can be written as: MATH_19 , (8) where J is a control coefficient. The background current density distribution along the flight trajectory is shown in Figure 2 [Figure 2: see original paper]. According to the design of the U. Engels current model, current density exhibits four concentrated current regions along the flight trajectory.

Figure 2. Distribution of current density strength on the orbit calculated by the model

The inversion error for ionospheric current density using multi-point magnetic field cooperative detection is influenced by numerous measurement factors. Under predetermined satellite formation and orbit conditions with typical values for measurement parameter errors (satellite attitude measurement error of 0.001°, satellite positioning error of 1 cm, magnetic field measurement error of 0.1 nT), the inverted current density distribution and inversion error distribution are shown below. Inversion error is defined as the mathematical expectation of the magnitude of the difference between the inverted current density and background current density under random error conditions. The inverted current density similarly exhibits four concentrated current regions, basically reflecting the overall current density distribution, with relatively low overall inversion error and larger errors appearing near 60°S latitude.

Figure 3. Distribution of inverted current density (left) and inversion error distribution (right) on orbit

4. Error Analysis and Discussion

Factors affecting magnetic field inversion current density error include satellite formation configuration, number of satellites, satellite attitude measurement error, satellite positioning error, magnetic field measurement error, external magnetic field intensity, and external current density intensity.

4.1. Satellite Formation Configuration

The eigenvalues of the volume tensor R of the satellite formation configuration, ordered from largest to smallest, are MATH_20 , MATH_21 , and MATH_22 , representing the spatial scales of the formation configuration in three directions. E denotes elongation: MATH_23 , and P denotes planarity: MATH_24 . During formation flight, the configuration continuously evolves. When the configuration gradually flattens in a particular direction, the calculation error of the spatial partial derivatives of the three magnetic field components in that direction increases, thereby affecting the calculation accuracy of the magnetic field curl in the other two directions and causing increased errors in the current density components in those directions. The typical evaluation index for configuration is the E-P combination. The figure below shows the current density error scatter plot obtained by Robert P et al. [16] for different configurations, where circle size and color represent relative error magnitude.

Figure 4. Relationship between E-P values of the formation shape and relative truncation errors of current density [16]

As shown in the figure, larger E and P values correspond to larger inversion errors, with inversion errors roughly distributed in an arc pattern with E and P values. Therefore, this paper adopts MATH_25 as the configuration index to evaluate satellite formation configuration. Smaller d indicates a more robust configuration, while larger d indicates a flatter configuration. Generally, more robust satellite formation configurations yield smaller current inversion errors, whereas flatter configurations produce larger errors.

4.2. Number of Satellites

The robustness of the satellite formation configuration significantly affects current density inversion error. To invert three-dimensional current density vectors, at least a four-satellite formation maintaining a robust tetrahedral configuration is required. However, without orbit control, a four-satellite formation with relatively close spacing typically experiences flattened configuration regions during orbital periods. Within these flattened regions, the tetrahedral configuration can degenerate into a planar configuration, dramatically increasing spatial current density inversion error. Adding a fifth satellite can improve the configuration index in flattened regions.

The figure below shows the configuration index variation (left) and current density inversion error variation (right) for different numbers of satellites over three orbital periods.

Figure 5. E-P values and errors of inverted current density with different

numbers of satellites over three orbital periods

In the figure, the four-satellite formation' s configuration index is larger in polar regions and smaller near the equator, while the five-satellite formation' s configuration index is smaller in polar regions and larger near the equator. Within the orbital segment where the four-satellite formation' s configuration index is minimal (near the equator), the four-satellite formation exhibits an ideal configuration, and adding a fifth satellite temporarily increases the configuration index. However, in most other regions, particularly within the flattened regions of the four-satellite formation, the five-satellite formation' s inversion error is significantly smaller than that of the four-satellite formation. Therefore, the five-satellite formation' s configuration index is clearly superior to that of the four-satellite formation. Near polar regions, the four-satellite formation's configuration index increases rapidly, as does its inversion error, while the five-satellite formation' s configuration index decreases, correspondingly reducing its inversion error. Near the equator, the configuration indices and inversion errors of four- and five-satellite formations are similar. Thus, satellite formation inversion error variation exhibits strong positive correlation with configuration index variation.

