Rasterization of Celestial Sphere Detection Coverage Based on Coordinate System Transformation and QTM Encoding: Postprint

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Abstract

This paper proposes a rasterization analysis method for celestial sphere detection coverage based on coordinate system transformation and QTM encoding, and conducts experimental analysis on its computational accuracy and efficiency. Compared with traditional algorithms and latitude-longitude grid encoding methods, this method demonstrates significant advantages in both efficiency and accuracy, and can meet the requirements of payload detection coverage analysis and mission planning for space science satellites that target the celestial sphere.

Full Text

A Fast Calculation Method for Coverage Analysis of Space Telescope Based on Coordinate Transformation and QTM Code

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Abstract

In sky coverage analysis, we present a fast algorithm based on coordinate transformation to calculate the points on the boundary of detection field. Based on this algorithm, QTM, a coding method that has many advantages can be used to manage the data calculated. The main contents of this topic include: 1)

Build the quantitative calculation model for sky coverage; 2) To establish the rasterized expression method of sky coverage analysis results.

Key words: celestial surface, coverage analysis, coordinate transformation, QTM, coding

Introduction

During the "Twelfth Five-Year Plan" period, the Chinese Academy of Sciences' Space Science Strategic Pioneer Program will launch five space science satellites under the "Innovation 2020"initiative. Among them, the Hard X-ray Modulation Telescope and the Dark Matter Particle Detection Satellite will observe cosmic radiation such as hard X-rays, high-energy electrons, and gamma rays using various observation modes including sky survey and fixed-point observations. During ground operation control of these satellites, it is necessary to frequently calculate the coverage of detection fields by payload instruments over specific time periods under different positions, attitudes, and working modes. This coverage analysis serves as input for mission planning and is a prerequisite for seeking optimal control schemes through mission planning methods.

Observational objects and coverage ranges in astronomical observations are generally described using the celestial coordinate system, which is commonly employed in astronomy. In this system, the celestial sphere is an imaginary sphere with theoretically infinite radius and Earth' s center as its center. All objects in the sky are projected onto the celestial sphere's surface. Based on the geocentric equatorial inertial coordinate system, the celestial sphere corresponds to Earth and possesses celestial equators and celestial poles. The celestial equator is the projection of Earth' s equator onto the celestial sphere, while celestial poles are projections of Earth' s poles. Figure 1 [Figure 1: see original paper] illustrates a celestial sphere projection example. In astronomical terminology, RA (Right Ascension) represents the angular distance from a celestial object' s projection point to the vernal equinox, expressed in hours, minutes, and seconds. Declination represents the angular distance from a celestial object' s projection point to the celestial equator, expressed in degrees, minutes, and seconds.

In near-Earth satellite celestial sphere coverage analysis, the satellite' s position change is negligible relative to distant stars, so the shape and size of the detection field' s projection on the celestial sphere remain constant. Therefore, only the right ascension and declination of the observation vector' s projection point on the celestial sphere need to be determined to solve for instantaneous coverage range.

However, domestic research on celestial sphere coverage analysis is limited. Existing mature Earth-oriented coverage calculation models have limitations when applied to celestial sphere coverage analysis, particularly in terms of efficiency and computational accuracy, making them inadequate for rapid celestial sphere coverage calculations. Specialized coverage analysis methods are needed. The key technical challenges in designing such methods include:

1. **Computational efficiency**: Mission planning involves extremely long time periods (months or even years). Within such large time spans, frequent calculations are required based on Earth' s revolution, satellite orbital position, attitude parameters, and working modes at certain time steps, resulting in massive computational loads. Improving computational efficiency and speed through algorithmic improvements while ensuring accuracy is a critical issue in engineering implementation.
2. **Rasterized expression of coverage results**: While payload detection fields are generally described using analytical geometric shapes, mission planning algorithms require rasterized coverage results as input. To balance celestial sphere subdivision coding efficiency, data processing accuracy, and computational speed, appropriate spherical grid methods must be selected for managing detection targets and regions while enabling rapid conversion of calculation results.

1.1 Mathematical Description of Space Detection Payload Fields

Satellite payload detection fields for cosmic space observation can be classified into two categories based on their spatial properties:

1. **Rectangular cone field**: The field shape can be described using two angular parameters. The four edges of a rectangular cone detection field project as four great circle arcs between vertices on the celestial sphere. The coverage area on the celestial sphere is the region bounded by these four great circle arcs, where the great circle minor arc length is and the great circle minor arc length is , as shown in Figure 2 [Figure 2: see original paper].
2. **Circular cone field**: The field shape can be described using the cone' s axis and half-angle . The intersection of the cone' s axis with the celestial sphere is point , and the intersection of the cone' s generatrices with the celestial sphere forms a small circle centered at . The coverage area on the celestial sphere is the region bounded by this small circle with center at point and radius , as shown in Figure 3 [Figure 3: see original paper].

