Unified Huynen Phenomenological Decomposition of Radar Targets and Its Classification Applications (Postprint)

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Abstract

Huynen decomposition (HD) as the first formalized target decomposition has not been widely accepted. The preference for symmetry and regularity restricts not only its application but also its unification with other target dichotomies. The nonuniqueness issue then arises because we may have different dichotomies of radar targets, but we have no idea on how to select them. In this paper, a unified Huynen dichotomy is developed by extending HD for a full preference for symmetry and regularity, nonsymmetry, irregularity, and their couplings. It covers all of the existing dichotomies and provides a unified selection mechanism for them. Scattering preference is identified as a main feature of target dichotomy, and its concise description is devised by relating each dichotomy to a canonical scattering. A scattering degree of preference (SDoP) parameter is defined to measure the preference of each dichotomy. In virtue of an adaptive combination and permutation of SDoPs, a scattering pyramid description of the mixed scattering is developed, which has better discrimination of target than entropy/alpha. An SDoP/alpha classification is further proposed by statistical modeling of the unified dichotomy, which is a competent alternative to entropy/alpha. The excellent performance of unified dichotomy makes us believe that the existing concerns on HD are well treated and the Huynen-Cloude controversy, in a sense, may be ended. 2015 IEEE.

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Preamble

Unified Huynen Phenomenological Decomposition of Radar Targets and Its Classification Applications

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ABSTRACT

Huynen decomposition, as the first formalized target decomposition, has not been widely accepted. Its preference for symmetry and regularity restricts not only its application but also its unification with other target dichotomies. The non-uniqueness issue then arises because we may have different dichotomies of radar targets, but we have no idea how to select among them. In this paper, a unified Huynen dichotomy is developed by extending Huynen decomposition to fully accommodate preferences for symmetry and regularity, non-symmetry, irregularity, as well as their couplings. It covers all existing dichotomies and provides a unified selection mechanism for them. Scattering preference is identified as a main feature of target dichotomy, and its concise description is devised by relating each dichotomy to a canonical scattering. A scattering degree of preference (SDoP) parameter is defined to measure the preference of each dichotomy. Through an adaptive combination and permutation of SDoPs, a scattering pyramid description of mixed scattering is devised, which provides better target discrimination than entropy/alpha. An SDoP/alpha classification is further obtained by statistical modeling of the unified dichotomy, which proves to be a competent alternative to entropy/alpha.

The excellence of the unified dichotomy leads us to believe that existing concerns regarding Huynen decomposition have been well addressed and that the Huynen-Cloude controversy may, in a sense, be resolved.

Key Words: Huynen decomposition, radar polarimetry, target extraction, target decomposition, unsupervised classification.

NOMENCLATURE AND ABBREVIATIONS

- U_ϕ, U_τ, and U_α: Unitary matrices accounting for antenna roll transform, ellipticity transform, and absolute phase transform.
- λ_j and u_j: The jth ($j = 1, 2, 3$) eigenvalue and eigenvector of mixed target T.
- T_{Scd} and k_{Scd}: Dominant single target and its Pauli vector extracted by CD from T.
- T_{Si} and T_{Ni}: Equivalent single target and remnant N-target extracted by D_i from T.
- $SDoP_3$ and $SDoP_9$: Average SDoP parameters of CHD and UHD.
- $SDoP_s$, $SDoP_d$, and $SDoP_v$: The SDoP for canonical surface, dihedral, and volume scatterers.
- α_{cp} and α_{lz}: Cloude-Pottier alpha angle and the proposed Li-Zhang alpha angle.
- H/α_{cp} and $SDoP_9/\alpha_{lz}$: Entropy/alpha classification and the devised SDoP/alpha classification.

I. INTRODUCTION

The concept of target decomposition was first formalized by Huynen in 1970. In his Ph.D. dissertation on "Phenomenological Theory of Radar Targets" [?], Huynen not only demonstrated that radar targets can be decomposed like waves, but also indicated that polarimetric decomposition is a feasible method for understanding complex targets. Pioneered by this work, numerous other decomposition techniques have been developed, and intense attention has been paid to this field over the past four decades. Comprehensive reviews of existing decompositions were presented in [?]-[?].

Despite its theoretical importance, Huynen decomposition (HD) has not received wide attention and application. It is considered one of Huynen' s main visionary concepts that is not widely accepted today [?], [?]. HD is often mentioned in the literature because it was the first decomposition or because it represents a typical target dichotomy. However, only a minority of reviews provide a particular introduction to or focus on HD [?]-[?], [?]-[?]. Three main factors impacting the application of HD can be summarized here.

The first factor arises from the findings of Barnes and Holm that HD is not unique because there are two other dichotomies that possess the same roll-invariance around the line of radar sight [?], [?]. Barnes-Holm decomposition (BHD) relaxes Huynen' s preference for symmetry & regularity (SR) [?], but was criticized for providing little insight into the physics of scattering because it creates two "exotic" worlds with imbalanced preferences for left or right helices [?], [?]. The second factor stems from the Huynen-Cloude controversy in 1992, as simply depicted in [?] and [?]. Cloude pointed out that HD cannot provide "global" invariance—that is, invariance under all unitary transforms—and mathematically there are infinite target dichotomies if Huynen' s restriction on roll-invariance is removed. However, the eigenvector-based Cloude decomposition (CD) can ensure satisfactory results [?], [?], [?], [?]. He therefore concluded that there is no target dichotomy but only one unique decomposition, namely CD [?]. Huynen argued that the physical significance of CD is lost in the conduct of eigendecomposition, and the related parameter proliferation problem makes it very "dubious," whereas HD has clear physical significance because it caters to the world of basic symmetry in which we live [?], [?]. He concluded that the only decomposition corresponding to the real world of symmetry is HD [?]. Huynen' s justification for HD received support from Pottier [?] and Holm [?], [?]. However, Cloude' s concerns about HD gained more followers [?], [?], [?]-[?], which have greatly impacted the use of HD [?], [?], [?]-[?]. The last factor was contributed by Yang et al., who found that HD cannot extract a desired target because it is not always stable [?], [?], and thus proposed a modification to HD. However, the first two factors regarding HD are so prominent that Yang decomposition (YD) received only limited notice [?], [?].

Huynen' s concerns about CD have been properly addressed: the success of entropy/alpha classification reveals its physical significance [?], and the lossless and sufficient roll-invariant decomposition solves the parameter proliferation problem [?]. However, concerns about HD, mainly regarding the non-uniqueness issue, have not been handled thus far. This issue manifested itself in two forms in the existing literature. First, there may mathematically exist infinite ways to decompose a mixed target into the sum of a single target and an N-target, but the preference for SR reduces them to HD only. We do not believe such non-uniqueness of HD warrants particular attention, because it also exists in other decompositions. Take CD, for example: as indicated by Cloude et al. [?], there are infinite ways to decompose a mixed target into the sum of three single targets, but orthogonality restricts this infinity to only Cloude' s solution. In this sense, the uniqueness controversy between Huynen and Cloude is trivial because they held different views on how to treat the same problem. Second, since BHD and YD can also conduct target dichotomy like HD, they together provide three different points of view: Huynen' s preference for SR, Barnes-Holm' s concentration on roll-invariance, and Yang' s concerns about stability. The dichotomy realities of these three views are all reasonable, but a common consensus has not been reached. We thus have no idea how to select among them, especially between BHD and YD. Such non-uniqueness is the underlying reason restricting the application of Huynen-type target dichotomies (HTD). A good way to solve this is to generalize these dichotomies to a unified Huynen dichotomy (UHD) and devise a fair mechanism to select the most appropriate one, which is detailed in Section II.

Regarding the concern that HD cannot provide "global" invariance, we find it is not always necessary. This is a specific attribute of CD, based on which Cloude and Pottier devised the entropy/alpha classification in response to Huynen' s concern about the physical significance of CD. Clearly, HTD only enables "local" invariance—that is, invariance under a certain unitary transform. Nevertheless, such local invariance gives each HTD a specific attribute: scattering preference, a term we use to depict the observation that certain target scattering information is preserved in the extracted single target by each HTD. As for HD, Huynen qualitatively explained its preference using three phenomenological characters: SR, non-symmetry (NS), and irregularity (IR). Although having practical relevance, it is unclear how useful this description is for detailed interpretation of the data [?]. We also find the information this description conveys to be limited.

