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Authors: Bu,YY, Yang,JM

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Full Text

Preamble

Structure Function of Holographic Quark-Gluon Plasma: Sakai-Sugimoto Model versus its Non-Critical Version

Yan Yan Bu∗ and Jin Min Yang†

Institute of Theoretical Physics, Academia Sinica, Beijing 100190, P. R. China

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Abstract

Motivated by recent studies of deep inelastic scattering (DIS) off the $\mathcal{N} = 4$ super-Yang-Mills (SYM) plasma holographically dual to $AdS_5 \times S^5$ black hole, we in this note use the spacelike flavor current to probe the internal structure of holographic quark-gluon plasma described by the Sakai-Sugimoto model at high temperature phase (i.e., the chiral symmetric phase). The plasma structure function is extracted from the retarded flavor current-current correlator. Our main aim is to explore the effect of non-conformality on these physical quantities. As usual, our study is under the supergravity approximation and the limit of large color number. Although the Sakai-Sugimoto model is non-conformal, which makes the calculations more involved than the well-studied $\mathcal{N} = 4$ SYM case, the result seems to indicate that the non-conformality has little essential effect on the physical picture of the internal structure of holographic plasma, consistent with the intuition from asymptotic freedom of QCD at high energy. While the physical picture underlying our investigation is the same as DIS off the $\mathcal{N} = 4$ SYM plasma with(out) flavor, the plasma structure functions are quantitatively different, especially in their scaling dependence on temperature, which can be recognized as model-dependent. As a comparison, we also perform the same analysis for the non-critical version of the Sakai-Sugimoto model, which is conformal in the sense that it has a constant dilaton vacuum. The result for this non-critical model is much more similar to the conformal $\mathcal{N} = 4$ SYM plasma. We therefore attribute the above difference to the effect of non-conformality of the Sakai-Sugimoto model.

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∗ yybu@itp.ac.cn

† jmyang@itp.ac.cn

Introduction

In heavy ion collisions, which are now experimentally studied at RHIC and LHC, the so-called quark-gluon plasma appears to be strongly interacting and behaves like a perfect liquid, which is greatly different from the previously recognized weakly-coupled picture. This brings non-perturbative investigations of hadronic matter at high temperature and high density produced in heavy ion collisions into an urgent stage. While the lattice method can calculate some properties of strongly-coupled systems, it is still constrained to extracting static quantities such as hadron mass spectrum and thermodynamical behavior. What is worse is that when adding finite density or chemical potential into thermal QCD, lattice calculations usually confront the notorious sign problem. Therefore, improvement in theoretical understanding of strongly-coupled quark-gluon plasma (sQGP) should not only go beyond the traditional perturbative QCD (pQCD) approach but also reveal some properties out of equilibrium, such as transport properties, dispersion relations, high-energy scattering, and so on.

Gauge/gravity duality states that a strong coupling gauge theory can be mapped to weak coupling gravity with a negative cosmological constant in the limit of large 't Hooft coupling and large N_c, where N_c is the number of colors in the gauge theory. Although the gravity dual of realistic QCD has not been established till now, one expects this duality to be of great importance in understanding some non-perturbative properties of QCD or at least some universal features of strongly-coupled systems. In fact, over the last decade, using gauge/gravity duality techniques to study properties of sQGP has achieved great success. However, most studies have focused on static or hydrodynamic properties at large scales or long times compared to the inverse temperature of the system. Therefore, it is of great interest to study hard probes of the plasma and reveal its internal structure, which should be contrasted with the parton picture of a single hadron in pQCD. Studies on this topic were originally proposed for $\mathcal{N} = 4$ SYM plasma with(out) flavor, and the main lesson from these investigations is that at high energy the flavor current probes the partonic behavior of the plasma, giving nonvanishing plasma structure function; while at low energy the current is not absorbed by the plasma, indicating vanishing contribution to the plasma structure function. Besides the plasma structure function, another important quantity is the so-called plasma saturation momentum, which is the critical energy defining the transition from weakly quasi-elastic scattering to high-energy deep inelastic scattering. In other words, the partonic picture for holographic plasma exists only when all partons have transverse momenta below the saturation momentum.

Since previous studies mainly focused on the D3-brane geometry, which is dual to the conformal $\mathcal{N} = 4$ SYM, we in this note use one non-conformal gravity dual model of QCD to probe the effect of non-conformality on the plasma structure. To be specific, the model under consideration here is the transversely-intersecting D4/D8/$\overline{\text{D8}}$ brane system, now usually referred to as the Sakai-Sugimoto model. It is one of the most successful holographic QCD models from the top-down approach in realizing some phenomena of low-energy QCD such as confinement/deconfinement phase transition, non-Abelian chiral symmetry breaking, and vector meson dominance. The temperature under gauge/gravity duality approach can be realized by extending the color brane geometry to a black hole. The high temperature phase means that chiral symmetry is restored, which is denoted by parallel profile of flavor D8/$\overline{\text{D8}}$ branes. Note that the temperature will be much smaller than the four-momentum of the flavor current, since in this note we are focusing on high-energy scattering to probe the internal structure of holographic plasma. The setup in this note is similar to previous work, but due to the fact that the induced metric on the flavor worldvolume reduces to $\text{AdS}_5 \times \text{S}^4$ black hole, it is essentially the same as that of the $\mathcal{N} = 4$ SYM plasma. Thus, the procedure here is similar to earlier studies and much simpler than some other setups.

