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Abstract

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Full Text

Preamble

Spectral function and quark diffusion constant in non-critical holographic QCD

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Abstract: Motivated by recent studies of intersecting D-brane systems in critical string theory and phenomenological AdS/QCD models, we present a detailed analysis for the vector and scalar fluctuations in a non-critical holographic QCD model in the high temperature phase, i.e., the chiral symmetric phase. This model is described by N_f pairs of D4 and $\bar{D}4$ probe branes in a non-critical AdS₆ black hole background. Focusing on the hydrodynamic as well as the high frequency limit, we analytically obtain spectral functions for vector and scalar modes on the flavor probe. Then we extract the light quark diffusion constant for flavor current using three different methods and find that different methods give the same results. We also compute the heavy quark diffusion constant for comparison with the light quark case.

Keywords: Gauge-gravity correspondence, Holographic dual of QCD, Thermal Field Theory

1 Introduction

Gauge/gravity duality [1–4] has become a powerful tool in studying strongly coupled non-Abelian gauge theories. Since its invention, it has been expected to be helpful in understanding some mysterious behaviors of low energy QCD. Although the gravity dual of realistic QCD has not yet been established, the Sakai-Sugimoto model [5, 6] has successfully realized certain aspects of low energy phenomena of QCD: the confinement/deconfinement transition has been recognized as the Hawking-Page phase transition [7], and the non-Abelian chiral symmetry breaking/restoring phase transition also has a geometrical realization [5, 8, 9]. However, this model has undesired KK modes because it is based on critical string theory and has unwanted internal space. What is worse is that these KK modes and hadrons have the same mass scale and there is no way to disentangle the two scales. One effective way to overcome this serious problem is to consider string theory in non-critical dimensions, and some non-critical string models of QCD have been constructed in [10–31].

In this paper we consider a model of non-critical AdS_6 black hole solution [11, 12, 14] (corresponding to the deconfined phase), in which flavor quarks can be introduced by adding D4 and D4 probe branes [32, 33] into this black hole background. As the non-critical version of the Sakai-Sugimoto model, this model has also reproduced many properties of low energy QCD. Some studies about this model have been performed in the literature, e.g., the thermal aspects have been studied in [35] and the chiral phase transition with external electromagnetic fields and chemical potential has been examined in [36–38]. The chiral phase transition can be represented by the embedding profile of flavor branes: the U-shape configuration stands for the chiral breaking phase while parallel branes state the chiral symmetric phase [35]. Hadron physics related to low energy QCD has been intensively studied in [12, 14, 35, 39].

One motivation for studying such a non-critical holographic QCD model lies in that this model is different from well-studied intersecting D-brane systems in critical string theory and some phenomenological AdS/QCD models which lack theoretical completions. Special features of this model are that the dilaton is constant, which is the same as the well-studied D3 brane geometry, and supersymmetry is completely broken, which is the same as the Sakai-Sugimoto model. Given that studies on hadron physics and chiral phase transition in this model have reproduced many similar features to its critical version, we here focus on the non-equilibrium state physics (not far from equilibrium) of the quark sector—the retarded Green’s function of mesonic operators and the corresponding spectral functions. We mainly probe differences and similarities between the Sakai-Sugimoto model and its non-critical version. Meanwhile, a byproduct of this study is to further demonstrate universal features of gauge/gravity duality and eliminate non-universal ones peculiar to some models.

Specifically, in this work we study the vector/scalar fluctuations on the flavor probe, calculate their correlation functions to reproduce the meson spectral func-

tions, and then extract the quark diffusion constant. We work in the quenched approximation, i.e., we do not take into account the backreaction of flavor probe branes on the AdS_6 black hole background. For simplification we focus on the chiral symmetric phase, also referred to as the high temperature phase. In this phase mesons should not be confined or bound together, and they are no longer long-lived in contrast to mesons in the chiral broken phase. Instead, we expect them to be quasi-normal modes in the black hole backgrounds. This phenomenon can be described as meson melting [42, 43] in the chiral symmetric phase (though this idea is subtle when considering the processes of chiral phase transition because mesons prior to transition do not correctly correspond to quasi-normal modes after transition [44]). In general, such study usually needs extensive numerical work, and applicability of results depends on the precision of numerical computation. Therefore, in this paper we consider only the hydrodynamic and high frequency limits, both of which allow for analytical calculations.

In the hydrodynamic limit, the frequency and spatial momentum are much smaller than the temperature of the holographic system, and the system reduces to hydrodynamics, allowing us to probe its hydrodynamic properties. Using gauge/gravity duality to study hydrodynamic properties of strongly coupled non-Abelian gauge theories has achieved great success [45–68] (for reviews see [40, 69]). In our study we use the method proposed in [45] to obtain the hydrodynamic limit for meson spectral functions, whose form agrees with expectations from boundary thermal field theory, and then we extract the quark diffusion constant from the correlation function in the diffusive channel.