A novel description of scattering preference is thus developed in Section III by directly relating each HTD to a canonical scattering. By this means, we can see that HTD are born with physical significance.

An excellent decomposition should not only be able to extract the single target, but also be able to describe the mixed target, which can no longer be represented by a dominant single target. It was stated in [?] and [?] that the performance of HD deteriorates as entropy increases—that is, it cannot be used to analyze random mixed targets as often occur in natural scenes. This in fact also exists in CD, but was successfully solved by Cloude and Pottier using the concept of average target. To advance the application of UHD, a scattering pyramid scheme is devised in Section V by adaptively permutating and combining several HTD of different preferences to describe the scattering of mixed targets. Comparative experiments with entropy/alpha overturn the general impression that HTD is inferior to CD.

Another task of this paper is to investigate the potential consistency between HTD and CD, which is conducted in two aspects. The first arises in Section IV on the extraction of dominant single targets. A mathematical convergence to CD is clearly displayed by extending HD to the canonical Huynen dichotomy (CHD, which provides a simplification to YD, as indicated in the Appendix), and by further extending BHD and CHD to UHD. UHD can obtain extractions very consistent with CD. The second arises in Section VI by using UHD to further model mixed target scattering based on the average target concept of Cloude-Pottier, which enables a Huynen scattering degree of preference (SDoP)/alpha classification and is demonstrated to be a competent alternative to entropy/alpha. The effects of target orientation and speckle filtering on the classifications are also evaluated in Subsections V.B and VI.C, and Section VII.

HD was once treated as a decomposition procedure without much practical value. This paper is dedicated to unifying the existing HTD and applying them to advanced classification applications. Our work leads us to believe that concerns about HD have been well addressed, that the performance of HTD has been well established, and that the Huynen-Cloude controversy may be resolved. The paper is concluded in Section VIII.

II. THE UNIFICATION OF HTD

A. The Canonical Huynen Dichotomy (CHD)

HD is built on the physical realizability conditions of the Kennaugh matrix K, and these conditions can be concisely expressed by the target coherence matrix T. If the scattering of a target through a reciprocal propagation medium is measured by a monostatic radar, then the matrix T can be expressed as:

$$T = \langle kk^H \rangle = \begin{bmatrix} 2A_0 & C - jD & H + jG \\ C + jD & B_0 + B & E + jF \\ H - jG & E - jF & B_0 - B \end{bmatrix}$$

where $\langle\cdot\rangle$ denotes ensemble average, k is the Pauli vector, and superscript H indicates conjugate transpose. The parameters on the right side are the Huynen parameters because Huynen bridged them to the phenomenological characters of radar targets [**?**]. As shown in [Figure 1: see original paper], $2A_0$, $B_0 + B$, and $B_0 - B$ are the generators of SR, IR, and NS, respectively, while the others denote the pairwise couplings between these generators.

For a mixed target subjected to spatial and/or temporal variations, the matrix T is positive semidefinite, so the three second-order principal minors should be nonnegative:

$$\begin{cases}(2A_0)(B_0+B)-(C^2+D^2)\geq 0\\(2A_0)(B_0-B)-(H^2+G^2)\geq 0\\(B_0+B)(B_0-B)-(E^2+F^2)\geq 0\end{cases}\tag{2}$$

For a mixed target, rank(T) = 3. Matrix T becomes rank-one for a single target. Nine equations are then obtained, but only four of them are independent because a single target has five degrees of freedom. We can thus extract three equation groups:

$$\begin{cases}(2A_0)(B_0+B)=C^2+D^2\\(2A_0)(B_0-B)=H^2+G^2\\(B_0+B)(B_0-B)=E^2+F^2\end{cases}\tag{3}$$

[Figure 1: see original paper] illustrates the phenomenological significance of the nine Huynen parameters. Each group in (3) is self-contained, and from them we obtain the following three equations, respectively:

$$\begin{cases}2A_0=\frac{C^2+D^2}{B_0+B}\\2A_0=\frac{H^2+G^2}{B_0-B}\\B_0+B=\frac{E^2+F^2}{B_0-B}\end{cases}\tag{4}$$

For a single target, rank(T) = 1. Equations (2-1) and (4-1) reveal the different behaviors of parameters (B_0, B, E, F) in the two target scenarios, which inspired Huynen to use a target dichotomy to decompose the mixed target T as the sum of an equivalent single target T_S and a remnant N-target T_N, by analogy to wave dichotomy:

$$T=T_S+T_N=\begin{bmatrix}2A_{0S}&C-jD&H+jG\\C+jD&B_{0S}+B_S&E_S+jF_S\\H-jG&E_S-jF_S&B_{0S}-B_S\end{bmatrix}+\begin{bmatrix}2A_{0N}&0&0\\0&B_{0N}+B_N&E_N+jF_N\\0&E_N-jF_N&B_{0N}-B_N\end{bmatrix}\tag{5}$$

The decomposed parameters (B_{0S}, B_S, E_S, F_S) are inverted from (3-1) based on parameters $(2A_0, C, D, G, H)$, which are completely reserved in T_S because Huynen insisted that the real world physically prefers SR and that related scattering information should not appear in T_N [?]. This worldview restricts the applicability of HD to ideal SR scatterers only—that is, HD fails to analyze complex IR and NS scatterers. This has been independently validated by Barnes and Holm [?], Yang et al. [?], and Paladini [?] on the decomposition of a distributed dihedral scatterer. In response, Huynen indicated that our attention should turn from T_S to T_N in these cases [?]. However, HD decomposes the IR- and NS-related parameters (B_0, B, E, F) into both T_N and T_S. Attention to either T_S or T_N is inappropriate for integrated characterization of such radar targets.

Hence, our first attempt is to extend HD to preferences for IR and NS. This can be simply achieved because we still have two mixed target inequations in (2) with their corresponding single target equations appearing in (4). As for parameters $(2A_0, B_0 - B, H, G)$, if we define:

$$\begin{cases} \tilde{B}_0 + \tilde{B} = B_0 - B \\ \tilde{C} = H, \quad \tilde{D} = -G \\ \tilde{E} = E, \quad \tilde{F} = F \end{cases} \tag{6}$$

this enables us to write (2-2) and (4-2) into forms similar to (2-1) and (4-1), respectively:

$$\begin{cases} (2A_0)(\tilde{B}_0 + \tilde{B}) \geq \tilde{C}^2 + \tilde{D}^2 & \text{for mixed target} \\ (2A_0)(\tilde{B}_0 + \tilde{B}) = \tilde{C}^2 + \tilde{D}^2 & \text{for single target} \end{cases} \tag{7}$$

Analogous to HD, we then have:

$$T = T'_S + T'_N = \begin{bmatrix} 2A_{0S} & H + jG & C - jD \\ H - jG & B_{0S} - B_S & E_S + jF_S \\ C + jD & E_S - jF_S & B_{0S} + B_S \end{bmatrix} + \begin{bmatrix} 2A_{0N} & 0 & 0 \\ 0 & B_{0N} - B_N & E_N + jF_N \\ 0 & E_N - jF_N & B_{0N} + B_N \end{bmatrix} \tag{8}$$

Dichotomy (8) prefers IR because it reserves the IR-related Huynen parameters $(B_0 + B, C, D, E, F)$ in the single target matrix. Based on the reserved parameters, the decomposed parameters $(2A_{0S}, B_{0S} - B_S, G_S, H_S)$ can be easily retrieved from (3-2). Likewise, as for parameters $(2A_0, B_0 + B, C, D)$, if we define:

$$\begin{cases} \tilde{B}_0 - \tilde{B} = B_0 + B \\ \tilde{G} = C, \quad \tilde{H} = D \\ \tilde{E} = E, \quad \tilde{F} = F \end{cases} \tag{9}$$

from (2-3) and (4-3) we then have:

$$\begin{cases} (2A_0)(\tilde{B}_0 - \tilde{B}) \geq \tilde{G}^2 + \tilde{H}^2 & \text{for mixed target} \\ (2A_0)(\tilde{B}_0 - \tilde{B}) = \tilde{G}^2 + \tilde{H}^2 & \text{for single target} \end{cases} \tag{10}$$

Hence we can also obtain the following target dichotomy:

$$T = T_S'' + T_N'' = \begin{bmatrix} 2A_{0S} & C-jD & H+jG \\ C+jD & B_{0S}+B_S & 0 \\ H-jG & 0 & B_{0S}-B_S \end{bmatrix} + \begin{bmatrix} 2A_{0N} & 0 & 0 \\ 0 & B_{0N}+B_N & E_N+jF_N \\ 0 & E_N-jF_N & B_{0N}-B_N \end{bmatrix} \tag{11}$$

Dichotomy (11) prefers NS because it reserves the NS-related parameters ($B_0 - B, E, F, G, H$) in the single target coherence matrix, and the decomposed parameters ($2A_{0S}, B_{0S} + B_S, C_S, D_S$) can be obtained from (3-3) based on the preserved parameters.