To extract the structure function, we need to study the flavor current propagation in holographic plasma. According to gauge/gravity duality, this can be achieved by studying the U(1) flavor gauge field in curved spacetime. The corresponding action and equation of motion determining this dynamics are encoded in a Maxwell theory in curved five-dimensional spacetime. On the other hand, the parton structure function has a standard definition in quantum field theory, which is encoded in the retarded current-current correlator. One main task of gauge/gravity duality is to calculate the retarded Green function for finite temperature field theory from dual gravity following the standard recipe. In this sense, the information of the structure function is totally encoded in the solution of the above-mentioned Maxwell equation in five-dimensional curved spacetime of asymptotic AdS type. One additional key point under this prescription is that the solution should obey incoming wave boundary condition at the horizon, reflecting the full-absorption characteristic of black holes. Once the equation of motion for the flavor U(1) gauge field is turned into Schrödinger type, it will be found that their behavior is very similar to that of the $\mathcal{N} = 4$ SYM case. This is natural since the setup of holographic models is very general. This may also be recognized as one universal characteristic of gauge/gravity duality.

Another motivation for our study is to probe some universal features of the structure of holographic plasma described by intersecting D-brane systems in conformal or non-conformal cases. We have learned that the procedure and the physical picture for these calculations are universal, which is in part due to the unified approach of gauge/gravity duality in dealing with strongly-coupled problems. The specific form of the structure function is model-dependent, which may allow us to resort to experiments to judge which model is closer to physical reality.

For the two structure functions, we find that in the non-conformal background they also satisfy the Callan-Gross relation in the limit of large Bjorken variable, which is the same as in the $\mathcal{N} = 4$ SYM case. Therefore, it is reasonable to conjecture that this relation should be universal for holographic plasma described by intersecting D-brane systems, having nothing to do with the conformality of the holographic background.

Since the flavor gauge field we are focusing on can also be considered as the gravity dual of vector mesons, our study can be regarded as the completion of previous studies of mesonic quasinormal modes. Here, we extend these studies to high momentum and high frequency limit, in contrast to previous hydrodynamic behavior or just high frequency limit. However, we will not go into detailed numeric extraction of mesonic quasinormal frequency in high frequency and high momentum limit, leaving this task for future work.

We will also perform the same analysis for the non-critical version of the Sakai-Sugimoto model for comparison. This non-critical model was proposed to overcome some drawbacks of the critical Sakai-Sugimoto model, which are general in critical string theory. This model can be thought of as conformal, so we expect the related results should be more similar to the $\mathcal{N} = 4$ SYM case. This expectation has been confirmed by the fact that the structure function for the non-critical model has the same scaling dependence on temperature as that of the $\mathcal{N} = 4$ SYM plasma. This should be another difference between the Sakai-Sugimoto model and its non-critical version.

Another striking point is that stringy imprints have appeared in the final results of the structure functions for both models considered in this note. Seemingly, this would sway the successful aspects of the Sakai-Sugimoto model. However, if we recall that the holographic plasma is quite different from realistic sQGP, we should just take these unsatisfactory features as non-universal ones and focus on universal features emergent from different holographic models.

The rest of this note is organized as follows. In Sec. II we first give a brief overview of the Sakai-Sugimoto model as well as its non-critical version, and then provide the basic equations for later calculations. In Sec. III we state detailed extraction of the structure function for the Sakai-Sugimoto model and list the main results for its non-critical version. Then we have a brief discussion about the results. Sec. IV is devoted to a short summary and some open questions.

Overview of Models and Basic Equations

In this section we first recapitulate the Sakai-Sugimoto model and its non-critical version. Then we turn to the definitions for the plasma structure function in terms of physical quantities in standard field theory. We also state the basic equations determining the flavor current propagation in these plasmas from the viewpoint of dual gravity, which are essential for later extraction of the structure function. We follow the notation conventions in previous literature.

Sakai-Sugimoto Model versus its Non-Critical Version

The bulk background geometry of the Sakai-Sugimoto model is given by a ten-dimensional supergravity description of N_c coincident D4-branes in type-IIA superstring theory compactified on a circle. According to Witten, there are two different metrics for this supergravity, representing two different phases of holographic QCD. The transition between these phases is interpreted as the deconfinement phase transition. Here, we focus on the high temperature deconfining phase, described by the following backgrounds:

$$ds^2 = \left(\frac{u}{R}\right)^{3/2} \left[-f(u)dt^2 + d\vec{x}^2 + dx_4^2\right] + \left(\frac{R}{u}\right)^{3/2} \left[\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right]$$

$$e^{\phi} = g_s \left(\frac{u}{R}\right)^{3/4}, \quad f(u) = 1 - \frac{u_T^3}{u^3}$$

The temperature of holographic plasma dual to the above background is $T = \frac{3}{4\pi}\frac{u_T^{1/2}}{R^{3/2}}$. The curvature radius R of the background is related to the string coupling g_s and string length l_s by $R^3 = \pi g_s N_c l_s^3$. Here in the second equality we have defined the 't Hooft coupling constant λ from the dual gravity side as $\lambda = g_s N_c$.

The above gravity background is dual to the gluon sector, and the quark sector can be introduced in quenched approximation by adding N_f pairs of D8 and $\overline{\text{D8}}$ flavor branes to the above geometry and making them transverse to the circle along x_4. In the quenched limit $N_f \ll N_c$, the backreaction of the flavor branes on the background geometry can be neglected. Dynamics of the flavor sector is encoded in the Dirac-Born-Infeld (DBI) plus Chern-Simons (CS) actions for the flavor branes in the above background. However, the CS term will be exactly zero in this note as there is no background for the U(1) gauge field on the flavor branes.