To check the correctness of this extraction, we use two additional different ways to compute this diffusion constant and find that all three methods give the same results. This confirms the universality of the membrane paradigm, proposed in [48] and further developed in [61, 62] for studying transport coefficients in strongly coupled thermal field theories from gauge/gravity duality. Because the quark introduced by probe D-branes is light (here, precisely speaking, massless), we refer to this quark diffusion as light quark diffusion, which differs from the heavy quark case discussed later.

We also study the high frequency limit using the WKB approximation. In this regime, we analytically obtain spectral functions for different modes. Their behaviors are not the same: for vector modes they scale as ω^2 , consistent with previous results [64], while for the scalar mode they scale as ω^6 , which has not been observed in previous studies. These scaling behaviors can also be extracted from dimensional analysis for dual operators.

For completeness, we calculate the heavy quark diffusion constant in this model. The heavy quark can be modeled by an open string stretching from the boundary to the bulk. This configuration has been used in studying jet quenching parameter [70–72], drag force [73], and energy loss [74, 75] for heavy quarks moving through quark-gluon plasma (for reviews, see [69, 76]). We consider a static string and extract the diffusion constant from the correlation function of

the string's embedding coordinate, following the proposal in [77] and studies in [77–79] for $\mathcal{N} = 4$ super-Yang-Mills plasma. The same calculation for the Sakai-Sugimoto model was done in [80]. Our result is similar to [77] but different from [80]. A recent elaborate study of heavy quark diffusion in a non-conformal background can be found in [81].

The remainder of this paper is organized as follows. In Section 2, we set up backgrounds for our study, introducing the holographic model and notation conventions, calculating the charge susceptibility for later convenience, and working out equations of motion for flavor vector and scalar fluctuations. Section 3 is the main part of our study, where we intensively analyze the fluctuations under the hydrodynamic limit and extract the light quark diffusion constant in three ways. Section 4 concentrates on the study of spectral functions in the high frequency limit using the WKB method. In Section 5 we turn to the calculation of the heavy quark diffusion constant. The final section gives discussions, conclusions, and some open questions.

2.1 Non-critical AdS₆ black hole background

We take N_c coincident D4-branes wrapped on a circle in six dimensions. In the large N_c and near-horizon limits, this brane system is described by classical solutions to type IIA supergravity in six dimensions. To reproduce the confinement/deconfinement phase transition, the color branes wrap one circle with periodic boundary conditions for bosons and antiperiodic ones for fermions along the wrapped circle to explicitly break supersymmetry [7]. The gravitational background is dual to a five-dimensional gauge theory, which is effectively four-dimensional when the energy scale is much smaller than the inverse circle radius. Due to antiperiodic boundary conditions for fermions, the remaining low-energy degrees of freedom are just adjoint gluons. Thus we obtain a non-supersymmetric pure gauge theory with gauge group $SU(N_c)$ in the low-energy limit. To describe quarks in the fundamental representation, we introduce N_f pairs of D4 and $\bar{D}4$ branes into this background following [32, 33]. As mentioned in the introduction, we assume $N_f \ll N_c$, so the backreaction of flavor probe branes on the AdS₆ black hole background can be ignored (quenched approximation). As in the Sakai-Sugimoto model, the flavor probe is transverse to the wrapped circle, and the brane configuration is summarized in Table 1.

We directly write the background geometry for the deconfined phase using the notation conventions of [35]:

$$ds^2 = \left(\frac{u}{R}\right)^2 (-f(u)dt^2 + \delta_{ij}dx^i dx^j + dx_4^2) + \left(\frac{R}{u}\right)^2 \frac{du^2}{f(u)} \quad (2.1)$$

$$f(u) = 1 - \frac{u_T^5}{u^5} \quad (2.2)$$

$$F_6 = Q_c dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \wedge du \wedge dx_4 \quad (2.3)$$

where $i = \{1, 2, 3\}$, F_6 is the 6-form field strength of the 5-form field coupled with color branes, Q_c corresponds to the number of color branes N_c , and ϕ is the dilaton vacuum. The space spanned by t and u has cigar topology with minimum u_T at the tip. To avoid a conical singularity, t must be periodic:

$$t \sim t + \frac{4\pi R^2}{5 u_T} \quad (2.4)$$

We can take $T = 1/\beta$ as the temperature of the black hole background geometry and interpret it as the temperature of the dual boundary field theory. The corresponding background geometry for the confined phase can be found in [35].

