

Random Similarity Between Two Mixed Scatterers (Postprint)

Authors: Li, Dong, Zhang, Yunhua

Date: 2017-01-04T00:00:00+00:00

Abstract

Scattering similarity was first proposed by Yang et al. to measure the similarity between two single scatterers. It was extended by Chen et al. to measure the similarity between a mixed scatterer and a single scatterer. This letter develops a random similarity parameter to further measure the similarity between two mixed scatterers. The parameter not only covers Yang's and Chen's similarities by providing a general scattering similarity measurement, but also is useful for scattering randomness description by enabling a fast alternative and a competent complementary to the entropy parameter. A novel model-based characterization scheme of mixed scatterer is then proposed by parallel combining the random similarities between the mixed scatterer and three canonical mixed volume scatterers. By further fusing with the SPAN, the scheme can characterize both the texture and the scattering information regarding a target. Comparative experiment with Chen's approach on L-band ESAR Oberpfaffenhofen data demonstrates its excellent discrimination of radar targets. 2004-2012 IEEE.

Full Text

Preamble

Random Similarity Between Two Mixed Scatterers

Dong Li, Member, IEEE and Yunhua Zhang, Member, IEEE

ABSTRACT

Scattering similarity was first proposed by Yang et al. to measure the similarity between two single scatterers. It was later extended by Chen et al. to measure the similarity between a mixed scatterer and a single scatterer. This letter develops a random similarity parameter to further measure the similarity between two mixed scatterers. The parameter not only encompasses Yang's

and Chen' s similarities by providing a general scattering similarity measurement, but also proves useful for scattering randomness depiction by enabling a fast alternative and competent complement to the entropy parameter. A novel model-based characterization scheme for mixed scatterers is then proposed by parallel combining the random similarities between the mixed scatterer and three canonical mixed volume scatterers. By further fusing this with SPAN, the scheme can characterize both the texture and scattering information regarding a target. Comparative experiments with Chen' s approach on L-band ESAR Oberpfaffenhofen data demonstrate its excellent discrimination of radar targets.

Key Words: Mixed scatterer, polarimetric synthetic aperture radar (PolSAR), scattering randomness, scattering similarity, volume scatterer.

I. INTRODUCTION

Remote sensing of the Earth using polarimetric synthetic aperture radar (PolSAR) has received intensive attention in the past two decades. PolSAR can acquire both the geometrical and physical information of a scatterer, storing them in the scattering matrix (for single scatterers) or coherence matrix (for mixed scatterers). An understanding of the scatterer is then obtained by processing these matrices. Multiple approaches enable us to perform this, among which polarimetric target decomposition is the most popular. It interprets mixed target scattering by identifying the dominant scattering (the Huynen-type target dichotomies) or average scattering (the eigenvector-based decompositions), or by expanding the mixed scattering onto canonical scatterings (the model-based decompositions). A comprehensive review of existing target decompositions was presented in [1]. Another approach was proposed by Yang et al. in terms of scattering similarity, which measures the correlation between two scattering matrices [2]. By checking the similarity between a given (unknown) scatterer and other (known) canonical scatterers, direct characterization of the scatterer is obtained. Nevertheless, Yang' s parameter can only measure similarity between two single scatterers. For general mixed scatterers created by a mixture of single scatterers, this parameter fails to analyze their scattering. Chen et al. extended Yang' s parameter to measure similarity between a mixed scatterer and a single scatterer [3]. Discrimination of the mixed scatterer is then obtained by investigating similarities between it and canonical scatterers such as surface, dihedral, and $\pi/4$ -rotated dihedral. The $\pi/4$ -rotated dihedral is utilized to model volume scattering and account for cross-polar HV scattering. However, volume scattering usually arises from the decorrelation of odd-bounce or even-bounce scatterings and is thus often modeled as a mixed scatterer instead of a simple rotated dihedral [4]. Many canonical mixed volume models have been developed hitherto, which have been widely used in model-based target decompositions [5]-[7]. However, a problem arises when these models are used, because Chen' s parameter cannot measure similarity between two mixed scatterers.