Chiral phase transition for the flavor sector in this deconfined phase has a beautiful geometric explanation: parallel profile of the flavor D8 and $\overline{\text{D8}}$ branes stands for chiral restoring phase while connected U-shaped profile for chiral broken phase. In general, high temperature corresponds to chiral restoring phase while low temperature to chiral broken phase. In the high temperature phase, the induced metric on the flavor branes has the following standard AdS form:

$$ds^2 = \left(\frac{u}{R}\right)^{3/2}\left[-f(u)dt^2 + d\vec{x}^2\right] + \left(\frac{R}{u}\right)^{3/2}\left[\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right]$$

The other model we consider here is the non-critical version of the above one. It is based on the supergravity description of N_c coincident D4-branes in six dimensions with one dimension compactified on a circle as in the Sakai-Sugimoto model. The corresponding background geometry takes the following form in high temperature phase:

$$ds^2 = \left(\frac{u}{R}\right)^{2}\left[-f(u)dt^2 + d\vec{x}^2 + dx_4^2\right] + \left(\frac{R}{u}\right)^{2}\frac{du^2}{f(u)}$$

$$e^{\phi} = g_s, \quad f(u) = 1 - \frac{u_T^4}{u^4}, \quad R^2 = \frac{g_s N_c l_s^2}{2}$$

The flavor quark sector can be introduced by adding N_f pairs of D4 and $\overline{\mathrm{D4}}$ branes into the above background geometry. One important feature of this model is that it does not have the undesired internal space, which may introduce unwanted KK modes. Another striking characteristic is that it has constant dilaton vacuum as for the D3-brane geometry in ten dimensions, which signals that this model is conformal. As in the Sakai-Sugimoto model, chiral restoring phase means the induced metric on the flavor branes takes the form:

$$ds^2 = \left(\frac{u}{R}\right)^{2}\left[-f(u)dt^2 + d\vec{x}^2\right] + \left(\frac{R}{u}\right)^{2}\frac{du^2}{f(u)}$$

For convenience in later calculations, we now rescale the radial coordinate u to make it dimensionless by the transformation $u_T/u = r$. Then the induced geometry on the flavor branes takes the following simplified versions:

$$ds^2 = \frac{R^2}{r^2}\left[-f(r)dt^2 + d\vec{x}^2\right] + \frac{R^2}{r^2}\frac{dr^2}{f(r)} + R^2 d\Omega_4^2$$

$$e^{\phi} = g_s, \quad f(r) = 1 - r^3$$

for the Sakai-Sugimoto model, and

$$ds^2 = \frac{R^2}{r^2}\left[-f(r)dt^2 + d\vec{x}^2\right] + \frac{R^2}{r^2}\frac{dr^2}{f(r)}$$

$$f(r) = 1 - r^4$$

for the non-critical model. Note that under the new coordinates, the interval for radial coordinate r is located in a finite regime: $r \in [0, 1]$, with the horizon at $r = 1$ and boundary at $r = 0$, which makes later analysis convenient.

Before concluding this subsection we give a short comment about one general feature of the above models. Whether there is background on flavor branes, fluctuations of the flavor gauge field and scalar mode in chiral restoring phase will always decouple, which together with exact AdS forms of the induced metrics will greatly simplify later calculations. This is also one reason why we choose the transversely intersecting D-brane systems for study. As mentioned in Sec. I, the plasma structure function is completely encoded in the dynamics of the flavor U(1) gauge field propagating through the above-mentioned geometry, which is described by the DBI actions on the flavor branes.

Deep Inelastic Scattering: Field Theoretical Definitions

Deep inelastic scattering in QCD is a powerful tool for exploring hadron structure. Here we focus on electromagnetic mediation between the charged lepton and the hadron. The basic objective of DIS is to compute the retarded current-current correlator defined by:

$$\Pi_{\mu\nu}(k) = i \int d^{4x} e^{-ik\cdot x} \theta(x^0) \langle [J_\mu(x), J_\nu(0)] \rangle$$

where k is the four-momentum of the electromagnetic current, $\langle \cdot \rangle$ means quantum vacuum expectation, and $J_\mu(x)$ is the mediated electromagnetic current. When considering lepton scattering off the plasma, the hadron should be replaced by the plasma system and the vacuum polarization tensor is modified to:

$$\Pi_{\mu\nu}(k, T) = i \int d^{4x} e^{-ik\cdot x} \theta(x^0) \langle [J_\mu(x), J_\nu(0)] \rangle_T$$

where the subscript T means thermal expectation value in the plasma system.

Although the gravity dual of the $\mathrm{SU}(N_c) \times \mathrm{U}(1)_{\mathrm{e.m.}}$ gauge theory has not been established, we can use a non-dynamical electromagnetic field to model the photon just as in condensed matter physics, given that the electromagnetic coupling constant is very small. In the present context, the electromagnetic current is replaced by the flavor U(1) current and also denoted as $J_\mu(x)$.

Now we list the general structure of the thermal polarization tensor. According to gauge symmetry and rotation symmetry of thermal field theory, it can be decomposed into two scalar functions as:

$$\Pi_{\mu\nu}(k,T) = \Pi_1(x,Q^2)\left(\eta_{\mu\nu} - \frac{k_\mu k_\nu}{Q^2}\right) + \Pi_2(x,Q^2)\left(n_\mu - \frac{n\cdot k}{Q^2}k_\mu\right)\left(n_\nu - \frac{n\cdot k}{Q^2}k_\nu\right)$$

where $\eta_{\mu\nu} = \mathrm{diag}(-1,1,1,1)$, $Q^2 = -k^\mu k_\mu$ is the current virtuality, and n^μ is the four-velocity of the plasma which we choose as $n^\mu = (1,0,0,0)$ to signal that the plasma is at rest. We have also defined the Bjorken variable as $x = Q^2/[2(n\cdot k)T]$. Then the DIS structure function of the plasma can be extracted from the polarization tensor as:

$$F_1 \equiv \frac{1}{2\pi}\mathrm{Im}\,\Pi_1, \quad F_2 \equiv \frac{1}{2\pi}\mathrm{Im}\,\Pi_2$$