Dichotomies (8) and (11) provide two useful supplements to HD, and together they offer three dichotomies preferring SR, IR, and NS, respectively. To unify them, a fair mechanism should be devised so that each has a chance to be selected. Our strategy is based on the total power (SPAN) of the extracted single target: use the three dichotomies to extract T_S, T_S', and T_S'' from T, and select the one with maximum SPAN as the final extraction. We name this decomposition the canonical Huynen dichotomy (CHD) because Huynen claimed the existence of T_N' and T_N''. They were termed as type I and type II symmetrical canonical targets but were abandoned for lacking the natural appeal of T_N [**?**].

Yang et al. indicated that HD cannot stably extract a desired single target when the $2A_0$ parameter of matrix K is small or even zero [**?**]. They proposed a modified HD based on the K matrix by constructing two simple transforms of matrix K for two new matrices K_1 and K_2, then identified K, K_1, or K_2 whose $2A_0$ parameter is maximum and used HD to decompose it for the final dichotomy. YD greatly improves the stability of HD but has not attracted much attention because its physical significance is somewhat unclear. Since $2A_0$ is the generator of SR, the instability of HD in the small $2A_0$ case is simply because HD is not applicable to SR-subordinate targets. As formulated in the Appendix, the $2A_0$ parameters of K_1 and K_2 are $B_0 + B$ and $B_0 - B$, which are respectively the generators of IR and NS. Thus, the adaptive selection in YD is to judge the dominant character of the target as SR, IR, or NS. It is also shown in the Appendix that the K_1- and K_2-related decompositions are just dichotomies (8) and (11), except that CHD identifies the final dichotomy based on SPAN instead of $2A_0$. CHD thus simplifies YD and reveals its physical significance from a different starting point.

B. The Unified Huynen Dichotomy (UHD)

Besides the physical meaning, Barnes and Holm found that the N-target T_N in (5) is mathematically invariant to the antenna roll transform U_ϕ [**?**], [**?**]. Given vector q belonging to the null space of T_N, we can have:

$$U_\phi T_N U_\phi^H q = 0 \quad \text{with} \quad U_\phi = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 2\phi & \sin 2\phi \\ 0 & -\sin 2\phi & \cos 2\phi \end{bmatrix} \tag{12}$$

The Pauli vector k_S of the equivalent single target T_S can be reconstructed by [**?**]:

$$k_S \propto Tq \tag{13}$$

Equation (12) indicates that q is the eigenvector of U_ϕ, as listed in Table I. Thus we can obtain three estimations of k_S under the roll-invariance of T_N [**?**]:

$$\begin{cases} k_{S1} \propto [2A_0, C - jD, H + jG]^T \\ k_{S2} \propto [C + jD, B_0 + B, E + jF]^T \\ k_{S3} \propto [H - jG, E - jF, B_0 - B]^T \end{cases} \tag{14}$$

where k_{S1} corresponds to the single target extracted by HD, and the dichotomies corresponding to k_{S2} and k_{S3} are respectively BHD I & II.

The roll-invariance condition in BHD relaxes the SR preference of HD and indicates two other dichotomies. The preference of BHD can likewise be inferred. The introduction of q_2 and q_3 (from Table I) into $Tq = T_S q$ of (13) shows the reservation of combinations of the 2nd and 3rd columns of T, which reveals BHD' s preference for the coupling of IR and NS because these two columns physically relate to IR and NS, respectively, as illustrated in [Figure 1: see original paper]. This is necessary for describing helix-like scatterers. Likewise, we can also have the coupling of SR and IR, as well as that of SR and NS. Our second attempt is thus to unify CHD and BHD by generalizing CHD to preferences for the pairwise couplings of SR, IR, and NS. This can be easily achieved because T_N' and T_N'' are found invariant to the following two unitary transforms, respectively:

$$U_\tau = \begin{bmatrix} \cos 2\tau & \sin 2\tau & 0 \\ -\sin 2\tau & \cos 2\tau & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad U_\alpha = \begin{bmatrix} \cos 2\alpha & 0 & \sin 2\alpha \\ 0 & 1 & 0 \\ -\sin 2\alpha & 0 & \cos 2\alpha \end{bmatrix} \tag{15}$$

where U_τ refers to the ellipticity transform, such as when replacing a linearly polarized antenna with a circularly polarized antenna, and U_α corresponds to the absolute phase transform resulting from slant range variation or change of antenna phase center. From the invariance of T_N', we obtain:

$$U_\tau T_N' U_\tau^H q = 0 \tag{16}$$

which signifies that q is the eigenvector of U_τ. Analogous to BHD, we obtain the dichotomies in (17), where the vector k_{S4} corresponds to dichotomy (8). Likewise, the invariance of T''_N indicates that q is also the eigenvector of U_α. Hence we further obtain (18), where the vector k_{S7} corresponds to dichotomy (11).

$$\begin{cases} k_{S4} \propto [C + jD, B_0 + B, E + jF]^T \\ k_{S5} \propto [2A_0 + C + jD, B_0 + B + E + jF, H + jG + E + jF]^T \\ k_{S6} \propto [2A_0 - C - jD, B_0 + B - E - jF, H + jG - E - jF]^T \end{cases} \tag{17}$$

$$\begin{cases} k_{S7} \propto [H - jG, E - jF, B_0 - B]^T \\ k_{S8} \propto [2A_0 + H - jG, C - jD + E - jF, B_0 + B + E - jF]^T \\ k_{S9} \propto [2A_0 - H + jG, C - jD - E + jF, B_0 + B - E + jF]^T \end{cases} \tag{18}$$

By combining (14), (17), and (18), we finally have nine dichotomies based on the invariance of N-targets of CHD. For convenience, we denote the dichotomy corresponding to vector k_{Si} as D_i ($i = 1, 2, \dots, 9$). Using the analysis method applied to BHD, the preferences of D_5 and D_6 for the coupling of SR and NS, as well as the preferences of D_8 and D_9 for the coupling of SR and IR, can likewise be inferred. To unify the nine dichotomies, we adopt an adaptive strategy to select the dichotomy among the nine whose single target has the maximum SPAN. We call this decomposition the unified Huynen dichotomy (UHD) because it covers all existing HTD such as HD (D_1), BHD (D_2 and D_3), and YD (D_4 and D_7), and provides an adaptive dichotomy for all mixed radar targets dominated by SR, IR, NS, or their couplings.

III. SCATTERING PREFERENCE ANALYSIS OF UHD

The nine sub-dichotomies in UHD can be generally expressed as:

$$T = T_{Si} + T_{Ni}, \quad i = 1, 2, \dots, 9 \tag{19}$$

where T_{Si} denotes the single target extracted by D_i with Pauli vector k_{Si}, q_i is the eigenvector of U_ϕ, U_τ, and U_α, as summarized in Table I, and it satisfies:

$$Tq_i = T_{Si}q_i = \mathrm{SPAN}_{Si}q_i \tag{20}$$

The significance of (20) is underlined because it mathematically shows an important characteristic of HTD: D_i intactly reserves certain target information into T_{Si}. We term this reservation scattering preference, and following Huynen' s convention, we qualitatively describe it in terms of the phenomenological characters SR, IR, and NS, as listed in Table I. Although having practical relevance, it is unclear how useful this description is for detailed data interpretation [?].