We briefly describe some physical aspects of this non-critical holographic QCD model. The main conclusions can be found in [35, 39]. As mentioned in Section 1, there are two kinds of phase transitions for this model—confinement/deconfinement and chiral phase transition—similar to the critical Sakai-Sugimoto model. The former is realized as two different background geometries according to Witten [7]. The latter is geometrically realized as a thermal phase transition on the dual gravity side, marked by the shape of the embedding profile of flavor D4/ $\bar{D}4$ branes. This can be justified by comparing on-shell actions (related to free energies for the two phases) for different flavor embedding profiles. Parallel D4/ $\bar{D}4$ branes represent the chiral symmetric phase, while the U-shape flavor profile indicates chiral symmetry breaking. This identification is confirmed by studying the meson spectrum under these two embedding profiles. As in the critical Sakai-Sugimoto model, the meson spectrum is discrete and gapped for U-shape embedding, while for parallel embedding there is no discrete meson spectrum. Generally, there is no parallel flavor profile in the confined phase due to U-shape topology of the confined background geometry in the x_4 - u plane. When temperature increases to the deconfined phase, two flavor embedding profiles are possible—parallel versus U-shaped. Therefore, high temperature corresponds to chiral symmetry restoring phase while low temperature corresponds to chiral broken phase.

One striking feature of non-critical holographic QCD models is that the curvature of the supergravity background is of order one and cannot be suppressed by taking large N_c limit. This is necessary to avoid an unwanted gap between low and high spin hadron spectra. However, unlike critical supergravity, there are possible stringy corrections. One might be concerned that large stringy corrections could affect calculations on the gravity side, but results in [12] indicate this non-critical supergravity approximation to low-energy behavior of non-critical superstring is meaningful. Another good feature is that the dilaton vacuum is constant and does not need elevation to M-theory setup in the ultraviolet region as in the critical Sakai-Sugimoto model.

For later convenience in extracting the light quark diffusion constant, we calculate the charge susceptibility in the chiral symmetric phase. We turn on chemical potential and its corresponding charge density, simulated by the time component of $U(1)$ gauge field on the flavor worldvolume (we do not consider the non-Abelian part of $U(N_f)$ flavor gauge group). We assume $A_0(u) \neq 0$ and set other components to zero. The total effective action for D-brane includes the Dirac-Born-Infeld (DBI) part and Chern-Simons part, but we omit the Chern-Simons term as explained in [35]: if included, the free energy difference between chiral symmetric and chiral breaking phases diverges. Omitting it, the action for flavor probe branes is:

$$S_{DBI} = -T_4 N_f \int d^4x du e^{-\phi} \sqrt{-\det(g_{ind} + 2\pi l_s^2 F)} \quad (2.5)$$

where g_{ind} is the induced metric, T_4 is D4-brane tension, l_s is string length, and the only nonzero field strength component is $F_{0u} = -F_{u0}$.

The induced metric is:

$$g_{ind} = \left(\frac{u}{R}\right)^2 (-f(u)dt^2 + \delta_{ij}dx^i dx^j + x_4'^2 du^2) + \left(\frac{R}{u}\right)^2 \frac{du^2}{f(u)} \quad (2.6)$$

where prime means derivative with respect to radial coordinate u and $x_4(u)$ is flavor brane embedding profile. Focusing on chiral symmetric phase, we set $x_4' = 0$, simplifying the action:

$$S_{DBI} = V_4 \tilde{S}_{DBI} \quad (2.7)$$

$$\tilde{S}_{DBI} = - \int du \left(\frac{u}{R}\right)^3 \sqrt{1 - (2\pi l_s^2 A_0')^2} \quad (2.8)$$

where V_4 is four-dimensional Minkowski spacetime volume and \tilde{S}_{DBI} is action density. The equation of motion for $A_0(u)$ is:

$$\partial_u \left[\left(\frac{u}{R}\right)^3 \frac{(2\pi l_s^2)^2 A_0'}{\sqrt{1 - (2\pi l_s^2 A_0')^2}} \right] = 0 \quad (2.9)$$

Choosing integration constant as physical charge density d (clear from holographic dictionary [82]), we define rescaled density $\tilde{d} = d/(2\pi l_s^2 T_4 N_f e^{-\phi})$ for convenience. The relation between chemical potential μ and charge density is:

$$2\pi l_s^2 \mu = \int_{u_T}^{\infty} du \frac{\tilde{d}}{\sqrt{\tilde{d}^2 + (u/R)^6}} \quad (2.11)$$

From these equations, physical charge density d is proportional to rescaled \tilde{d} , and $\mu = 0$ is equivalent to $d = 0$. The charge susceptibility Ξ is:

$$\Xi \equiv \left. \frac{\partial d}{\partial \mu} \right|_{\mu=0} = \left. \frac{\partial d}{\partial \tilde{d}} \frac{\partial \tilde{d}}{\partial \mu} \right|_{\tilde{d}=0} = \frac{(2\pi l_s^2)^2 T_4 N_f e^{-\phi}}{R^3} u_T^2 \quad (2.12)$$