For later convenience, we now express these two functions in terms of longitudinal and transverse polarization tensors Π_{LL} and Π_{yy} introduced in Sec. III:

$$\mathrm{Im}\,\Pi_{yy} = \mathrm{Im}\,\Pi_{zz} = \frac{2\pi}{\tilde{q}^2}F_1$$

$$\mathrm{Im}\,\Pi_{LL} = \frac{8\pi x}{\tilde{q}^2}\left(F_2 - 2xF_1\right)$$

In obtaining these expressions we have used the flavor current momentum defined later and assumed the plasma is at rest. From these relations we can see that in the interesting kinematic regime $\omega^2/q^2 \gg 1 \Rightarrow Q^2/q^2 \ll 1$, F_2 can be simplified further to $F_2 \simeq 2xF_1 + \frac{\tilde{q}^2}{8\pi x}\mathrm{Im}\,\Pi_{LL}$. If the first term could be negligible, we would straightforwardly obtain the familiar Callan-Gross relation $F_2 \simeq 2xF_1$.

In particle physics, the structure function has been studied by operator product expansion techniques for specific hard processes. The parton model is suitable for the weak coupling regime of high-energy QCD, while in the present case for holographic quark-gluon plasma, which is thought of as strongly-coupled, we resort to its gravity dual for calculations of these quantities. There is a standard prescription for calculating the retarded Green function such as the polarization tensor under the approach of gauge/gravity duality. The current J_μ couples to its source $A_\mu(x, r=0)$ as $S_{\mathrm{int}} = \int d^{4x} J_\mu A^\mu(x, r=0)$, where $A_\mu(x,r)$ is the flavor U(1) gauge field introduced in the next subsection. The main idea behind this prescription is to invert the operator Green function on the field theory side to the dual field Green function on the gravity side, whose calculation only needs classical gravity action.

Now we explicitly write down the expression for the polarization tensor defined above in terms of variables on the dual gravity side:

$$\Pi_{\mu\nu}(k,T) = \frac{\delta^2 S}{\delta A_\mu(k)\delta A_\nu(-k)}\bigg|_{r=0}$$

where S is the on-shell action defined later. Due to coupling of A_x and A_t, we have to express the on-shell action in terms of longitudinal mode A_L and transverse modes A_y, A_z defined in the next subsection.

Basic Equations: Flavor Current Propagation in Plasma

As in many works on applications of gauge/gravity duality to strong coupling problems, we choose radial gauge for the gauge potential, i.e., $A_r = 0$. This is sufficient for the non-critical case while for the Sakai-Sugimoto model we also have internal symmetry on Ω_4 space. For brevity, we also set the gauge potential along it to zero and assume that the gauge potential does not depend on the internal coordinate. As to the action, we retain only terms quadratic in the gauge field fluctuation, which is sufficient for the propagation of the flavor current. In the following, we write down the main equations for the gauge fields.

For the Sakai-Sugimoto model, the DBI action for the fluctuation of the flavor U(1) gauge field takes the following form after integrating out the Ω_4 space:

$$S^{(2)}_{\text{DBI}} = -\frac{T_8 N_f V_{S^4}}{2g_s^2}\int d^{4x}dr\sqrt{-g_5}\, g^{MN}g^{PQ}F_{MP}F_{NQ}$$

where $F_{MP} = \partial_M A_P - \partial_P A_M$ is the field strength, g_{MN} is the induced metric, and the indices M, N run over the five dimensions. The equation of motion for the gauge field A_M follows from this action:

$$\partial_M\left(\sqrt{-g_5}g^{MN}g^{PQ}F_{NQ}\right) = 0$$

Now we turn to momentum space by performing the following partial Fourier transformation:

$$A_\mu(x,r) = \int \frac{d^{4k}}{(2\pi)^4}e^{ik\cdot x}A_\mu(k_\mu,r), \quad k_\mu = (\omega, q, 0, 0)$$

Without loss of generality, we have chosen the spatial momentum along just one spatial direction as done in the literature. Then in partial momentum space, the on-shell action turns into:

$$S_{\text{on-shell}} = -\frac{T_8 N_f V_{S^4}}{2g_s^2}\int \frac{d^{4k}}{(2\pi)^4}\left[g^{rr}g^{tt}A_t(k,r)\partial_r A_t(-k,r) + g^{rr}g^{ii}A_i(k,r)\partial_r A_i(-k,r)\right]$$

while the equations of motion can be explicitly cast into three equations:

$$\partial_r\left(\sqrt{-g_5}g^{rr}g^{tt}\partial_r A_t\right)-\sqrt{-g_5}g^{tt}g^{xx}q(\omega A_x+qA_t)=0$$

$$\partial_r\left(\sqrt{-g_5}g^{rr}g^{yy}\partial_r A_y\right)+\sqrt{-g_5}g^{tt}g^{yy}\left(\frac{\omega^2}{f(r)}-q^2\right)A_y=0$$

$$\partial_r\left(\sqrt{-g_5}g^{rr}g^{tt}\partial_r(\omega A_x+qA_t)\right)+\sqrt{-g_5}g^{tt}g^{xx}\frac{\omega^2-q^2f(r)}{f(r)}(\omega A_x+qA_t)=0$$

The component A_z satisfies the same equation as the A_y component, and we refer to them as transverse modes. Another important relation between A_x and A_t is:

$$\sqrt{-g_5}g^{rr}g^{tt}\partial_r A_t+\sqrt{-g_5}g^{tt}g^{xx}\frac{q}{\omega}(\omega A_x+qA_t)=0$$