We also find the information this description conveys to be limited. Taking the two BHDs as examples, their preferences are expressed as the two couplings of IR and NS following this description, but we cannot further distinguish their difference. To advance the application of UHD, a better description of scattering preference information is necessary.

In fact, (20) can provide further information if we transform it into the following form:

$$q_i^H T q_i = \mathrm{SPAN}_{Si} \tag{21}$$

If q_i is treated as a Pauli vector corresponding to canonical scatterers, such as surface, helix, dihedral, dipole, and volume scatterer, as listed in Table I, then the left side of (21) indicates the scattering similarity between q_i and the random scatterer T [?], and the right side shows the similarity between q_i and the deterministic scatterer k_{Si} [?]. The scattering similarity with q_i is thus reserved by D_i, and we infer that D_i naturally prefers scattering q_i. This description directly bridges D_i to a canonical scattering, from which we immediately attach the preferences of D_2 and D_3 to left- and right-wound helices, respectively—consistent with Huynen' s comments on BHD from the viewpoint of power received by an antenna [?]. It should be noted that the preferred scattering q_i is merely a physical description of the mathematical preservation by D_i; it cannot replace k_{Si} to determine the scattering of the extracted single target. As formulated in (19), only when q_i is the eigenvector of T do q_i and k_{Si} correspond to the same scattering. Hence, although q_5, q_6, q_8, and q_9 correspond to different rotated dipoles, it does not mean that k_{S5}, k_{S6}, k_{S8}, and k_{S9} are also canonical dipoles if T is deoriented beforehand.

In addition to the qualitative description in terms of canonical target scattering, we further define a scattering degree of preference (SDoP) parameter to quantitatively evaluate the preference degree of D_i:

$$\mathrm{SDoP}_{D_i} = \frac{\mathrm{SPAN}_{Si}}{\mathrm{SPAN}}, \quad i = 1, 2, \ldots, 9 \tag{22}$$

where SPAN_{Si} and SPAN denote the power of single target T_{Si} and mixed target T, respectively. SDoP thus measures relative power and is comparable to the wave degree of polarization, which measures the relative power between the fully polarized part and the whole wave. As a key feature of HTD, the SDoP information will be used in Sections V and VI to provide two novel classifications of radar targets.

UHD is used to process the 4-look NASA/JPL L-band AIRSAR San Francisco data, which covers various scatterers such as ocean, urban area, and vegetation. Figure 2: see original paper shows the Pauli image of the original data, while Figure 2: see original paper to (i) exhibit the Pauli images of the single targets extracted by D_i, respectively.

The dark blue in Figure 2: see original paper reflects the preference of D_1 (i.e., HD) for SR scatterers like rough ocean surface. Figure 2: see original paper shows the extraction of D_4. The wide distribution of red color demonstrates its preference for IR dihedral scatterers. The distributed dihedral scatterer utilized by Barnes and Holm [?], Yang et al. [?], and Paladini [?] to demonstrate the failure of HD can be successfully decomposed by D_4. This dichotomy is useful for building detection because buildings generally contribute the most dihedral scattering in urban areas. D_7 prefers NS volume scatterers, and the green forests in Figure 2: see original paper clearly reflect this, making it suitable for forest detection. The images of the other six dichotomies prefer the pairwise hybrids of red, green, and blue. For example, Figure 2: see original paper and (c) prefer red and green because the combinations of the 2nd and 3rd columns of T are reserved by D_2 and D_3. However, it is difficult to visually relate them to real targets because the helix scatterings they prefer are tiny in this scene. The preferences of D_5 and D_6, as well as D_8 and D_9, can be likewise validated. Target extractions on Alcatraz Island, framed in Figure 2: see original paper, are detailed in [Figure 3: see original paper]. All these clearly illustrate the scattering preference of D_i.

UHD selects the most preferable extraction among the nine dichotomies as the final single target extraction. Figure 2: see original paper and Figure 3: see original paper illustrate UHD extractions on the two scenes, respectively, which are much closer to the original mixed scattering than the extraction by D_i. Furthermore, UHD makes the target scattering signature much clearer by adaptively filtering unwanted noisy scatterings. The island scattering in Figure 3: see original paper is strengthened because the "blur cover" in Figure 3: see original paper is removed, improving capability for target identification. By counting the contribution of D_i to the final extraction, UHD also provides a coarse classification of terrain. The SR scatterer is found covering only 41% of the San Francisco scene–that is, the majority of the scene is inappropriate for HD. This validates the restricted application of HD in a sense.

IV. MATHEMATICAL EVALUATION OF UHD

The mixed target coherence matrix T can be eigendecomposed as follows:

$$T = \sum_{j=1}^{3} \lambda_j u_j u_j^H = \lambda_1 u_1 u_1^H + \lambda_2 u_2 u_2^H + \lambda_3 u_3 u_3^H \tag{23}$$

where λ_j is the eigenvalue associated with eigenvector u_j. Cloude interpreted u_j as a single target and chose u_1 as the dominant scattering because it is the optimal estimation of the unit target vector [?], [?]. T_{Scd} in (23) denotes the coherence matrix of dominant scattering, and k_{Scd} is its Pauli vector. CD can thus secure an optimal scattering estimation, and the performance of UHD can be well demonstrated by comparison with it.

Figure 2: see original paper and Figure 3: see original paper display the dominant scatterings extracted by CD from the San Francisco and Alcatraz Island scenes, respectively. They look nearly identical to the UHD extractions in Figure 2: see original paper and Figure 3: see original paper. To validate this, the Pauli vector k_S extracted respectively by HD, BHD, CHD (YD), and UHD is compared with k_{Scd} on the San Francisco scene as follows. Figure 4: see original paper to (c) show the relationship between the moduli of $k_S(i)$ and that of $k_{Scd}(i)$, respectively, where $k_S(i)$ is the ith element ($i = 1, 2, 3$) of k_S, and the meaning of $k_{Scd}(i)$ can be likewise inferred. The relationship on Frobenius norm (F-norm) is given in Figure 4: see original paper. A convergence to CD is clearly revealed from HD and BHD to CHD, and further to UHD. For quantitative demonstration, we calculate the relative residues (rr) between k_S and k_{Scd} on F-norm and moduli as follows:

$$\begin{cases} rr_i = \left| \frac{|k_S(i)| - |k_{Scd}(i)|}{|k_{Scd}(i)|} \right|, \quad i = 1, 2, 3 \\ rr_F = \left| \frac{\|k_S\|_F - \|k_{Scd}\|_F}{\|k_{Scd}\|_F} \right| \end{cases} \tag{24}$$

In this way, a sample of rr_1, rr_2, rr_3, and rr_F can be obtained at each image pixel. The mean and variance of all final rr_1 samples, and those of rr_2, rr_3, and rr_F are calculated and listed in [TABLE:II]. It is indicated that HD and BHD I & II perform equivalently but deviate quite substantially from CD. This deviation is compensated by CHD to some extent, but their differences remain clear. UHD reduces the residue the most, leaving only tiny differences. Particularly, the average rr_F between UHD and CD is merely 0.99%, and their F-norm relation in Figure 4: see original paper is nearly linear. If the confidence thresholds of rr_1, rr_2, rr_3, and rr_F are all fixed at 20%, then 91.73%, 91.91%, 90.20%, and 100% of pixels support the consistency between UHD and CD, respectively. To remove potential dataset bias, the two decompositions are also compared on DLR L-band ESAR data of Oberpfaffenhofen, and a similar convergence trend is observed. It is exhibited that only 47.16% of the Oberpfaffenhofen scene is appropriate for HD, and the average rr_F is also as tiny as 1.3854%. All these not only demonstrate UHD' s superiority over existing HTD but also indicate the potential unifiability of UHD and CD. UHD does not appear as straightforward as CD because it needs to perform nine dichotomies for final target extraction. In fact, it can be achieved quickly because the nine extractions in (14), (17), and (18) are just the columns of T or their pairwise combinations. Therefore, unlike CD, eigendecomposition is avoided in UHD.