A random similarity parameter is proposed in this letter to provide a general similarity measure for any two scatterers. It encompasses both Yang' s and Chen' s

s similarities and proves competent in scattering randomness characterization because it enables a fast alternative and good complement to the widely-used parameter of polarimetric entropy. By parallel combining the similarities between a mixed scatterer and several canonical mixed volume scatterers, a characterization of the mixed scatterer is devised, and comparison with Chen' s approach on ESAR Oberpfaffenhofen data demonstrates its excellent performance.

The rest of this letter is arranged as follows. Section II presents background on polarimetric scattering description. The random similarity and its properties are given in Section III. Section IV uses the parameter to depict scattering randomness by devising a self-similarity parameter and a mirror-similarity parameter. A characterization of scattering mechanism is then presented in Section V and compared with Chen' s approach. Section VI discusses the meaning of the characterization. The letter is concluded in Section VII.

II. POLARIMETRIC SCATTERING DESCRIPTION

The scattering of a single scatterer can be described by the scattering matrix S . We have $SHV = SVH$ in the monostatic backscattering case. For mixed scatterers subjected to spatial or temporal variations, we cannot model their scattering by a fixed S matrix anymore. The coherence matrix T is then formed in terms of the statistical average of all acquired scattering information:

$$T = \langle kk^H \rangle = \begin{bmatrix} T_{11} & T_{12} & T_{13} \\ T_{12}^* & T_{22} & T_{23} \\ T_{13}^* & T_{23}^* & T_{33} \end{bmatrix}$$

where $\langle \cdot \rangle$ is the ensemble average operation and k is the Pauli vector:

$$k = \frac{1}{\sqrt{2}} [S_{HH} + S_{VV}, S_{HH} - S_{VV}, 2S_{HV}]^T$$

The superscripts “T” and “H” in (3) and (2) denote transpose and conjugate transpose operations, respectively. The average in (2) can be removed in the single scatterer case.

III. RANDOM SCATTERING SIMILARITY

For a given mixed scatterer T and a canonical mixed scatterer T_c , their random scattering similarity r_{rr} is defined as:

$$r_{rr}(T, T_c) = \frac{\text{Tr}(TT_c)}{\sqrt{\text{Tr}(T^2)}\sqrt{\text{Tr}(T_c^2)}}$$

where $\text{Tr}(\cdot)$ denotes the matrix trace operation. Based on (4), it is easy to validate that the random similarity r_{rr} satisfies the following properties:

1) **Commutativity and size invariance**

$$r_{rr}(aT, bT_c) = r_{rr}(T, T_c)$$

where a and b are two arbitrary complex numbers.

2) **Unitary transform invariance**

$$r_{rr}(U_r T U_r^H, U_r T_c U_r^H) = r_{rr}(T, T_c)$$

where U_r is an arbitrary $SU(3)$ matrix. This property shows that the similarity is unchanged even if we replace the target coherence matrices with the corresponding target covariance matrices.

3) **Traceless additivity**

If there are three canonical scatterers T_{ci} ($i = 1, 2, 3$) satisfying:

$$\sum_{i=1}^3 T_{ci} = I$$

and $\text{Tr}(T_{ci}) = 1$, where I is the 3×3 identity matrix, then we have:

$$\sum_{i=1}^3 r_{rr}(T, T_{ci}) = \text{Tr}(T)$$

4) **Finite value range**

$$r_{rrl} \leq r_{rr} \leq r_{rru}$$

where r_{rrl} and r_{rru} denote the lower and upper bounds of r_{rr} , respectively, which relate to the eigenvalues λ_i ($\lambda_3 \leq \lambda_2 \leq \lambda_1$) and λ_{ci} ($\lambda_{c3} \leq \lambda_{c2} \leq \lambda_{c1}$) of T and T_c . The upper bound is approached when:

$$T = U \Lambda U^H, \quad T_c = U_c \Lambda_c U_c^H$$

where U and U_c are unitary matrices composed of the eigenvectors u_i and u_{ci} of T and T_c , respectively, and ϕ_i is an arbitrary phase that can be omitted. T and T_c have the same eigenvector matrix in this case.