In the above equations, prime denotes derivative with respect to the radial coordinate r. Once the induced metric is inserted, they can be simplified to the following forms:

$$a''+\frac{f'(r)}{f(r)}a'+\left[\frac{\tilde{\omega}^2-\tilde{q}^2f(r)}{rf^2(r)}-\frac{f'(r)}{2rf(r)}\right]a=0$$

$$A_y''+\frac{f'(r)}{f(r)}A_y'+\frac{\tilde{\omega}^2-\tilde{q}^2f(r)}{rf^2(r)}A_y=0$$

where $(\tilde{\omega},\tilde{q})=\frac{R^{3/2}}{4\pi T u_T^{1/2}}(\omega,q)$ are dimensionless variables, a denotes A_t', and A_L is the longitudinal mode defined as $A_L\equiv qA_t+\omega A_x$. Note that we have written these equations in a general form so that they should also be suitable for the non-critical model studied later.

Similarly, we can write down analogous equations for the non-critical version of the Sakai-Sugimoto model. For brevity we just list the main results. The on-shell action is:

$$S_{\text{on-shell}}=-\frac{T_5N_f}{2g_s^2}\int\frac{d^{4k}}{(2\pi)^4}\left[r^{-1}f(r)A_i(k,r)\partial_r A_i(-k,r)+q^{-1}A_L(k,r)a(-k,r)\right]$$

The equations of motion become:

$$a''+\frac{f'(r)}{rf(r)}a'+\left[\frac{\tilde{\omega}^2-\tilde{q}^2f(r)}{r^2f^2(r)}-\frac{f'(r)}{rf(r)}\right]a=0$$

$$A_y'' + \frac{f'(r)}{rf(r)}A_y' + \frac{\tilde{\omega}^2 - \tilde{q}^2 f(r)}{r^2 f^2(r)}A_y = 0$$

Here the only difference from the Sakai-Sugimoto model in notation lies in the specific definitions of the dimensionless variables $\tilde{\omega}$ and $\tilde{q}$: $(\tilde{\omega}, \tilde{q}) = \frac{R}{0.8\pi T}(\omega, q)$.

We now express the flavor brane actions in terms of A_L, A_y, A_z and a as mentioned in the last subsection. Because similar calculations have been done many times in the literature, we here merely list the final results. For the Sakai-Sugimoto model we have:

$$S_{\text{on-shell}} = -\frac{T_8 N_f V_{S^4}}{2g_s^2}\int \frac{d^{4k}}{(2\pi)^4}\left[g^{rr}g^{tt}q^{-1}A_L(k,r)a(-k,r) + g^{ii}A_i(k,r)\partial_r A_i(-k,r)\right]$$

and for the non-critical model:

$$S_{\text{on-shell}} = -\frac{T_5 N_f}{2g_s^2}\int \frac{d^{4k}}{(2\pi)^4}\left[r^{-1}f(r)A_i(k,r)\partial_r A_i(-k,r) + q^{-1}A_L(k,r)a(-k,r)\right]$$

Some remarks are due about these equations which will determine later calculations. Explicitly, these equations are more involved than the related ones in previous work on D3/D7 brane setup. We can attribute this complexity to the introduction of flavor branes and the non-conformality of the model. However, once these equations of motion are turned into standard Schrödinger types and taking high momentum and high frequency limits, we find that these complications automatically disappear and they are qualitatively similar to those of earlier studies.

We now proceed by following previous work to turn these equations of motion into standard Schrödinger types and discuss general features of effective potentials. These general discussions will reveal physical pictures for the DIS processes considered in this note. The equations obeyed by new fields have the following common form irrespective of the models:

$$\psi_L''(r) + V_L(r)\psi_L(r) = 0 \quad \text{(for longitudinal mode)}$$

$$\psi_T''(r) + V_T(r)\psi_T(r) = 0 \quad \text{(for transverse mode } A_y \text{ or } A_z\text{)}$$

For the Sakai-Sugimoto model, the explicit field transformations and effective potentials are:

$$\psi_L = r^{-1/4}(1-r^3)^{1/2}a, \quad V_L(r) = \frac{9}{16r^2} + \frac{9r}{8(1-r^3)} + \frac{9r^4}{16(1-r^3)^2} + \frac{\tilde{\omega}^2 - \tilde{q}^2(1-r^3)}{r(1-r^3)^2}$$

$$\psi_T = r^{-1/4}A_y, \quad V_T(r) = \frac{9}{16r^2} + \frac{3r}{8(1-r^3)} + \frac{9r^4}{16(1-r^3)^2} + \frac{\tilde{\omega}^2 - \tilde{q}^2(1-r^3)}{r(1-r^3)^2}$$

While for the non-critical model, we have:

$$\psi_L = r^{-1/2}(1-r^4)^{1/2}a, \quad V_L(r) = \frac{3}{4r^2} + \frac{2r^2}{1-r^4} + \frac{r^6}{(1-r^4)^2} + \frac{\tilde{\omega}^2 - \tilde{q}^2(1-r^4)}{r^2(1-r^4)^2}$$

$$\psi_T = r^{-1/2}A_y, \quad V_T(r) = \frac{3}{4r^2} + \frac{r^6}{(1-r^4)^2} + \frac{\tilde{\omega}^2 - \tilde{q}^2(1-r^4)}{r^2(1-r^4)^2}$$

In these equations, we have defined the dimensionless current virtuality as $K^2 = \tilde{q}^2 - \tilde{\omega}^2$. We take this virtuality to be space-like, which amounts to saying that the process considered here is like lepton deep inelastic scattering off the proton. But as noted in previous work, the final results will also be suitable for time-like virtuality if we take the high momentum limit.