V. UNSUPERVISED CLASSIFICATION BASED ON ADAPTIVE PERMUTATION OF UHD

Besides application to scattering extraction, another utility of UHD lies in its evident reflection of target scattering because D_i has its own scattering preference. This preference arises from the "misfortune" that D_i only provides invariance under a certain unitary transform U_ϕ, U_τ, or U_α, which enables us to directly relate D_i to a fixed canonical scattering q_i. By replacing q_i with eigenvector u_j and substituting $\lambda_j u_j u_j^H$ for T_{Si}, we see that CD also satisfies (20). Nevertheless, q_i is no longer stationary here but depends on T due to CD' s pursuit of invariance under all unitary transforms. The scattering preference of CD is thus unclear.

A. The Scattering Pyramid Classification Scheme

A simple classification can be achieved by directly comparing $SDoP_{D_i}$ to identify the most preferable scattering as the dominant scattering preference of the target. For instance, the target is labeled as "more preferable to surface" if $SDoP_{D_1}$ is larger than other $SDoP_{D_i}$. This classification has been used in Sections III and IV to indicate the restricted application of HD. It is based on the radical assumption that there is always a dominant scattering preference in mixed scattering. This works well for low-randomness targets but is no longer tenable for medium- and high-randomness targets because these targets may have several comparable preferences, and we cannot extract a significantly stronger one from them. To extend UHD for such targets, we need to devise an advanced scheme. Chen et al. [?] recently gave another interpretation to CD. They proposed describing low-entropy targets using only scattering u_1, medium-entropy targets using both u_1 and u_2, and high-entropy targets using all u_1, u_2, and u_3. However, since u_j cannot evidently reflect target scattering like k_{Si}, Chen et al. used the similarity parameter to indicate the scattering mechanism instead. Here this method is improved to a scattering pyramid by adaptively permutating and combining several HTD with different scattering preferences to jointly characterize mixed target scattering.

The pyramid is composed of several layers to indicate different degrees of scattering randomness. Each layer is further subdivided into several blocks to reflect different scattering mechanisms. A special characteristic of this scheme is that both layers and blocks are determined by the scattering preferences of HTD—that is, scattering preferences can model both scattering mechanism and randomness. More dichotomies indicate more layers and blocks, signifying better description but requiring more computation. An adaptive strategy is used to select scene-related dichotomies from the nine based on some prior information. For instance, ocean, building, and forest dominate the San Francisco scene; D_1, D_4, and D_7 can thus provide enough information on the scene due to their preferences for surface, dihedral, and volume scatterings, respectively. Here we exemplify the scheme based on D_1, D_4, and D_7. By bringing the Pauli vectors of these three dichotomies into (22), we obtain the SDoP for surface ($SDoP_s$), dihedral ($SDoP_d$), and volume scatterer ($SDoP_v$) as follows:

$$\begin{cases} SDoP_s = \frac{2A_0}{\text{SPAN}} \\ SDoP_d = \frac{B_0+B}{\text{SPAN}} \\ SDoP_v = \frac{B_0-B}{\text{SPAN}} \end{cases} \tag{25}$$

These three parameters can be obtained quickly because they are directly related to each column of T. They then enable a three-layer scattering pyramid, as shown in Figure 5: see original paper. Each layer is composed of several blocks, with each block indicating a different scattering class and expressed as a permutation or combination of $SDoP_s$, $SDoP_d$, and $SDoP_v$. The number of blocks in each layer differs because each layer is designed to denote a different scattering scenario. Scenario I, located in the 3rd layer, concerns only significantly strong preference. This results in three potential permutations $\binom{1P}{3}$ of $SDoP_s$, $SDoP_d$, and $SDoP_v$. By inquiring which is strongest, three classes (blocks) preferring surface (simplified as S), dihedral (simplified as D), and volume scattering (simplified as V) are obtained, respectively. Scenario II in the 2nd layer appears if the contribution of the second strongest preference is also prominent. Six permutations $\binom{2P}{3}$ then arise, signifying six different scattering mechanisms. The target is indexed as "more preferable to surface and dihedral" if $SDoP_s \geq SDoP_d > SDoP_v$ (simplified as SD). The other five classes−SV, DS, DV, VS, and VD−can be likewise inferred. Scenario III in the 1st layer indicates a chaotic state where the three preferences are comparable. Only one combination $\binom{3C}{3}$ is obtained, and the target is wholly labeled as "random scatterer" (simplified as R). Nevertheless, a confusion emerges when $SDoP_s = SDoP_d = SDoP_v$. When the target is fully determined, $SDoP_{D_i}$ in (22) equals 1 because a single target K matrix cannot be decomposed further [?]. On the other hand, when the target is fully noisy, $SDoP_{D_i}$ also equal each other but change to 1/3. Hence both fully determined and fully random targets are labeled as R by comparing $SDoP_s$, $SDoP_d$, and $SDoP_v$. A solution can be obtained if the following average SDoP ($SDoP_3$) is formed:

$$SDoP_3 = \frac{SDoP_s + SDoP_d + SDoP_v}{3} \tag{26}$$

This parameter is 1 for a single target, 1/3 for a noisy target, and resides between 1/3 and 1 for other targets. Hence it can measure target randomness and be used to distinguish the three scattering scenarios. A target with high $SDoP_3$ has low randomness, and a dominantly preferable scattering can represent it, corresponding to scenario I. A target with medium $SDoP_3$ has medium randomness, and the second strongest preference is added, as depicted in scenario II. The target approaches completely random when $SDoP_3$ is close to 1/3, then scenario III appears. By statistically counting different scatterers on widely-used PolSAR data like San Francisco and Oberpfaffenhofen, the values of 2/5 and 2/3 are determined as the boundaries of the three scenarios, and a total of ten classes are finally obtained, as illustrated in Figure 5: see original paper and summarized in [TABLE:III].

Figure 5: see original paper shows the classification of the San Francisco scene. Before applying UHD, the refined Lee filter with a 7$×$7 aligned window (RLF7) is used to suppress speckle. The meaning of RLFn can be likewise inferred. The influences of RLFn on classification will be investigated in Section VII. We can

see that typical targets such as the ocean, buildings, forest, avenue, beach, polo field, golf course, mountain area, the Golden Gate Bridge, the Sunset Reservoir Park, and Alcatraz Island are all well identified.

B. Influence of Target Orientation

The same target can be presented differently by a simple rotation about the line of radar sight [**?**]. As a result, a building may be identified as forest because orientation increases cross-polarized scattering [**?**], [**?**]. The coherence matrix of an oriented target can be expressed as $U_\phi T U_\phi^H$, which impacts both target extraction in (19) and $SDoP_{D_i}$ calculation in (22). To avoid these influences, deorientation should be conducted first.

The classification of the deoriented San Francisco scene is given in Figure 5: see original paper. The chocolate color in Figure 5: see original paper denotes DV, but it changes to dark red denoting DS in Figure 5: see original paper. To reveal the change quantitatively, [TABLE:IV] lists the percentages of appearance of the ten classes with and without deorientation, respectively. The changes of DS and DV are clearly presented. They account for 19.09% and 8.61% of the scene without deorientation but turn to 27.23% and 0.66% with deorientation. The decrease of DV is 7.95%, approaching the increase of DS (8.14%). Increases of S and D, as well as the decrease of V, can also be found. Deorientation thus effectively decreases cross-polarized scattering resulting from misalignment between radar and target, making classification much closer to ground truth.

C. Comparison with Entropy/Alpha

Cloude and Pottier described mixed target scattering using the concept of average target [**?**]. They interpreted CD as three single target scatterings u_j occurring with probability proportional to λ_j. An entropy parameter H was used to depict target randomness, attributing mixed scattering to three scenarios: low entropy, medium entropy, and high entropy. By parameterizing u_j with a revised Bragg scattering model, an average scattering u_0 was obtained to indicate scattering mechanism. Particularly, the alpha angle (α_{cp}) of u_0 can directly reflect the physical property of the target and was utilized to subdivide the three H-scenarios into eight effective zones, indexed by Z_i ($i = 1, 2, \ldots, 8$) in Figure 5: see original paper. The H/α_{cp} classification of the San Francisco scene is shown in Figure 5: see original paper, which is compared with the classification in Figure 5: see original paper below.