1.2 Fast Solution Algorithm: Coordinate Transformation Method

Analysis of the mathematical description shows that the coverage problem can be simplified into solving great circle arcs and small circles with known centers and spherical radii. Regarding great circle solutions, mature algorithms exist in aviation and navigation fields.

Domestic scholar Hong Deben proposed an analytical method for great circle route planning suitable for computer calculation. This method solves for coordinates, headings, and distances of points along great circle routes. It uses the spherical cosine theorem to calculate great circle distances, then divides the distance into segments at certain intervals. The spherical sine theorem yields coordinates of division points (waypoints). While simple and adjustable for precision, this algorithm calculates heading angles at each point, increasing computational load. It also assumes constant heading within each arc segment, causing cumulative errors unsuitable for celestial sphere coverage calculations. For small circle solutions, coordinates are typically calculated in 3D Cartesian coordinates then converted to latitude/longitude.

To address these limitations, this paper proposes a fast celestial sphere coverage algorithm based on coordinate transformation. In the celestial coordinate system, an arbitrary point (with coordinates: declination , right ascension in the celestial coordinate system) is selected as the new celestial pole. The great circle arc passing through this point is defined as 0° right ascension, establishing a new spherical coordinate system. The relationship between this new system and the original celestial coordinate system is shown in Figure 4 [Figure 4: see original paper].

The new spherical coordinate system is similar to the celestial coordinate system but differs in that its Z' -axis points toward point , with being the celestial coordinates of point . The great circle plane perpendicular to this is the "equator" of the spherical coordinate system, with X' and Y' axes in its equatorial plane. The meridian determined by point and the north celestial pole is the zero-degree meridian.

A point on the celestial sphere is identified by latitude (the angle between the line connecting the point to the sphere' s center and the "equator") and longitude (the angle between the meridian passing through the point and the meridian passing through the north celestial pole).

For any point on the celestial sphere, the conversion relationship between coordinates in the new spherical coordinate system and the celestial coordinate system is:

$$\begin{cases} \cos\lambda'\cos\varphi' = \cos\lambda\cos\varphi\cos\lambda_0\cos\varphi_0 + \sin\lambda\sin\lambda_0 \\ \cos\lambda'\sin\varphi' = \cos\lambda\sin\varphi - \cos\lambda\cos\varphi\sin\lambda_0\cos\varphi_0 \\ \sin\lambda' = \cos\lambda\cos\varphi\sin\lambda_0\sin\varphi_0 - \sin\lambda\cos\lambda_0 \\ \csc\lambda'\cos\varphi' = \cos\lambda\cos\varphi\cos\lambda_0 - \sin\lambda\sin\lambda_0\cos\varphi_0 \\ \csc\lambda'\sin\varphi' = \cos\lambda\sin\varphi\cos\lambda_0 - \sin\lambda\sin\lambda_0\sin\varphi_0 \end{cases}$$

By establishing a new celestial coordinate system based on the geometric characteristics of the detection field boundary, computational complexity can be significantly reduced.

1.3 Algorithm Implementation

For rectangular cone detection fields, a vertex of one rectangular edge is selected as the north pole in the new spherical coordinate system. The great circle line containing that edge is defined as 0° right ascension in the new system, making that edge have 0° right ascension and declination ranging from to 90°. Using Equation 1, the right ascension and declination coordinates of points on this edge can be quickly calculated in the celestial coordinate system (as shown in Figure 5 [Figure 5: see original paper]).

For circular cone detection fields, the field center point is selected as the north pole in the new spherical coordinate system. The boundary line then has right ascension ranging from 0° to 360° and declination of . Using Equation 1, coordinates of boundary points can be quickly calculated (as shown in Figure 6 [Figure 6: see original paper]).

In implementation, the sine function in Equation 1 is positive in both first and second quadrants, sometimes producing ambiguous solutions. The spherical four-part formula can replace the sine formula to resolve this uncertainty and improve calculation accuracy.

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2 Rasterized Expression of Celestial Sphere Coverage Based on QTM Encoding

Spherical grid systems are equal-area projection systems whose precision increases with grid refinement, making them suitable for coverage analysis and mission planning. Among various spherical grid algorithms, the Quaternary Triangular Mesh (QTM) proposed by Dutton, based on a regular octahedron, has become an effective method for managing global multi-scale massive data structures due to its hierarchical nature.