Since we are interested in the internal structure of holographic quark-gluon plasma, we should use high-energy probes to explore this, just as in DIS processes in pQCD. One basic difference between the plasma and a single hadron is that the former has an intrinsic scale (temperature). In short, we focus on the following kinematic parameter space:

$$\tilde{q} \gg 1, \quad \tilde{\omega} \gg 1, \quad K^2 \ll \tilde{q}^2, \tilde{\omega}^2$$

Under the above kinematics, these effective potentials can be further approximated as follows. For the Sakai-Sugimoto model we have:

$$V_L(r) \simeq \frac{15}{4r^2} + \frac{\tilde{q}^2}{r^3}, \quad V_T(r) \simeq \frac{15}{4r^2} + \frac{\tilde{q}^2}{r^3}$$

while for the non-critical one:

$$V_L(r) \simeq \frac{3}{4r^2} + \frac{\tilde{q}^2}{r^5}, \quad V_T(r) \simeq \frac{3}{4r^2} + \frac{\tilde{q}^2}{r^5}$$

These are the main ingredients for later extraction of the structure functions of holographic plasma.

We have seen that in the interesting kinematic regime, for both the Sakai-Sugimoto model and its non-critical version, the effective potentials for longitudinal as well as transverse modes are qualitatively similar to those of the $\mathcal{N} = 4$ SYM case with(out) flavors. More explicitly, the maximum of the longitudinal potential can be positive (corresponding to potential barrier), negative (with no barrier), or zero according to the value of $\tilde{q}/K^4$ (Sakai-Sugimoto model) or $\tilde{q}^2/K^7$ (non-critical model). The effective potentials for transverse modes are more involved because they start from positive infinity and then fall to negative infinity very rapidly, which may complicate later analysis using WKB approximation. Recalling that the dilaton vacuum for the Sakai-Sugimoto model is not constant while for the non-critical version it is constant, these facts together seem to indicate that non-conformality of the Sakai-Sugimoto model is not essential for the physical picture of high-energy scattering processes. This is a byproduct of the general behavior analysis for the effective potentials, which we will confirm by direct extraction of the structure functions for both holographic models.

Before concluding this section, we briefly summarize the physical picture governing high-energy DIS from the viewpoint of non-relativistic quantum mechanics. For the longitudinal mode, when a potential barrier builds up (corresponding to small spatial momentum case), the wave function cannot satisfy the incoming wave condition at the horizon due to the high and narrow barrier, indicating zero structure function. When taking into account non-perturbative tunneling effects, a small exponentially suppressed structure function can be obtained. In the following sections we therefore focus on the high spatial momentum limit. In this regime, the wave function will be complex and the incoming wave condition can be imposed at the horizon. Moreover, in this high-energy regime, a partonic picture for the plasma exists.

Structure Function of Holographic Quark-Gluon Plasma

Now we have all the elements to calculate the polarization tensor. As mentioned above, we should focus on the high momentum kinematics. This means that the K^2 terms in effective potentials can be ignored, which makes semi-analytical solutions for these Schrödinger equations possible. In this section we follow the standard WKB approach in non-relativistic quantum mechanics to construct these solutions. We present the calculations in detail for the Sakai-Sugimoto model and then list the final results for its non-critical version.

We first discuss the longitudinal mode. Under the high momentum approximation, the Schrödinger equation for the longitudinal mode takes the following simple form near the boundary:

$$\psi_L''(r) + \frac{15}{4r^2}\psi_L(r) + \tilde{q}^2 r^2 \psi_L(r) = 0$$

Its general solution is a linear combination of Bessel and Neumann functions:

$$\psi_L(r \to 0) = c_1 \tilde{q}^{1/4} r^{1/2} J_{1/8}\left(\frac{\tilde{q}r^2}{2}\right) + c_2 \tilde{q}^{1/4} r^{1/2} N_{1/8}\left(\frac{\tilde{q}r^2}{2}\right)$$

where the constants c_1, c_2 will be determined by imposing incoming wave boundary condition at the horizon, which requires matching solutions in different regimes.

Near $r \to 1$, the Schrödinger equation can be approximated as:

$$\psi_L''(r) + \frac{3}{(1-r)^2}\psi_L(r) = 0$$

Its general solution is:

$$\psi_L(r \to 1) = c_3(1-r)^{3/2} + c_4(1-r)^{-1/2}$$

Imposing incoming wave boundary condition at the horizon leads to $c_4 = 0$, leaving the general solution near the horizon as $\psi_L(r \to 1) = c_3(1-r)^{3/2}$.

We also need to study the solution in the intermediate regime far from the singularities at $r = 0$ and $r = 1$. In this regime, the Schrödinger equation is approximated as:

$$\psi_L''(r) + \tilde{q}^2 r^2 \psi_L(r) = 0$$

For convenience, we define the canonical momentum $p(r)$ and action $s(r)$:

$$p(r) = \tilde{q}r, \quad s(r) = \int p(r)dr = \frac{\tilde{q}r^2}{2}$$

The two linearly independent solutions in the intermediate regime under WKB approximation are:

$$\psi_L^{(1)}(r) = \frac{1}{\sqrt{p(r)}}\cos[s(r) + \phi_1], \quad \psi_L^{(2)}(r) = \frac{1}{\sqrt{p(r)}}\sin[s(r) + \phi_2]$$

The asymptotic behaviors for $p(r)$ and $s(r)$ at singularities are necessary for matching solutions in different regimes:

$$p(r \to 0) \sim \tilde{q}r, \quad s(r \to 0) \sim \frac{\tilde{q}r^2}{2}$$

$$p(r \to 1) \sim \tilde{q}, \quad s(r \to 1) \sim \tilde{q}r + \text{constant}$$