Our intuitive impression is similarity, arising from the fact that both schemes use scattering randomness to coarsely differentiate targets and divide the low-randomness scenario into three classes for surface, dihedral, and volume scatterings. Nevertheless, their difference is also obvious, with two major differences observed. First, the forest (dark green) in Figure 5: see original paper is much clearer, particularly in the avenue area (circles 1 and 2) and the Sunset Reservoir Park (circle 3), due to different treatments of high-entropy target. The target is totally labeled as R in the scattering pyramid but is split into two zones in H/α_{cp}. The good separability in Figure 5: see original paper reveals that it is not always necessary to further distinguish random targets. Second, the beach area in rectangle 4 appears as volume scattering (green) in Figure 5: see original paper but turns to SD (yellow) in Figure 5: see original paper. We believe the latter is more consistent with ground truth because beaches generally comprise sand. Similar situations are observed in the polo field (circle 5) and golf course (circle 6). Moreover, the Reservoir Park and Golden Gate Bridge in circles 3 and 7 also appear as volume scattering in Figure 5: see original paper but turn to DS (dark red) in Figure 5: see original paper. The classification in Figure 5: see original paper is found more credible by further referring to the optical image and the Wishart H/α_{cp} classification [**?**] of the San Francisco scene. The deficiency of H/α_{cp} originates from its statistical average modeling of mixed scattering. It can represent mixed scattering in a total sense but also introduces certain obscurity. To explain this, a quantitative overlapping evaluation of the two schemes on the division of medium-entropy scenario II is further conducted by surveying how many scenes of Z_3, Z_4, or Z_5 of H/α_{cp} are also attributed to classes SD, SV, DS, DV, VS, and VD of the scattering pyramid. As listed in [TABLE:V], Z_5 appears in both SD and SV. These two classes not only reveal the importance of surface scattering like Z_5 but also identify the secondary preferable scattering. The extensive distribution of Z_3 in DS and DV is also observed, providing more scattering information beyond attaching importance to dihedral scattering like Z_3. The clearest illustration of H/α_{cp} obscurity is found in Z_4, which characterizes volume scattering. Nevertheless, [TABLE:V] indicates that only limited scenes of Z_4 correspond to volume scattering-related VS and SV. The majority of Z_4 emerges in DS and SD, which denote stronger preferences for surface and dihedral scatterings than for volume scattering. The alpha angles of surface and dihedral scatterings are less than 45° and greater than 45°, respectively, but their average approximates 45° (corresponding to volume scattering) in DS and SD because $SDoP_s$ and $SDoP_d$ are comparable. This misleads H/α_{cp} to identify DS and SD as volume scattering.

VI. UNSUPERVISED CLASSIFICATION BASED ON THE STATISTICAL UNDERSTANDING OF UHD

Besides work on theoretical unification and practical application of HTD, another aim of this paper is to investigate potential consistency between HTD and CD. Instead of treating them as two competitive approaches, we focus on their consistency. The consistency on dominant single target extraction has been investigated in Section IV, and this section further explores consistency in describing mixed scattering. The Cloude-Pottier statistical modeling of random scatterers is extended to UHD, enabling another UHD application to classification.

A. Statistical Modeling of UHD

The single target scattering k_{Si} in UHD is independently extracted by nine dichotomies preferring different scatterings. UHD thus provides nine potential understandings of mixed target scattering. We normalize k_{Si} into a unit vector k_{nSi} and parameterize k_{nSi} in accordance with the revised Bragg scattering α-β model [**?**]:

$$k_{nSi} = \begin{bmatrix} \cos\alpha_i \\ \sin\alpha_i \cos\beta_i e^{j\delta_i} \\ \sin\alpha_i \sin\beta_i e^{j\gamma_i} \end{bmatrix} e^{j\phi_i}, \quad i = 1, 2, \ldots, 9 \tag{27}$$

where ϕ_i is an absolute phase. k_{nSi} is treated as a potential form of real scattering occurring with probability:

$$P_i = \frac{\mathrm{SPAN}_{Si}}{\mathrm{SPAN}} = SDoP_{D_i}, \quad i = 1, 2, \ldots, 9 \tag{28}$$

We term this the nine-symbol Bernoulli scattering model. Then each parameter in (27), such as α, can be treated as a random sequence $\{\alpha_i\}$ with occurrence probability P_i. Their mean estimations are obtained by:

$$\bar{\alpha} = \sum_{i=1}^{9} P_i\alpha_i, \quad \bar{\beta} = \sum_{i=1}^{9} P_i\beta_i, \quad \bar{\delta} = \sum_{i=1}^{9} P_i\delta_i, \quad \bar{\gamma} = \sum_{i=1}^{9} P_i\gamma_i \tag{29}$$

By replacing each parameter with its average, a mean estimation of target scattering is achieved according to (27). The Bernoulli model also provides another estimation of the average SDoP ($SDoP_9$):

$$SDoP_9 = \sum_{i=1}^{9} P_i SDoP_{D_i} = \sum_{i=1}^{9} P_i^2 \tag{30}$$

$SDoP_9$ changes to $SDoP_3$ if vectors k_{S1}, k_{S4}, and k_{S7} are used for estimation only. It also measures target randomness because it is 1 for a single target, 1/3 for a noisy target, and between 1/3 and 1 for other targets.

$SDoP_9$ is compared with entropy H below. Figure 6: see original paper and (b) show the obtained H and $SDoP_9$ on the San Francisco scene. They behave completely inversely, and Figure 6: see original paper further displays their relationship. Good correspondence is observed for targets with high H, but poor correspondence arises for low- and medium-H targets. H is only related to eigenvalue λ_j and is independent of eigenvector u_j. However, given λ_j, a series of T are obtained by varying u_j, and so is $SDoP_9$. This is the main factor leading to ambiguous mapping from $SDoP_9$ to H. The black contour in Figure 6: see original paper shows the maximum range of λ_j influence when

u_j is excluded, where outside areas are from the additional contribution of u_j. Nevertheless, u_j influence is controlled by randomness and does not function when the target is fully determined or noisy. u_1 is the dominant scattering in low-randomness scenarios. Fluctuation of u_1 then results in a wide range of $SDoP_9$ and signifies ambiguity. The contributions of u_1, u_2, and u_3 are comparable in high-randomness cases, and their orthogonality then restricts u_j influence, resulting in good mapping.

Another contribution to ambiguous mapping is the nonlinear logarithmic operation in H calculation. To illustrate this, we define an average SDoP for CD by substituting λ_j into (30) for SPAN_{Si}:

$$SDoP_{cp} = \sum_{j=1}^{3} \left(\frac{\lambda_j}{\sum_{k=1}^{3} \lambda_k} \right)^2 \tag{31}$$

The $SDoP_{cp}$ of the San Francisco scene is shown in Figure 6: see original paper. It performs similarly to $SDoP_9$. Figure 6: see original paper shows the relationship between $SDoP_{cp}$ and H, where the influence of nonlinearity is obviously revealed. Both H and $SDoP_{cp}$ depend only on λ_j, so their correspondence is confined by the black contour. Because the logarithmic operation is avoided, the relationship between $SDoP_9$ and $SDoP_{cp}$ given in Figure 6: see original paper presents the clearest display of target randomness influence on mapping. We can conclude that if H is a global feature measuring target randomness, then $SDoP_9$ is just a local one revealing more details. Based on these observations, we further use $SDoP_9$ to differentiate the three scattering scenarios. The values of 2/3 and 2/5 are still used as boundaries because they also correspond well to H boundaries, as shown in Figure 6: see original paper. [TABLE:VI] lists the updated three scattering scenarios.