2.1 QTM Construction and Encoding

QTM is constructed based on a regular octahedron whose top and bottom vertices coincide with the north and south celestial poles, and whose edges align with projections of 0°, 90°, 180°, 270° right ascension and 0° declination. As shown in Figure 7 [Figure 7: see original paper], the corresponding codes are 0-7.

Triangles are then coded layer by layer. In each subdivision, one triangle is divided into four new triangles: the central triangle is coded 0, the left triangle 2, the right triangle 3, and the upper or lower triangle 1, as shown in Figure 8 [Figure 8: see original paper].

A k-layer QTM code is represented by , where is an octant code representing one face of the initial octahedron, and are k quaternary codes representing the triangle patch at layer k. The QTM network division result for k=6 is shown in Figure 9 [Figure 9: see original paper].

2.2 Conversion Algorithm from Right Ascension/Declination to QTM Code

This paper implements fast conversion from right ascension/declination to QTM code based on the "Cavalcade Approach Method" proposed by Zhao Xuesheng and Chen Jun. This is a spherical small circle subdivision method that first determines the maximum row and column numbers of QTM according to the subdivision level, then calculates the row and column numbers of the corresponding triangular grid based on the point' s latitude and longitude, and finally recursively derives the QTM code layer by layer.

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3 Experimental Results Analysis

Based on the proposed coordinate transformation and QTM encoding method, the authors developed a software system for rasterized celestial sphere coverage analysis. The correctness of the analysis results was verified (as shown in Figure 10 [Figure 10: see original paper]), and the method' s precision and efficiency were evaluated.

3.1 Computational Efficiency Analysis

The experimental platform used was a desktop computer (CPU: Intel(R) Core™ 2 Duo E8400 @ 3.0 GHz, Memory: 4 GB RAM, OS: Windows XP).

For rectangular cone field efficiency, the spherical coordinate transformation algorithm was compared with the great circle route analytical method used in navigation. Calculating 1000 temporal fragments yielded the results shown in Table 1 (resolution based on a 10° angle case).

The great circle route method, which calculates heading at each step, shows the lowest efficiency. The coordinate transformation approach first calculates coverage in the satellite body coordinate system, then uses attitude matrix transformation to compute coverage in the celestial coordinate system based on the fact that coverage moves with the satellite. This provides significant efficiency improvement. Using spherical coordinate transformation, as a 2D-to-2D coordinate conversion compared to attitude matrix transformation, offers additional efficiency gains.

For circular cone fields, the attitude matrix transformation method is typically used as a general approach. However, this method doesn' t exploit special properties of the field. In the new spherical coordinate system, a circular cone field forms a declination circle with complete symmetry, allowing calculation of only the 0-180° range. Efficiency comparison is shown in Table 2 (resolution based on a 10° cone angle).

3.2 QTM Encoding Precision

The edge length of a triangular grid at a certain QTM level represents the resolution at that level. In celestial sphere coverage analysis, resolution can be expressed as the central angle corresponding to the triangular grid edge length. Based on Zhao Xuesheng's fast conversion algorithm using the "latitude-longitude bisection method" for spherical triangle subdivision, some deformation in triangle size and shape is inevitable. Using the edge at 0° declination as the resolution standard, a k-layer QTM has edges on the 0° declination line, corresponding to a central angle of . Thus, k-layer QTM resolution is .

In the "Cavalcade Approach Method," only odd columns are calculated, introducing an additional half-grid error, so the comprehensive resolution is approximately . When k=19, QTM encoding achieves arcsecond-level resolution.

3.3 Comprehensive Precision and Efficiency of Rasterized Coverage Analysis

In practical engineering applications, QTM encoding layer selection should meet actual coverage analysis precision requirements. For example, when the satellite field angle reaches 30°, using k=12 yields a rasterized coverage analysis precision of 0.03°, sufficient for daily mission analysis and planning. Overall efficiency is shown in Table 3 .

Experimental verification demonstrates that the proposed rasterized celestial sphere coverage analysis method offers significant advantages over non-rasterized methods, including high computational efficiency, controllable precision, easy storage and expression of results, and straightforward implementation of intersection/union operations. In practical applications, detection targets distributed on the celestial sphere (area targets or point targets) can be encoded using QTM and expressed as one or a set of codes (a spherical triangle or a region formed by multiple triangles). This enables rapid retrieval of detection opportunities for specific targets from coverage analysis results through code lookup, well satisfying mission planning algorithm input requirements. The method can be further optimized in practical applications based on engineering constraints, particularly in storage management efficiency of analysis results.

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Note: Figure translations are in progress. See original paper for figures.