The lower bound is achieved when:

$$U_c = [u_3, u_2, u_1] \cdot \text{diag}(e^{j\phi_1}, e^{j\phi_2}, e^{j\phi_3})$$

Matrices T and T_c then have the same eigenvectors u_i , but the arrangement of u_i in U_c mirrors that in U —i.e., if $U = [u_1, u_2, u_3]$, then $U_c = [u_3, u_2, u_1]$. We thus name U_c as the mirror of U .

5) Generalized form

The similarity in (4) is valid even if T and T_c have different ranks and is also appropriate for bistatic scattering. It covers both Yang's and Chen's similarity parameters and extends them to the general case.

If T_c is a rank-1 canonical mixed scatterer with $T_c = k_c \cdot k_c^H$, then T_c is actually a single scatterer and (4) can be arranged as:

$$r_{rr}(T, T_c) = \frac{|k_c^H T k_c|}{\sqrt{\text{Tr}(T^2)} \|k_c\|_2^2}$$

where $\|\cdot\|_2$ denotes the 2-norm. Parameter r_{rr} thus changes to Chen's parameter r_r measuring the similarity between a mixed scatterer and a single scatterer [3]. The range in (9) then becomes:

$$0 \leq r_r \leq \frac{\lambda_1}{\sqrt{\sum_{i=1}^3 \lambda_i^2}}$$

This range is more accurate than Chen's range of $[0, 1]$ if scatterer T is a general mixed scatterer.

If scatterer T is also a rank-1 mixed scatterer and $T = k \cdot k^H$, then (4) can be further written as:

$$r_{rr}(T, T_c) = \frac{|k_c^H k|^2}{\|k\|_2^2 \|k_c\|_2^2}$$

This is just Yang's parameter r measuring the similarity of two single scatterers [2]. Then (9) becomes:

$$0 \leq r \leq 1$$

This range is in accordance with Yang's result.

6) Scattering randomness description

Scattering randomness arises from nonstationary target scattering, system noise, and environmental clutter, which makes target scattering decorrelated. By checking the coherence of obtained polarimetric scattering information, a measure of scattering randomness can be achieved. The similarity in (4) is in fact a correlation-like parameter; hence it can be used to measure randomness, as

detailed in the next section. This is precisely why we name it the random scattering similarity.

IV. CHARACTERIZATION OF SCATTERING RANDOMNESS

A mixed scatterer can be concisely characterized by SPAN, randomness, and scattering mechanism. SPAN denotes the total power, which can be directly obtained once target scattering is acquired. The scattering mechanism and randomness indicate what the scatterer is, representing two valuable parameters for target recognition and classification [8]. This section demonstrates that the random similarity r_{rr} can effectively depict randomness, illustrated by relating the self-similarity parameter and mirror-similarity parameter with the polarimetric entropy parameter H , which has proven successful in randomness depiction.

A. Self-Similarity Parameter

The self-similarity parameter r_{rrs} is obtained when $T = cT_c$, where c is an arbitrary complex number:

$$r_{rrs} = \frac{\sum_{i=1}^3 \lambda_i \lambda_{ci}}{\sqrt{\sum_{i=1}^3 \lambda_i^2} \sqrt{\sum_{i=1}^3 \lambda_{ci}^2}}$$

Here r_{rrs} is not always 1 but relates to the eigenvalues, with $r_{rrs} \leq 1$. Matrix T has only one nonzero eigenvalue in the single scatterer case, so r_{rrs} approaches the upper bound. T has three equal eigenvalues in the randomly noisy scatterer case, so the lower bound is attained. For general mixed scatterers, r_{rrs} is between 1/3 and 1. We can thus use r_{rrs} to measure the randomness of target scattering.