The average α angle estimation in (29) is detailed here:

$$\bar{\alpha} = \arccos(|k_{nSi}(1)|) = \arccos\left(\sum_{i=1}^{9} P_i \cos\alpha_i \right) \tag{32}$$

To differentiate from the Cloude-Pottier alpha angle α_{cp}, we call (32) the Li-Zhang alpha angle (α_{lz}) hereafter. One can validate that α_{lz} corresponds to a flat change from surface scattering ($\alpha_{lz} = 0^\circ$) to dihedral scattering ($\alpha_{lz} = 90^\circ$). It thus performs similarly to α_{cp} in characterizing target scattering mechanism. Figure 7: see original paper and (d) illustrate these two angles on the San Francisco scene. They behave nearly identically, and their strong relationship in Figure 7: see original paper clearly shows this, where the black curve gives λ_j contribution when u_j is excluded. Once again, correspondence is impacted by u_j and controlled by randomness. α_{lz} equals α_{cp} when the target is completely determined, and good mapping is maintained through low-randomness scenario I, as exhibited in Figure 7: see original paper, though it changes in scenarios

II and III, as illustrated in Figure 7: see original paper and (f). Nevertheless, good correspondence is still observed. [TABLE:VII] lists the mean and standard deviation of absolute residue between the two angles on each scenario and on the total scene to quantify randomness influence. The total mean and standard deviation are only 2.44° and 1.91°, respectively.

B. SDoP/Alpha Classification Scheme

The consistency between α_{lz} and α_{cp} enables us to similarly use α_{lz} to subdivide the three $SDoP_9$ scenarios into eight zones. To validate this, we first transfer the 2D H/α_{cp} plane of San Francisco in Figure 8: see original paper into $SDoP_9$-α_{lz} space, as shown in Figure 8: see original paper. The zones are still clearly separable with slight mixtures near borders. Hence the combination of $SDoP_9$ and α_{lz} has good separability. By referring to α_{cp} boundaries in H/α_{cp} and the α_{lz}-α_{cp} correlation in [Figure 7: see original paper], the α_{lz} partition of each scenario is obtained and listed in [TABLE:VI]. The corresponding 2D $SDoP_9/\alpha_{lz}$ plane of San Francisco is displayed in Figure 8: see original paper. By further mapping the classification to H-α_{cp} space, we get Figure 8: see original paper, where horizontal borders are straight due to good α_{lz}-α_{cp} correspondence, but vertical borders are somewhat askew, particularly the one between Z_5 and Z_8, because $SDoP_9$-H correspondence is not as good. Nevertheless, the majority of Figure 8: see original paper is correctly aligned to the H/α_{cp} plane in Figure 8: see original paper. Figure 5: see original paper and Figure 9: see original paper exhibit the two classification results. Typical scatterers like ocean, city, and vegetation are clearly separated with nice consistency achieved. The confusion matrix [?], a statistical table recording consistency between two classifications, is used to quantitatively evaluate them. As listed in [TABLE:VIII], where each row reflects how many pixels of Z_i in $SDoP_9/\alpha_{lz}$ are reclassified into different zones of H/α_{cp}. The majority of Z_i in $SDoP_9/\alpha_{lz}$ has been successfully classified into the corresponding Z_i in H/α_{cp}. [TABLE:IX] shows total consistency is 94.04%. The main visual difference appears in the mountain area of rectangle 8, where some volume scatterings (green) in Figure 5: see original paper change to rough surface scatterings (yellow) in Figure 9: see original paper, but it is not easy to identify which is better. By referring to the classification in Figure 5: see original paper, the changing area belongs to SD or SV, denoting stronger preference for surface scattering. Thus $SDoP_9/\alpha_{lz}$ performs better. To eliminate potential dataset bias, the two classifications are further compared on Oberpfaffenhofen and Flevoland (NASA/JPL L-band AIRSAR) scenes. [TABLE:IX] lists their consistencies, still as high as 91.64% and 91.29%. Thus $SDoP_9/\alpha_{lz}$ is a competent alternative to H/α_{cp}.

C. Influence of Target Orientation

$SDoP_9$ and α_{lz} are not roll-invariant, so better classification may be obtained if deorientation is performed first. However, the classification of deoriented San Francisco in Figure 9: see original paper does not differ much from Figure 9:

see original paper. To show this, [TABLE:X] counts how many pixels of the scene are attributed to Z_i by H/α_{cp} and $SDoP_9/\alpha_{lz}$ with and without deorientation. Both $SDoP_9/\alpha_{lz}$ results are consistent with H/α_{cp}, but changes after deorientation are not as significant as in [TABLE:IV]. This is further reflected in [TABLE:IX] in terms of total consistency between H/α_{cp} and deoriented $SDoP_9/\alpha_{lz}$ on different scenes. These reveal that despite $SDoP_{D_i}$ and α_i not being roll-invariant, certain invariance is achieved when they are averaged into $SDoP_9$ and α_{lz}. Deorientation then has limited improvement and is not always necessary. This is also shown in [TABLE:IV]. Changes after deorientation are remarkable for the ten classes, which are identified by directly comparing $SDoP_s$, $SDoP_d$, and $SDoP_v$. However, changes are only 1.97%, -2.20%, and 0.23% on the three scenarios because they are determined by $SDoP_3$, which is the statistical average of $SDoP_s$, $SDoP_d$, and $SDoP_v$. A comparison of $SDoP_3$ and $SDoP_9$ is also achieved if we further count changes on the three $SDoP_9$ scenarios. They are -0.15%, 0.11%, and 0.04%, respectively−gentler than the former case. $SDoP_9$ is thus more stable because it involves averaging all nine single target scatterings.

VII. DISCUSSION

Sections V and VI provide two descriptions for mixed target scattering in terms of adaptive permutation and statistical modeling of UHD, respectively. They enable not only a novel description of target randomness ($SDoP_3$ or $SDoP_9$) but also two different yet consistent indications of scattering mechanism (α_{lz} and permutation of $SDoP_{D_i}$). Scattering preference plays a pivotal role in both approaches.

Besides target orientation influence, another factor to note when using these descriptions is speckle filtering. The proposed classifications are based on parameters $SDoP_{D_i}$, $SDoP_3$, $SDoP_9$, and α_{lz}, which are retrieved by UHD from T. Incoherent averaging of many neighboring pixels of T is thus required because insufficient averaging produces biased retrieval and hence biased classification. Lopez-Martinez et al. [**?**] and Lee et al. [**?**] investigated speckle filtering effects on H/α_{cp} and suggested using 7\$$\times 7 and 5\times$\$5 or larger averaging windows, respectively, for estimating H and α_{cp}. Lee and Pottier [**?**] further found that H estimation from RLF7 performs even better than from boxcar filter, although RLF7 may not provide enough averages for reliable estimates.

This section gives a simple evaluation of RLFn influences on the scattering pyramid and $SDoP_9/\alpha_{lz}$. [Figure 10: see original paper] illustrates classifications of the San Francisco scene when RLF1 (i.e., without filtering), RLF3, RLF5, and RLF9 are respectively used to filter the data. Since orientation impacts scattering pyramid performance, deorientation is also carried out after speckle suppression. The corresponding H/α_{cp} classifications are also shown for comparison. The performances under RLF7 were given in [Figure 5: see original paper] and [Figure 9: see original paper]. One can find that consistency between $SDoP_9/\alpha_{lz}$ and H/α_{cp} is independent of speckle filtering, revealing border determination stability in $SDoP_9/\alpha_{lz}$. Although original San Francisco data were averaged by a 4$×$4 window in coherence matrix construction, such filtering is insufficient for target classification. Without further filtering, classifications in Figure 10: see original paper, (e), and (i) are still heavily speckled, preventing us from bridging classification results with ground truth. Nevertheless, the scattering pyramid scheme still enables better target discrimination. The avenue, park, polo field, and golf course are clear in Figure 10: see original paper but hard to identify in Figure 10: see original paper and (i). The three classifications are greatly strengthened by simply filtering with RLF3, RLF5, or RLF7, which increases yellow- and green-coded areas and improves identifiability of beaches, parks, and avenues. This improvement mainly results from increased scattering randomness. It is observed that H value increases while $SDoP_3$ and $SDoP_9$ values decrease with averaging amount. But besides introducing square imprints, filtering influence on $SDoP_{D_i}$, α_{lz}, and α_{cp} is limited. Similar findings on H and α_{cp} were presented in [?] and [?]. Hence although $SDoP_3$ and $SDoP_9$ are more independent of target orientation, $SDoP_{D_i}$ is less dependent on speckle filtering.