We change notation via radial coordinate transformation $r = u_T/u$. The background geometry becomes:

$$ds^2 = \left(\frac{u_T}{R}\right)^2 \frac{1}{r^2} (-f(r)dt^2 + \delta_{ij}dx^i dx^j) + \left(\frac{R}{r}\right)^2 \frac{dr^2}{f(r)} \quad (2.14)$$

$$f(r) = 1 - r^5 \quad (2.15)$$

The induced metric on flavor worldvolume in chiral symmetric phase is:

$$g_{ind} = \left(\frac{u_T}{R}\right)^2 \frac{1}{r^2} (-f(r)dt^2 + \delta_{ij}dx^i dx^j) + \left(\frac{R}{r}\right)^2 \frac{dr^2}{f(r)} \quad (2.16)$$

Under this transformation, horizon $u = u_T$ maps to $r = 1$ and boundary $u = \infty$ maps to $r = 0$, facilitating later analytical studies.

2.2 Equations of motion for vector and scalar fluctuations

We extract equations of motion for vector and scalar fluctuations, key ingredients for later calculations. We focus on the vector $U(1)$ part of $U(N_f) \times U(N_f)$ gauge group for N_f flavor brane-antibrane pairs. The DBI action expands to quadratic order in field fluctuations. Vector and scalar modes do not couple when vector background is off, allowing separate analysis.

First, consider $U(1)$ gauge field fluctuation and expand DBI action to quadratic order:

$$S_{gauge} = -T_4 N_f \int d^4x dr e^{-\phi} \sqrt{-\det(g_{ind} + 2\pi l_s^2 F)} \quad (2.17)$$

We choose radial gauge $A_r = 0$. Indices are contracted using induced metric g_{ind} . The equation of motion is:

$$\partial_\alpha \left(\sqrt{-\det g_{ind}} F^{\alpha\beta} \right) = 0 \quad (2.18)$$

Inserting into on-shell action yields:

$$S_{gauge} = -\frac{(2\pi l_s^2)^2}{4} \int d^4x dr \sqrt{-\det g_{ind}} F_{\alpha\beta} F^{\alpha\beta} \quad (2.19)$$

After Fourier transformation with spatial momentum along x-direction:

$$A_\mu(x^\mu, r) = \int \frac{d^4k}{(2\pi)^4} e^{ikx} A_\mu(k^\mu, r), \quad k^\mu = (-\omega, q, 0, 0) \quad (2.20)$$

The on-shell action becomes:

$$S_{gauge} = -\frac{(2\pi l_s^2)^2}{2} \int \frac{d^4k}{(2\pi)^4} [f(r) A_i(-k, r) \partial_r A_i(k, r) - A_t(-k, r) \partial_r A_t(k, r)]_{r=0}^{r=1} \quad (2.22)$$

Choosing $\beta = r, x, y$ in the equation of motion gives coupled equations for A_t and A_x , and decoupled equations for A_y, A_z . Defining longitudinal and transverse electric fields:

$$E_L \equiv \omega A_x + q A_t, \quad E_T \equiv E_{y,z} = \omega A_{y,z} \quad (2.23)$$

They obey decoupled equations:

$$E_T'' + \frac{f'(r)}{f(r)} E_T' + \frac{\tilde{\omega}^2 - \tilde{q}^2 f(r)}{f^2(r)} E_T = 0 \quad (2.26)$$

$$E_L'' + \frac{f'(r)}{f(r)} \frac{\tilde{\omega}^2}{\tilde{\omega}^2 - \tilde{q}^2 f(r)} E_L' + \frac{\tilde{\omega}^2 - \tilde{q}^2 f(r)}{f^2(r)} E_L = 0 \quad (2.27)$$

where we rescaled frequency and momentum:

$$\tilde{\omega} = \frac{\omega}{4\pi T/5}, \quad \tilde{q} = \frac{q}{4\pi T/5} \quad (2.29)$$

For scalar mode fluctuation around parallel embedding $x'_4 = 0$, described by $\phi(x^\alpha, r)$, the action to quadratic order is:

$$S_{scalar} = -T_4 N_f \int d^4x dr e^{-\phi} \sqrt{-\det(g_{ind} + g_2)} \quad (2.30)$$

where g_2 encodes scalar fluctuation. The equation of motion is:

$$\partial_\alpha \left(\sqrt{-\det g_{ind}} g^{\alpha\beta} \partial_\beta \phi \right) = 0 \quad (2.32)$$

After Fourier transformation, the on-shell action and equation become:

$$S_{scalar} = \frac{(2\pi l_s^2)^2}{2} \int \frac{d^4 k}{(2\pi)^4} \left[\frac{f(r)}{r^5} \phi(-k, r) \partial_r \phi(k, r) \right]_{r=0}^{r=1} \quad (2.35)$$

$$\phi'' + \left(\frac{f'(r)}{f(r)} - \frac{5}{r} \right) \phi' + \frac{\tilde{\omega}^2 - \tilde{q}^2 f(r)}{f^2(r)} \phi = 0 \quad (2.36)$$

The transverse electric field E_T and scalar mode ϕ have similar behaviors, differing only in $1/r$ versus $5/r$ terms affecting near-boundary solutions.