To validate this, we provide a simple comparison between r_{rrs} and entropy H , defined as:

$$H = - \sum_{i=1}^3 p_i \log_3 p_i, \quad p_i = \frac{\lambda_i}{\sum_{j=1}^3 \lambda_j}$$

Figs. 1(a) and 1(b) illustrate the obtained H and r_{rrs} on DLR L-band ESAR data of Oberpfaffenhofen. We can observe their inverse relationship, which is further reflected in Fig. 1(d). Good correspondence appears in high- and low- H scatterers, but somewhat poor correspondence arises for scatterers of medium H , owing to the nonlinear logarithmic operation in (18). The right side of (16) formulates another derivation of r_{rrs} independent of eigenvalue decomposition, which makes calculation of r_{rrs} considerably faster than that of H . Within a computer hardware environment of Pentium (R) 8.00 GB memory and 3.20

GHz CPU clock, as well as a software environment of Matlab R2012b, the time consumption for calculating r_{rrs} on the whole Oberpfaffenhofen scene is just 0.0203 s, while calculation of H costs 20.0336 s—about 985 times slower. We therefore consider self-similarity a fast alternative to entropy.

B. Mirror-Similarity Parameter

In the mirror-similarity scenario, $\lambda_{ci} = s\lambda_i$ ($i = 1, 2, 3$), where s is an arbitrary real number. T and T_c then have the same scattering randomness, but their eigenvector matrices are mirroring, as formulated in (11). Hence the Cloude-Pottier average α angles of T and T_c are also “mirroring” —i.e., if T ’s α angle is 0, then this angle of T_c will be $\pi/2$, mirroring about $\pi/4$. T and T_c thus correspond to two “mirroring” scatterings. Such T_c is termed the “mirror target” of T , denoted as T_m . The mirror-similarity r_{rrm} is then defined as:

$$r_{rrm} = r_{rr}(T, T_m) = \frac{\sum_{i=1}^3 \lambda_i \lambda_{3-i+1}}{\sum_{i=1}^3 \lambda_i^2}$$

We can also obtain that $r_{rrm} \leq 1/3$. Contrary to r_{rrs} , one can easily validate that parameter r_{rrm} is 0 for single scatterers, 1/3 for randomly noisy scatterers, and between 0 and 1/3 for other scatterers. Fig. 1(c) shows the obtained r_{rrm} on the scene of Fig. 1. It appears r_{rrm} can provide even better target discrimination than H , as the contrast in Fig. 1(c) looks better than that in Fig. 1(a). Figs. 1(e) and 1(f) further reveal this through histograms of H and r_{rrm} , respectively. The histogram in Fig. 1(f) is much flatter than that in Fig. 1(e), ensuring better contrast. Like anisotropy A , r_{rrm} can improve capability to distinguish different scattering types when H increases and reaches high values. Although named anisotropy, A is not related to spatial anisotropy of scatterers. It has no effective value in the single scatterer case and is zero when $\lambda_2 = \lambda_3$. Based on A , we cannot distinguish scatterings with associated eigenvalue spectra such as $(\lambda_1 = 0.4, \lambda_2 = 0.3, \lambda_3 = 0.3)$ and $(\lambda_1 = \lambda_2 = \lambda_3 = 1/3)$. It can be validated that r_{rrm} enables good discrimination of scattering when $p_2 = p_3$ and is always effective under different scattering processes. Hence we consider r_{rrm} a competent complement to entropy H for distinguishing different types of target scatterings.

V. CHARACTERIZATION OF SCATTERING MECHANISM

This section applies random similarity to characterizing the scattering mechanism of mixed scatterer T .

A simple scheme can be achieved by checking similarities between T and canonical scatterers and identifying the most similar canonical scattering as its scattering mechanism. This scheme begins with the assumption that T always has

a dominant scattering mechanism. It thus works well only for low-entropy scatterers but fails for medium-entropy and high-entropy scatterers, as these may have several comparably similar canonical scatterings without any significantly stronger one.