From RLF3 to RLF5 and further to RLF7, the dark green-coded class R in scattering pyramid classification, as well as white-coded Z_1 and dark green-coded Z_2 in $SDoP_9/\alpha_{lz}$ and H/α_{cp} classifications become much clearer, reflecting increased randomness. But changes from RLF7 to RLF9 are relatively weak. Larger windows provide better estimation of H, $SDoP_3$, and $SDoP_9$ but risk degrading spatial resolution. Considering the compromise between resolution and parameter estimation, RLF7 is sufficient for the three classifications.

The black line framed Patch 9 in Figure 10: see original paper denotes an urban area of San Francisco named South of Market (SoMa). [Figure 11: see original paper] displays a close-up view of class R in scattering pyramid classifications of SoMa under RLFn. Class R denotes the high-randomness scenario and is split into Z_1 and Z_2 in $SDoP_9/\alpha_{lz}$ and H/α_{cp}. It expands with increasing filtering size and stabilizes in RLF7, then dominates Patch 9 (covering about 52% of the area). Such classification may lead us to misidentify SoMa as vegetated area because vegetation usually possesses high randomness and volume scattering. One might attribute this misidentification to building orientation because it increases HV scattering and mistakes buildings for forest. A powerful technique to cure this is performing deorientation to minimize HV scattering. However, deorientation is no longer effective here because all classifications in [Figure 11: see original paper] are based on deoriented data.

SoMa is becoming a new test for target decomposition procedures because of its special city planning. Unlike other San Francisco areas, streets here are neither vertical nor horizontal but are about 40° tilted. We can roughly observe this from the classification in Figure 11: see original paper. Based on the fact that buildings are usually constructed along streets, such city planning results in misalignment (azimuth tilting) between building vertical walls and radar azimuth direction. The wall normal will not be within the radar incidence plane, thus creating orientation. Detailed analysis and modeling of polarization orientation in such built-up areas have been presented by Kimura [**?**]. Moreover, azimuth tilting in dense urban areas also increases scattering complexity and randomness, and the wide distribution of class R reflects the high randomness in SoMa. Patch 10 in Figure 10: see original paper is also an urban area adjacent to SoMa with dense buildings. Nevertheless, the wide distribution of dark red color (class DS) indicates its medium randomness because azimuth tilting is negligible here. Actually, instead of target orientation, it is high randomness ($H > 0.9$, $SDoP_3$ & $SDoP_9 < 2/5$) that enables the scattering pyramid, $SDoP_9/\alpha_{lz}$, and H/α_{cp} to attribute SoMa to class R, as well as to Z_1 and Z_2.

Azimuth tilting results in target orientation and high randomness, increasing the possibility of misidentifying buildings as vegetation. Deorientation can only compensate orientation-related misidentification but has tiny effects on that from randomness. As shown in Subsection VII.C, parameters $SDoP_3$, $SDoP_9$, and H possess strong invariance to U_ϕ. For such randomness-related misidentification, from the viewpoint of model-based target decompositions, we may need to develop modeling that better accounts for random scattering. The superiority of Singh' s decomposition over Yamaguchi' s and Sato' s decompositions in SoMa may be ascribed to this reason [**?**]. Class R in the scattering pyramid scheme denotes a chaotic state where the three preferences $SDoP_s$, $SDoP_d$, and $SDoP_v$ are comparable. It is thus determined only by $SDoP_3$ with no comparison of the three preferences. Nevertheless, tiny differences still exist among $SDoP_s$, $SDoP_d$, and $SDoP_v$, and this information may help discriminate buildings from forest. By treating $SDoP_d$, $SDoP_v$, and $SDoP_s$ respectively as red, green, and blue colors, the second row in [Figure 11: see original paper] shows the pseudo-color map of SoMa pixels belonging to class R under RLFn. Saturation of this characterization is relatively low because preferences are comparable here. Nevertheless, one can still discriminate hue differences and find more red than green colors. The average $SDoP_d$ in Figure 11: see original paper is 0.3618, larger than that of $SDoP_v$ (0.3094), indicating building preference over vegetation. Therefore, scattering preference analysis can be further applied to elaborating classifications of the scattering pyramid and $SDoP_9/\alpha_{lz}$. Together they provide a coarse-to-fine understanding of different scattering behaviors.

VIII. CONCLUSION

This paper addresses three aspects to respond to existing concerns about HD. The first aspect concerns non-uniqueness, treated by generalizing HD to UHD. UHD covers all existing HTD and provides a fair application mechanism for each HTD. The second aspect regards the practical value of HTD. Scattering preference is identified as a special characteristic of HTD, enabling us to relate each dichotomy directly to a canonical scattering. This information is adaptively used in mixed scattering description based on a pyramid scheme and promotes a scattering preference-based classification, demonstrated to have better target discrimination than entropy/alpha. This overturns the general impression that

HTD is inferior to CD. We thus conclude that HTD are born with physical significance and nice applicability, hoping this will promote their wide acceptance in the future.

The third aspect regards the uniqueness controversy between HTD and CD. Instead of treating them as competitive methods, their potential unification is investigated in two aspects: optimal extraction of single target and statistical description of mixed scattering. We show that UHD and CD can achieve consistent target extraction, and the devised $SDoP/\alpha$ classification is a competent alternative to entropy/alpha. These provide a good resolution to the controversy between Huynen and Cloude.

Four decades of development in this area have indicated that there is no unique decomposition but rather an infinity. Uniqueness arises only when one prefers a certain aspect. Each decomposition has its own usefulness but cannot provide all information regarding target scattering. Therefore, we need to combine all decompositions for an integrated understanding to make polarimetric radar viewing of the world more colorful.

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APPENDIX

The consistency between CHD and YD is illustrated here. Yang et al. [**?**] proposed a modified HD in terms of two transforms of K, which can be concisely expressed via matrix T as:

$$T_1 = U_\tau T U_\tau^H, \quad T_2 = U_\alpha T U_\alpha^H \tag{A.1}$$

These respectively correspond to matrices K_1 and K_2 mentioned in Subsection II.B, where:

$$U_\tau = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad U_\alpha = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \tag{A.2}$$

Based on these, YD performs as follows [**?**]:

If T_{11} is larger than T_{22} and T_{33}, then directly apply HD to T and denote:

$$T = T_S + T_N \tag{A.3}$$

If T_{22} is larger than T_{11} and T_{33}, then apply HD to T_1 and denote:

$$T_1 = T_{MS1} + T_{MN1} \tag{A.4}$$

and the modified decomposition of T is:

$$T = U_\tau^H T_{MS1} U_\tau + U_\tau^H T_{MN1} U_\tau \tag{A.5}$$

While if T_{33} is larger than T_{11} and T_{22}, then apply HD to T_2 and denote:

$$T_2 = T_{MS2} + T_{MN2} \tag{A.6}$$

and the modified decomposition of T is:

$$T = U_\alpha^H T_{MS2} U_\alpha + U_\alpha^H T_{MN2} U_\alpha \tag{A.7}$$

From the vector k_{S1} representation of HD in (14), we obtain that:

$$k_{S1} \propto [T_{11}, T_{12}, T_{13}]^T \tag{A.8}$$

Therefore, the modified single targets T_{MS1} and T_{MS2} in (A.5) and (A.7) can be further expressed as:

$$\begin{cases} U_\tau^H T_{MS1} U_\tau \propto U_\tau^H [T_{22}, T_{23}, T_{21}]^T = [T_{22}, T_{23}, T_{21}]^T \\ U_\alpha^H T_{MS2} U_\alpha \propto U_\alpha^H [T_{33}, T_{32}, T_{31}]^T = [T_{33}, T_{32}, T_{31}]^T \end{cases} \tag{A.9}$$

They correspond to the single target Pauli vectors k_{S4} and k_{S7} in (17) and (18), respectively:

$$\begin{cases} k_{S4} \propto [T_{22}, T_{23}, T_{21}]^T \\ k_{S7} \propto [T_{33}, T_{32}, T_{31}]^T \end{cases} \tag{A.10}$$

Hence, the three dichotomies (A.3), (A.5), and (A.7) in YD are the same as those in CHD. YD selects them based on the $2A_0$ parameters of T, T_1, and T_2 –i.e., T_{11}, T_{22}, and T_{33}—but CHD achieves this by comparing the SPAN of T_S, T_{MS1}, and T_{MS2}.

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Note: Figure translations are in progress. See original paper for figures.