4.3. Other Factors

Besides formation configuration and satellite number, factors affecting current density inversion error include satellite attitude measurement error, satellite positioning error, magnetic field measurement error, external magnetic field intensity, and external current density intensity. All these parameters have typical measurement values in practice. Figure 6 [Figure 6: see original paper] examines how variation of a single parameter within a certain range around its typical value affects current density inversion error for a specific orbital point (158.52°E, 17.94°S, 695.10 km altitude), without considering changes or effects of other factors.

Figure 6. Error curves of inverted current density influenced by different parameters

In Figure 6, the horizontal axis represents parameter values in corresponding units, the vertical axis represents the expected value of current density inversion error, and asterisks indicate typical parameter values. As shown, satellite attitude measurement error, magnetic field measurement error, external magnetic field intensity, and satellite positioning error exhibit essentially linear relationships with inversion error. Since external magnetic field intensity manifests its influence through satellite attitude measurement error, their typical values correspond to identical inversion errors. According to the error curves in Figure 6, satellite positioning error has the smallest impact on inversion error, while satellite attitude measurement error and external magnetic field intensity have the largest impact. The relative error of inverted current density remains essentially unchanged with variations in external current density intensity, indicating that this method can still achieve good relative inversion accuracy when strong currents appear in the external environment, such as intense field-aligned currents in polar regions.

Based on simulation results with the aforementioned error sources under typical measurement parameter and error conditions, the current density inversion errors caused by specific measurement parameters are: satellite positioning error of 0.01 m yields relative inversion error of approximately 2.8 × 10 %; satellite attitude measurement error of 0.001° and external magnetic field intensity of 3.3 × 10 nT yield relative inversion error of approximately 23.8%; magnetic field measurement error of 0.1 nT yields relative inversion error of approximately 6.7%; different external current density intensities yield relative inversion error of approximately 2.6%.

4.4. Main Error Sources

In Figure 3 (right), a region with relatively large errors appears in the Southern Hemisphere. Through error analysis in Sections 4.1–4.3, we know that inversion error relates to changes in external conditions. When satellite number and measurement factors such as satellite attitude measurement error, satellite positioning error, and magnetic field measurement error are determined, inversion error should relate to formation configuration and external magnetic field intensity. The figure below shows variation curves of relevant indices along an orbital segment crossing the region with relatively large errors.

Figure 7. Curves of inversion errors, E-P values, and magnetic field strengths on part of the orbit

In Figure 7, we can see that inversion error variation exhibits strong correlation with configuration index and external magnetic field intensity variation, with correlation coefficients reaching 0.6432 and 0.8586 respectively, and a multiple correlation coefficient of 0.9625. In the 60°S region, the primary reason for relatively large errors is high external magnetic field intensity, with the secondary reason being poor satellite formation configuration index.

5. Conclusions

This paper presents a method for ionospheric current density inversion using multi-point magnetic field cooperative detection. By calculating the magnetic field gradient tensor at the geometric center of the satellite formation and subsequently obtaining the current density at that location, this study investigates through numerical simulation and error analysis how factors such as satellite formation configuration, number of formation satellites, satellite positioning error, satellite attitude measurement error, magnetic field measurement error, external magnetic field intensity, and external current density intensity affect inversion error. The following conclusions are obtained:

1) More robust satellite formation configurations yield smaller inversion errors, and vice versa. A five-satellite formation shows significant improvement in inversion error compared to a four-satellite formation.
2) Satellite attitude measurement error and external magnetic field intensity constitute the main sources of inversion error. With attitude error of 0.001° and external magnetic field intensity of 3.3×10 nT, the relative inversion error is approximately 23.8%. To achieve accurate measurement of ionospheric current density, satellites must ensure high-quality attitude measurement.
3) When designing satellite formations, robust formation configurations should be maintained in strong magnetic field regions.

According to simulation results, using a five-satellite formation with attitude error of 0.001° and formation scale of approximately 100 km, the current density inversion error in the equatorial region is about 8.86 nA/m², with external current density of 37.22 nA/m², yielding a relative error of about 24%. The method described in this paper can effectively invert ionospheric current density.

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