3.1 Retarded Green's function from gauge/gravity duality

We summarize the prescription for calculating Minkowskian retarded Green's functions. The operator/field correspondence means boundary operators correspond one-to-one to bulk fields. This weak/strong duality transforms strongly-coupled field theory problems into weakly-interacting gravitational ones. The semi-classical approximation treats quantum field theory in curved spacetime by taking supergravity geometry as background. Physics is encoded in classical equations of motion.

The prescription [45] involves: (1) Find a solution to the equation of motion with Dirichlet condition (value = 1) at the boundary and incoming wave asymptotics at the horizon for timelike momenta (regular for spacelike). (2) The retarded Green's function is given by the second derivative of on-shell action with respect to boundary fluctuation, keeping only boundary contributions. The imaginary part of $F(k, z)$ is radial-coordinate independent, allowing calculation at any convenient radius.

For scalar operator $\varphi(x, z)$, the quadratic action is:

$$S = K \int_{z_H}^{z_B} dz \int d^4 x [g^{zz} (\partial_z \phi)^2 + g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + m^2 \phi^2] \quad (3.1)$$

The linearized equation after Fourier transformation is:

$$\partial_z (\sqrt{-g} g^{zz} \partial_z f_k) - (g^{\mu\nu} k_\mu k_\nu + m^2) \sqrt{-g} f_k = 0 \quad (3.5)$$

The on-shell action reduces to surface term:

$$F(k, z) = K \sqrt{-g} g^{zz} f_{-k}(z) \partial_z f_k(z) \quad (3.7)$$

With boundary value $f_k(z_B) = 1$, the retarded Green's function is:

$$G_R(k) = -2F(k, z)|_{z=z_B} \quad (3.8)$$

The solution must obey incoming wave boundary condition at horizon due to black hole's fully-absorbing feature.

3.2 Analytical result in hydrodynamic limit

Near boundary $r = 0$, exponents of equations are (0, 2) for vector modes and (0, 6) for scalar mode. General solutions are:

$$E_T(k, r) = A_T(\tilde{\omega}, \tilde{q})(1 + \dots) + B_T(\tilde{\omega}, \tilde{q})r^2 + \dots \quad (3.9)$$

$$E_L(k, r) = A_L(\tilde{\omega}, \tilde{q})(1 + \dots) + B_L(\tilde{\omega}, \tilde{q})r^2 + \dots \quad (3.10)$$

$$\phi(k, r) = A_\phi(\tilde{\omega}, \tilde{q})(1 + \dots) + B_\phi(\tilde{\omega}, \tilde{q})r^6 + \dots \quad (3.11)$$

Higher powers are omitted. The retarded Green's function connects to coefficients as:

$$G_R(\tilde{\omega}, \tilde{q}) \sim \frac{B(\tilde{\omega}, \tilde{q})}{A(\tilde{\omega}, \tilde{q})} \quad (3.12)$$

Hydrodynamic limit means $\omega \ll T$ and $q \ll T$, or equivalently $\tilde{\omega} \ll 1$ and $\tilde{q} \ll 1$. Near horizon $r = 1$, exponents are $\pm i\tilde{\omega}/5$. Incoming wave boundary conditions require:

$$E_{T,L}(k, r \rightarrow 1) \sim f(r)^{-i\tilde{\omega}/5}, \quad \phi(k, r \rightarrow 1) \sim f(r)^{-i\tilde{\omega}/5} \quad (3.13)$$

We define new variables:

$$E_T(k, r) = f(r)^{-i\tilde{\omega}/5} E_T(r), \quad E_L(k, r) = f(r)^{-i\tilde{\omega}/5} E_L(r), \quad \phi(k, r) = f(r)^{-i\tilde{\omega}/5} \phi(r) \quad (3.14-3.16)$$