A. Chen' s Approach

A similarity-based scattering characterization scheme was developed by Chen et al. based on a combination strategy [3], which treats mixed scattering T as the combination of canonical surface scattering T_{cs} , dihedral scattering T_{cd} , and volume scattering T_{cv1} . T_{cs} , T_{cd} , and T_{cv1} are listed in Table I; they satisfy (7), so according to the third property we have:

$$\text{Tr}(T) = r_{rrcs} + r_{rrcd} + r_{rrcv1}$$

where r_{rrcs} , r_{rrcd} , and r_{rrcv1} are the similarities between T and T_{cs} , T_{cd} , and T_{cv1} , respectively, as formulated in Table I. The combination of r_{rrcs} , r_{rrcd} , and r_{rrcv1} may thus provide good characterization of T . Based on this, Chen et al. constructed a pseudo-color map of mixed scattering by:

$$\text{Red} : r_{rrcs}, \quad \text{Green} : r_{rrcd}, \quad \text{Blue} : r_{rrcv1}$$

Chen' s approach is used for characterization of ESAR Oberpfaffenhofen data below. Fig. 2(a) shows the SPAN image of the data, depicting a scene around Oberpfaffenhofen Special Airport and covering several typical scatterers such as forest, buildings, farmland, bare land, runway, and parking apron.

It should be noted that real PolSAR data generally has a very large dynamic range and should be adjusted for clear display. A commonly-adopted method replaces SPAN with its normalized logarithmic value. The SPAN image of Oberpfaffenhofen in Fig. 2(a) exemplifies the necessity of this operation: nothing is visible without adjustment. Figs. 3(a) to 3(c) illustrate the obtained r_{rrcs} , r_{rrcd} , and r_{rrcv1} . Bare land and airport areas have the strongest surface scattering, while buildings exhibit the strongest dihedral scattering. These are consistent with ground truth. The strongest HV scattering appears in forest and some building areas; nevertheless, the surface and dihedral scatterings in these areas are much stronger. Such areas should possess stronger volume scattering instead. Therefore, using HV scattering to account for volume scattering is not always appropriate. Fig. 2(b) exhibits the final combination of r_{rrcd} , r_{rrcv1} , and r_{rrcs} . It can characterize the scattering mechanism of the scene well but cannot provide satisfactory discrimination of different typical scatterers. Particularly in the airport area, we cannot clearly distinguish the runway from the parking apron.

B. Proposed Approach

Our characterization begins with canonical modeling of volume scattering, which relates to HV scattering and is usually created by both even-bounce structures and odd-bounce objects. The random orientation of even-bounce structures such as building-ground and tree trunk-ground results in HV scattering. T_{cv1} used by Chen et al. models a special case when orientation is fixed to $\pi/4$. This model only fits low-randomness volume scatterers because the $\pi/4$ -rotated dihedral indicates a single scatterer. For complex random scattering cases, mixed modeling of volume scattering is necessary.

Many canonical mixed volume models have been devised and used in model-based decompositions [5]–[7]. This letter focuses on four of them, T_{cvi} ($i = 2 \sim 5$), as listed in Table I. We use T_{cv2} to model a cloud of $\pi/4$ -rotated dihedral scatterers with additional random orientation angles subject to cosine distribution. As an extension of T_{cv1} , T_{cv2} is competent to model highly random volume scattering resulting from even-bounce structures. T_{cv3} , T_{cv4} , and T_{cv5} are dedicated to odd-bounce-induced volume scatterings. T_{cv3} was first used by Freeman and Durden to model volume scattering from a cloud of dipole scatterers with uniform orientation angle distribution [5]. T_{cv4} and T_{cv5} were developed by Yamaguchi et al. for dipole scatterers with dominant horizontal or vertical structures and cosine orientation distribution [6].