The next step is to match these solutions to determine c_1, c_2 and c_3. In doing this, we need the asymptotic expansion for the Bessel or Neumann function for very large argument:

$$J_\nu(z) \sim \sqrt{\frac{2}{\pi z}} \cos\left(z - \frac{\nu\pi}{2} - \frac{\pi}{4}\right)$$

$$N_\nu(z) \sim \sqrt{\frac{2}{\pi z}} \sin\left(z - \frac{\nu\pi}{2} - \frac{\pi}{4}\right)$$

It is then easily found that if we choose $\phi_1 = \phi_2 = 5\pi/16$ and $c_2 = ic_1$, the matching of solutions in different regimes is accomplished. Note that the condition $c_2 = ic_1$ is the main result from the solution matching, which is also a direct reflection of the incoming wave boundary condition imposed at the horizon.

Now we easily obtain the boundary behavior for the wave function $\psi_L(r)$ as:

$$\psi_L(r \to 0) = c_1 \tilde{q}^{1/4} r^{1/2} J_{1/8}\left(\frac{\tilde{q}r^2}{2}\right) + ic_1 \tilde{q}^{1/4} r^{1/2} N_{1/8}\left(\frac{\tilde{q}r^2}{2}\right) = c_1 \tilde{q}^{1/4} r^{1/2} H^{(1)}_{1/8}\left(\frac{\tilde{q}r^2}{2}\right)$$

In the second line we have written the solution as the first kind Hankel function with order 1/8.

The effective potential for the transverse mode is more involved, but the analysis under WKB approximation is similar, so we just list the final results. Matching of solutions in three different regimes (near horizon, near boundary, and intermediate regime) results in the following boundary behavior for the transverse mode A_y and A_z:

$$\psi_T(r \to 0) = c_1^{(i)} \tilde{q}^{1/4} r^{3/2} H^{(1)}_{3/8}\left(\frac{\tilde{q}r^2}{2}\right)$$

The only undetermined constants c_1 and $c_1^{(i)}$ can be expressed in terms of boundary values of the gauge field $A_L(r=0) \equiv A_L(0)$ or $A_{y,z}(r=0) \equiv A_i(0)$ respectively. This can be achieved using the field transformations and the final results essential for extraction of structure functions are:

For longitudinal mode:

$$A_L(r \to 0) = c_1 R^3 q^{-1} \tilde{q}^{1/4} r^{3/2} (1 - r^3) r^{1/2} H^{(1)}_{1/8}\left(\frac{\tilde{q}r^2}{2}\right)$$

$$c_1 = -\frac{i\pi 2^{3/4}}{q\Gamma(1/8)\tilde{q}^{1/8}} A_L(0)$$

For transverse modes A_i $(i = y, z)$:

$$A_i(r \to 0) = c_1^{(i)} \tilde{q}^{1/4} r^{3/2} H_{3/8}^{(1)} \left(\frac{\tilde{q} r^2}{2} \right)$$

$$c_1^{(i)} = -\frac{i\pi 2^{3/4}}{\Gamma(3/8)\tilde{q}^{1/8}} A_i(0)$$

With these solutions, we can derive the expressions for the thermal polarization tensor:

$$\mathrm{Im}\,\Pi_{LL}(k,T) = \frac{12\Gamma^2(1/8)}{g_s \lambda N_f N_c T \tilde{q}^{1/4}}$$

$$\mathrm{Im}\,\Pi_{yy}(k,T) = \mathrm{Im}\,\Pi_{zz}(k,T) = \frac{54\Gamma^2(3/8)}{g_s \lambda N_f N_c T^3 \tilde{q}^{3/4}}$$

and other components are exactly vanishing. Referring to these results, we have just listed the imaginary parts which are directly related to the structure function. The real part of the polarization tensor is divergent and therefore needs regularization (we skip these details). Moreover, for simplicity we assume the regularization will not introduce new terms into the imaginary parts.

Therefore, the structure functions F_1 and F_2 are easily derived:

$$F_1(k,T) = \frac{108\Gamma^2(3/8)}{g_s \lambda N_f N_c T^3 \tilde{q}^{3/4}}$$

$$F_2(k,T) = \frac{12\Gamma^2(1/8)}{g_s \lambda N_f N_c T \tilde{q}^{1/4}} + 2xF_1$$

In the first line we can easily show that the first term can be ignored compared to the second one in the interesting kinematic regime $\tilde{q}/K^4 \gg 1$, so we have the approximate equality $F_2 \simeq 2xF_1$.

Following previous work, we now express the two structure functions in terms of the Bjorken variable x defined in Sec. II B and the flavor current virtuality Q^2:

$$F_1(x,Q^2) = \frac{108\Gamma^2(3/8)}{g_s \lambda N_f N_c T^3} \left(\frac{8\pi x T^2}{\lambda N_f N_c Q^2} \right)^{3/4}$$

$$F_2(x,Q^2) = 2xF_1(x,Q^2) = \frac{2\lambda N_f N_c T^3 x}{8\pi x T^2} \left(\frac{8\pi x T^2}{\lambda N_f N_c Q^2} \right)^{1/4}$$

In obtaining these equations, we have used the approximate relation $\tilde{\omega} \simeq \tilde{q}$ to express the spatial momentum as $q \simeq Q^2/(2xT)$.