Regularity at horizon gives equations for $E_T(r)$, $E_L(r)$, and $\phi(r)$. For transverse mode, double series expansion yields:

$$E_T(r) = F_0(r) + \tilde{\omega} F_1(r) + \dots \quad (3.20)$$

Solving with regularity conditions gives $F_0 = C$ (constant) and a lengthy expression for $F_1(r)$. Expanding to $O(r^2)$:

$$E_T(k, r) = C \left[1 + i\tilde{\omega} \left(\frac{\pi + 2\sqrt{5} \coth^{-1}(\sqrt{5}) - 5 \ln 5}{5\sqrt{5}} \right) + O(r^2) \right] \quad (3.24)$$

The retarded Green's function for transverse electric field is:

$$G_R^{TT}(\tilde{\omega}, \tilde{q}) = -2 \times \frac{(2\pi l_s^2)^2 T_4 N_f}{R} \frac{2B_T(\tilde{\omega}, \tilde{q})}{A_T(\tilde{\omega}, \tilde{q})} \quad (3.25)$$

For longitudinal mode, similar analysis with expansion:

$$E_L(r) = G_0(r) + \tilde{\omega} G_1(r) + \tilde{q}^2 G_2(r) + \dots \quad (3.27)$$

gives $G_0 = C'$ and $G_1 = F_1|_{C=C'}$. The solution for G_2 is:

$$G_2(r) = \frac{iC'}{10} (1 - r^2) \quad (3.34)$$

Expanding $E_L(k, r)$ to $O(r^2)$ yields retarded Green's function:

$$G_R^{LL}(\tilde{\omega}, \tilde{q}) = -2 \times \frac{(2\pi l_s^2)^2 T_4 N_f}{R} \frac{\tilde{\omega}^2}{\tilde{\omega}^2 - \tilde{q}^2} \frac{2B_L(\tilde{\omega}, \tilde{q})}{A_L(\tilde{\omega}, \tilde{q})} \quad (3.36)$$

Transforming to original gauge field A_μ gives nonzero components:

$$G_R^{tt}(\omega, q) = -NT \frac{q^2}{\omega + iDq^2}, \quad G_R^{xx}(\omega, q) = -NT \frac{\omega^2}{\omega + iDq^2} \quad (3.41)$$

$$G_R^{xt}(\omega, q) = G_R^{tx}(\omega, q) = -NT \frac{\omega q}{\omega + iDq^2}, \quad G_R^{yy} = G_R^{zz} = -NTi\omega$$

The diffusion constant is:

$$D = \frac{1}{1.6\pi T} \quad (3.42)$$

The common factor N is:

$$N = \frac{(2\pi l_s^2)^2 T_4 N_f}{R} = \frac{5Q_c N_f}{20\pi g_s l_s} \quad (3.43)$$

For scalar mode, similar procedure gives:

$$\phi(r) = H_0(r) + \tilde{\omega}H_1(r) + \dots \quad (3.44)$$

with solutions $H_0 = D'$ and H_1 yielding:

$$\phi(k, r) = D' \left[1 + i\tilde{\omega} \left(\frac{25 + 2\sqrt{5}\pi + 10\sqrt{5} \coth^{-1} \sqrt{5} - 100 + 5 \ln 5}{5\sqrt{5}} \right) r^5 - i\tilde{\omega}r^6 + O(r^7) \right] \quad (3.49)$$

The scalar correlator is:

$$G_R^{\phi\phi}(\tilde{\omega}, \tilde{q}) = 2 \times 6 \frac{B_\phi(\tilde{\omega}, \tilde{q})}{A_\phi(\tilde{\omega}, \tilde{q})} = -N'T^6 i\omega \quad (3.51)$$

with $N' = 3T_4 N_f R^5 (4\pi/5)^6$.

Spectral functions are proportional to imaginary part of retarded Green's functions:

$$\mathcal{R}(\omega, q) \equiv -2\text{Im} G_R(\omega, q) \quad (3.53)$$

Explicit expressions show vector modes have ω^2 scaling while scalar mode has ω^6 scaling at high frequency.

3.3 Light quark diffusion constant

We extract the light quark diffusion constant from results above. Transport coefficients for stress tensor or conserved charge have been widely studied, mainly focusing on gluon sector. Analysis in [64] studied flavor charge transport in D3/D7 systems. We follow [64] using three methods, finding agreement with D2-brane background results [48]. The black factor $f(r) = 1 - r^5$ appears directly related to diffusion constant.