Table I formulates the similarity r_{rrcvi} between T_{cvi} and T . The obtained r_{rrcvi} ($i = 2 \sim 5$) for Oberpfaffenhofen data are given in Figs. 3(d) to 3(g). Buildings and forest have the strongest r_{rrcv2} due to concrete building-ground and trunk-ground structures. Acting as the combination of r_{rrcd} and r_{rrcv1} , r_{rrcv2} can depict both dihedral scattering and even-bounce-induced volume scattering. r_{rrcv3} is consistent with r_{rrcs} in characterizing surface scattering. It is competent to depict odd-bounce-induced volume scattering with no orientation preference and is strongest in bare land and airport areas. The difference between r_{rrcv4} and r_{rrcv5} lies in the real part of T_{12} , i.e., $\text{Re}(T_{12})$. By combining (2) and (3), we obtain that r_{rrcv4} and r_{rrcv5} thus characterize scatterers with dominant HH or dominant VV scatterings, respectively.

Based on these two parameters, the runway is clearly discriminated from the parking apron because the runway has stronger HH scattering while the parking apron has stronger VV scattering, and different farmlands are also discriminated. The forest is found to possess larger r_{rrcv4} than r_{rrcv5} , indicating dominant horizontal branches. The sum of r_{rrcv4} and r_{rrcv5} is further found to be a good alternative to r_{rrcv3} . Fig. 3(h) exhibits the average of r_{rrcv4} and r_{rrcv5} , i.e., $(r_{rrcv4} + r_{rrcv5})/2$, which looks nearly identical to r_{rrcv3} . Fig. 3(i) illustrates the relationship between r_{rrcv3} and $(r_{rrcv4} + r_{rrcv5})/2$, and the nearly linear relationship supports their interchangeability. The orientation preference is thus eliminated when r_{rrcv4} and r_{rrcv5} are averaged.

Parameters r_{rrcvi} ($i = 2 \sim 5$) provide four perspectives on mixed scatterers. To achieve integrated characterization, their combination is necessary. Since

$(r_{rrcv4} + r_{rrcv5})/2$ can well replace r_{rrcv3} , we propose constructing a pseudo-color map of mixed scattering based on r_{rrcv2} , r_{rrcv4} , and r_{rrcv5} :

$$\text{Red} : r_{rrcv2}, \quad \text{Green} : r_{rrcv4}, \quad \text{Blue} : r_{rrcv5}$$

We further find that:

$$r_{rrcv2} + r_{rrcv4} + r_{rrcv5} = \text{Tr}(T)$$

because T_{cv2} , T_{cv4} , and T_{cv5} satisfy (7). Theoretically, this indicates that the combination of r_{rrcv2} , r_{rrcv4} , and r_{rrcv5} in (24) can achieve good coverage of the hue circle, thus guaranteeing good characterization. To demonstrate this, Fig. 4(a) illustrates the updated characterization of Oberpfaffenhofen, which clearly shows improvement compared with Chen's result in Fig. 2(b). The proposed characterization is more colorful and has better discrimination of radar targets. Particularly in the airport area, the runway in Fig. 4(a) is clearly discriminated from the surrounding parking apron because T_{cv4} and T_{cv5} are more sensitive to HH and VV scatterings than T_{cv1} .

VI. DISCUSSION

Both model-based decompositions and similarity-based characterization depict mixed target scatterings in terms of combinations of surface, dihedral, and volume scatterings. Their physical significance is thus clear because these canonical scatterings correspond to general scenes of land/ocean, urban areas, and vegetation. The canonical scatterings are serial in model-based decompositions because their sum should compose the mixed scattering. Balance equations result, and solving these yields the power of each scattering mechanism. However, the canonical scatterings in similarity-based characterization are independent of one another, providing a parallel model-based depiction of scattering from different viewpoints. Instead of balance equations, the contribution of scattering mechanisms is determined by simple similarity measurement. This permits flexible scattering characterization and allows us to try different types of canonical scattering models. We can therefore use T_{cv2} as an extension of T_{cd} to depict dihedral scattering and use T_{cv3} as a good extension of T_{cs} in depicting surface scattering. Volume scattering then covers surface and dihedral scatterings and extends them to random scattering cases. Hence, the volume scattering models in the proposed scheme are not competitive (i.e., if one is used, the others are ruled out) but cooperative. Together they provide good characterization of mixed scattering.

The proposed scheme uses similarity as the contribution of each scattering mechanism instead of power. The first property in (5) then tells us that the combination of r_{rrcv2} , r_{rrcv4} , and r_{rrcv5} in (25) cannot cover SPAN information, which depicts geometrical textural features. Better characterization is attained if we

can fuse SPAN and scattering similarities into a whole. In fact, this can be easily achieved by revising the pseudo-color map of mixed scattering in (24) as:

$$\text{Red} : \text{SPAN} \cdot r_{rrcv2}, \quad \text{Green} : \text{SPAN} \cdot r_{rrcv4}, \quad \text{Blue} : \text{SPAN} \cdot r_{rrcv5}$$

The final fusion is shown in Fig. 4(b). Both textural and scattering information can be directly exhibited. Excellent characterization of complex radar targets is therefore achieved.

VII. CONCLUSION

The random scattering similarity is a general similarity measure for any two scatterers and is also valid for bistatic scattering. It possesses properties of commutativity, size and unitary invariance, traceless additivity, finite range, and generalized form. Scattering randomness depiction and scattering mechanism characterization demonstrate its applications. Self-similarity is a fast alternative to entropy; mirror-similarity is a competent complement to entropy. The similarity-based characterization scheme provides a compromise between algebraic depiction and model-based decompositions. The canonical scatterings in it are independent of one another. Together they provide a parallel model-based characterization of target scattering. The contribution of each scattering mechanism is determined by algebraic similarity measurement.

REFERENCES

- [1] S. R. Cloude and E. Pottier, "A review of target decomposition theorems in radar polarimetry," *IEEE Trans. Geosci. Remote Sens.*, vol. 34, no. 2, pp. 498–518, May 1996.
- [2] J. Yang, Y.-N. Peng, and S.-M. Lin, "Similarity between two scattering matrices," *Electron. Lett.*, vol. 37, no. 3, pp. 193–194, Feb. 2001.
- [3] Q. Chen, Y.-M. Jiang, L.-J. Zhao, and G.-Y. Kuang, "Polarimetric scattering similarity between a random scatterer and a canonical scatterer," *IEEE Geosci. Remote Sens. Lett.*, vol. 7, no. 4, pp. 866–869, Oct. 2010.
- [4] S. R. Cloude, *Polarisation Applications in Remote Sensing*. Oxford, U.K.: Oxford Univ. Press, 2010.
- [5] A. Freeman and S. L. Durden, "A three-component scattering model for polarimetric SAR data," *IEEE Trans. Geosci. Remote Sens.*, vol. 36, no. 3, pp. 963–973, May 1998.
- [6] Y. Yamaguchi, T. Moriyama, M. Ishido, and H. Yamada, "Four-component scattering model for polarimetric SAR image decomposition," *IEEE Trans. Geosci. Remote Sens.*, vol. 43, no. 8, pp. 1699–1706, Aug. 2005.

[7] M. Arii, J. J. van Zyl, and Y. Kim, “Adaptive model-based decomposition of polarimetric SAR covariance matrices,” *IEEE Trans. Geosci. Remote Sens.*, vol. 49, no. 3, pp. 1104–1113, Mar. 2011.

[8] S. R. Cloude and E. Pottier, “An entropy based classification scheme for land applications of polarimetric SAR,” *IEEE Trans. Geosci. Remote Sens.*, vol. 35, no. 1, pp. 68–78, Jan. 1997.

Note: Figure translations are in progress. See original paper for figures.

Source: ChinaXiv –Machine translation. Verify with original.