3.3.1 From the pole of diffusive channel

Current-current correlator in thermal field theory has universal low-frequency behavior: transverse part is nonsingular while longitudinal part has pole at $\omega = -iDq^2$. For flavor current J^μ coupling to gauge field A_μ , the correlator is equivalent to A_μ retarded Green's function. Matching general properties with our results shows the longitudinal part has pole $\omega = -iDq^2$ with:

$$D = \frac{1}{1.6\pi T} \quad (3.62)$$

3.3.2 Membrane paradigm

This approach [48, 61, 62] expresses transport coefficients in terms of metric components. For flavor gauge field with action:

$$S = \frac{1}{4g_{eff}^2} \int d^{p+2}x \sqrt{-\det g} F^{\mu\nu} F_{\mu\nu} \quad (3.63)$$

the diffusion constant has universal formula:

$$D = \frac{\sqrt{-\det \tilde{g}} \tilde{g}^{xx} \tilde{g}^{rr}}{-\tilde{g}^{tt} \tilde{g}^{rr}} \Big|_{r=r_T} \int_{r_T}^{\infty} dr \frac{-\tilde{g}^{tt} \tilde{g}^{rr}}{\sqrt{-\det \tilde{g}}} \quad (3.64)$$

Applying to our model yields $D = 1/(1.6\pi T)$, matching previous result.

3.3.3 Green-Kubo formula

From non-equilibrium statistical physics, kinetic coefficients relate to current correlations:

$$D\Xi = \lim_{\omega \rightarrow 0} \frac{\mathcal{R}_{xx}(\omega, q = 0)}{2\omega} \quad (3.66)$$

Using charge susceptibility Ξ from Section 2.1 and spectral function from Section 3.2 gives $D = 1/(1.6\pi T)$, confirming universality.

4 High frequency: the WKB method

We study high-frequency behavior of retarded Green's functions and spectral functions. In high-frequency limit, we construct WKB solutions to Schrödinger-type equations and match asymptotic solutions to obtain boundary behaviors. The incoming wave boundary condition at horizon extends to near-boundary $r = 0$ because the effective Schrödinger potential is negative—no barrier prohibits propagation, so no reflected waves are generated.

Transforming equations to Schrödinger type:

$$E_T(k, r) = \sqrt{\frac{r}{1-r^5}} \psi_T(k, r) \quad (4.1)$$

$$E_L(k, r) = \sqrt{\frac{1-r^5}{\tilde{\omega}^2 - \tilde{q}^2(1-r^5)}} \psi_L(k, r) \quad (4.2)$$

$$\phi(k, r) = \sqrt{\frac{r}{1-r^5}} \psi_S(k, r) \quad (4.3)$$

For $q = 0$, vector and scalar equations simplify to:

$$\psi''_{T,L} + \left[\frac{-3 + 4r^2(9r^3 - 2r^8 + \tilde{\omega}^2)}{4r^2(1 - r^5)^2} \right] \psi_{T,L} = 0 \quad (4.7)$$

$$\psi''_S + \left[\frac{-35 + 4r^2(15r^3 + \tilde{\omega}^2)}{4r^2(1 - r^5)^2} \right] \psi_S = 0 \quad (4.8)$$

For $\tilde{\omega} \gg 1$, away from singularities:

$$\psi''_{T,L} + \frac{\tilde{\omega}^2}{(1 - r^5)^2} \psi_{T,L} = 0 \quad (4.9)$$

WKB solutions are:

$$\psi_{T,L}^{(1)} = \frac{\cos(s(r) + \phi_1)}{\sqrt{p(r)}}, \quad \psi_{T,L}^{(2)} = \frac{\sin(s(r) + \phi_2)}{\sqrt{p(r)}} \quad (4.11)$$

with $p(r) = \tilde{\omega}/(1 - r^5)$ and $s(r) = \int p(r) dr$.

Near $r = 0$, asymptotic solution is Bessel function combination. Matching with WKB solutions determines phases and yields boundary behavior. The retarded Green's functions at high frequency are:

$$G_R^{xx}(\omega) = G_R^{yy}(\omega) = G_R^{zz}(\omega) = -\frac{5\pi i + (5 + 10 \ln 2 - 10\gamma)}{5} N\omega^2 \quad (4.25)$$

with spectral functions:

$$\mathcal{R}_{xx}(\omega) = \mathcal{R}_{yy}(\omega) = \mathcal{R}_{zz}(\omega) = N\omega^2 \quad (4.26)$$

For scalar mode, similar analysis gives:

$$G_R^{\phi\phi}(\omega) = -\frac{6\pi i + (11 + 12 \ln 2 - 12\gamma)}{6} N'\omega^6 \quad (4.29)$$

$$\mathcal{R}_{\phi\phi}(\omega) = \frac{786432\pi^5}{5^6} N'\omega^6 \quad (4.30)$$

The ω^6 scaling for scalar meson differs from ω^2 for vector modes and can be extracted from operator dimensions.

5 Heavy quark diffusion

We compute heavy quark diffusion constant for comparison with light quark case. In gauge/gravity duality, heavy quark is described by an open string endpoint attached to flavor brane and extending into bulk. This picture has been used for jet quenching [70–72], drag force [73], and energy loss [74, 75]. We follow [77] to calculate diffusion constant from correlation function of string embedding coordinate, studied for $\mathcal{N} = 4$ SYM [77–79] and Sakai-Sugimoto model [80]. Our result resembles [77] but differs from [80].

Heavy quark motion through plasma is described by generalized Langevin equation. For time scales much larger than correlation time τ_c , the equation reduces to local form with friction and random noise. The diffusion constant κ and friction constant η relate to force correlations:

$$\kappa_{ij} \equiv \lim_{\omega \rightarrow 0} G_{ij}^{sym}(\omega) = - \lim_{\omega \rightarrow 0} \coth\left(\frac{\omega}{2T}\right) \text{Im} G_{ij}^R(\omega) \quad (5.7)$$

$$\eta_{ij} \equiv \int_0^\infty d\tau \gamma_{ij}(\tau) \quad (5.8)$$

On gravity side, we identify bulk field coupling to boundary operator F as string embedding coordinate $X^i(r, t)$. For static string in static gauge ($\sigma = r, \tau = t$), fluctuations $\zeta(t, r)$ in X^1 direction satisfy equation from Nambu-Goto action:

$$\ddot{\zeta} + \frac{f'(r)}{f(r)} \dot{\zeta} - r^2 f(r) \zeta'' - \left(2rf(r) + \frac{r^2 f'(r)}{2}\right) \zeta' = 0 \quad (5.12)$$

Fourier transforming and imposing incoming wave boundary condition at horizon yields solution:

$$\zeta(\omega, r) = c(1 + \dots) + c \left(\frac{r^3}{3} + \dots \right) \quad (5.22)$$

The retarded Green's function is:

$$G_R(\omega) = -2 \times \frac{12\pi R^2}{5} i\tilde{\omega} = -\frac{12\pi R^2}{5} i\omega \quad (5.23)$$

The noise correlation constant is:

$$\kappa \equiv - \lim_{\omega \rightarrow 0} \text{Im} G_R(\omega) = \frac{12\pi R^2}{5} \quad (5.24)$$

The heavy quark diffusion constant follows from Einstein relation:

$$D_{heavy} = \frac{2T^2}{\kappa} = \frac{5}{6\pi R^2} T \quad (5.25)$$

This matches $\mathcal{N} = 4$ SYM result [77] up to model-dependent factor, but differs from Sakai-Sugimoto model [80] which scales as T^4 . Both light and heavy quark diffusion constants depend on temperature as $1/T$ in this model, indicating universal features.

6 Conclusion

We investigated properties of chiral symmetric phase in non-critical holographic QCD, described by N_f pairs of D4 and D4 branes in near-horizon geometry of N_c coincident D4 branes in six dimensions (probe limit $N_f \ll N_c$). In this phase, flavor pairs extend parallel into black hole background, making induced geometry on flavor worldvolume similar to background but one dimension less. The embedding profile is trivial—parallel lines in x_4 - u plane—making the model simpler than D3/D7 systems yet similar to Sakai-Sugimoto.

We extracted charge susceptibility and analyzed flavor gauge and scalar mode fluctuations, including equations of motion, retarded Green's functions, and spectral functions in hydrodynamic and high-frequency limits. Holographic spectral functions and quasi-normal modes for mesonic operators have been studied extensively [42–44, 64, 65, 81, 84–100]. Generally, differential equations require numerical work, but we focused on two analytical limits.

In hydrodynamic limit, we obtained analytical results for retarded Green's functions. Vector meson operators decompose into transverse and longitudinal parts: transverse correlator has no pole while longitudinal has simple pole $\omega = -iDq^2$ responsible for flavor diffusion. Scalar meson correlator behaves like transverse vector mode—linear in frequency with no pole. Temperature dependence differs: vector modes scale linearly with T while scalar mode scales as T^6 , consistent with previous studies but model-dependent.

We extracted light quark diffusion constant from longitudinal pole using three methods: pole analysis, membrane paradigm, and Green-Kubo formula. Agreement across methods confirms universality. In high-frequency limit, we converted equations to Schrödinger type and used WKB approximation. Spectral functions become temperature-independent and frequency scaling follows from operator dimensions: ω^2 for vector modes, ω^6 for scalar mode.

For heavy quark diffusion, we followed trailing string picture [77] and found results similar to $\mathcal{N} = 4$ SYM but different from Sakai-Sugimoto model, signaling a key difference between models. Both light and heavy quark diffusion constants scale as $1/T$, suggesting universal gauge/gravity features.

Future directions include: studying relationship between stable mesons in chiral broken phase and quasi-normal modes in symmetric phase; introducing quark

mass effects; investigating external electromagnetic fields and chemical potential; and extending beyond probe limit using backgrounds with flavor backreaction [11].

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