Before closing this subsection, we briefly carry out similar analysis for the non-critical model. Since the general procedure for using the WKB method has been presented in detail, we here just list the key results. The solution for the longitudinal mode near boundary $r = 0$ behaves as:

$$A_L(r \to 0) = C_1 R^4 q^{-1} \tilde{q}^{1/7} r(1-r^4) H^{(1)}_{2/7}\left(\frac{\tilde{q} r^{7/2}}{2}\right)$$

$$C_1 = -\frac{i\pi 2^{2/7}}{q\Gamma(2/7)\tilde{q}^{1/7}} A_L(0)$$

and for the transverse mode $A_i(r)$ $(i = y, z)$:

$$A_i(r \to 0) = C_1^{(i)} \tilde{q}^{1/7} r^5 H^{(1)}_{5/7}\left(\frac{\tilde{q} r^{7/2}}{2}\right)$$

$$C_1^{(i)} = -\frac{i\pi 2^{2/7}}{\Gamma(5/7)\tilde{q}^{1/7}} A_i(0)$$

The polarization tensor can be derived by inserting these solutions into the on-shell action:

$$\operatorname{Im}\Pi_{LL}(k,T) = \frac{\sqrt{3} R N_f N_c}{56\sqrt{2}\pi g_s l_s} \tilde{q}^{2/7}$$

$$\operatorname{Im}\Pi_{yy}(k,T) = \operatorname{Im}\Pi_{zz}(k,T) = \frac{7\sqrt{6}\pi R N_f N_c}{25\Gamma^2(2/7) g_s l_s} \tilde{q}^{4/7}$$

Then the structure functions are:

$$F_1(k,T) = \frac{7\sqrt{6} R N_f N_c}{50\Gamma^2(2/7) g_s l_s} \tilde{q}^{4/7}$$

$$F_2(k,T) = \frac{\sqrt{3} N_f N_c}{56\sqrt{2}\pi g_s l_s} \tilde{q}^{2/7} + 2x F_1(k,T)$$

In the first line, one can easily show that the first term can be neglected in the interesting regime $\tilde{q}^{4/7} \gg K^2$, so we have the approximate equality $F_2 \simeq 2xF_1$ as in the Sakai-Sugimoto model. We now express these results in terms of the Bjorken variable x and flavor current virtuality Q^2:

$$F_1(x,Q^2) = \frac{7\sqrt{6}RN_fN_c}{50\Gamma^2(2/7)g_sl_s}\left(\frac{8\pi xT^2}{N_fN_cQ^2}\right)^{2/7}$$

$$F_2(x,Q^2) = 2xF_1(x,Q^2) = \frac{2N_fN_cT^2x}{8\pi xT^2}\left(\frac{8\pi xT^2}{N_fN_cQ^2}\right)^{3/7}$$

These results comprise the main findings of this note. We now have a short remark on these results and a brief comparison between the two models as well as the well-investigated $\mathcal{N}=4$ SYM case.

The first point is that we have an analogy of the Callan-Gross relation $F_2 \simeq 2xF_1$ in the interesting kinematic regime. In pQCD, this relation holds only at relatively large Bjorken variable x, where parton structures of hadrons are dominated by valence quarks. This relation has already been obtained in DIS off $\mathcal{N}=4$ SYM plasma with(out) flavors, and here it also holds for the Sakai-Sugimoto model as well as its non-critical version. Since the setups of holographic dual of sQGP are very general, and the physical picture underlying flavor current DIS off the plasma is quite simple and general, these surprising facts seem to indicate that it may be a general relation for holographic quark-gluon plasma.

The second key point concerns the non-conformal characteristic of the Sakai-Sugimoto model, which is the main motivation of the present study. Because in our interesting kinematic regime the two structure functions are related by the Callan-Gross relation, we mainly focus on F_1. It is clear that for the two models F_1 shows different scaling behavior in its dependence on temperature T and dimensionless spatial momentum $\tilde{q}$. The dependence on dimensionless momentum $\tilde{q}$ is approximately the same, while the temperature dependence is quite different: T^{-3} for the Sakai-Sugimoto model and T^{-2} for the non-critical one. Recalling that the latter scaling behavior T^{-2} has also been found for $\mathcal{N}=4$ SYM plasma, we may think of the essential effect of non-conformality of the Sakai-Sugimoto model as the T^{-3} scaling behavior of the F_1 structure function. However, this guess needs further confirmation because the non-conformality of the Sakai-Sugimoto model is not well-controlled. Moreover, since the gauge coupling constant of strong interaction has logarithmic running with energy scale and simple gravity realization of this kind of gauge theory has been established, it may be interesting to resort to such models to probe the effect of gauge coupling running on the internal structure of sQGP.

The last point concerns the pre-factors for the structure functions. Similar to previous work, N_fN_c counts the number of degrees of freedom of the plasma, and we here probe the quark sector. The models considered have left some stringy imprints on field-theoretical quantities (here the plasma structure function). This can be easily read from the results, which have explicit dependence on string coupling constant g_s and string length l_s. Moreover, the behavior concerning these two stringy parameters seems different between the Sakai-Sugimoto model and its non-critical version. These facts seem strange because we focused on field-side quantities which should not show explicit dependence on gravity or string side parameters. Compared to related results for $\mathcal{N}=4$ SYM plasma, these undesirable behaviors did not appear there, which seems to say that the D3-brane geometry is a much better gravity dual for describing field-theoretical physical quantities. However, if we recall that the models we use here are different from realistic QCD theory, then it is acceptable that the results are counter-intuitive from field-theoretical considerations.

Summary and Outlook

In this note, we have used the high-temperature version of the Sakai-Sugimoto model, a quite successful gravity dual model of QCD-like theory, to explore the internal structure of holographic quark-gluon plasma. The physical process we analyze is analogous to well-investigated DIS in standard QCD, but with the scattered proton replaced by the plasma system and the mediated electromagnetic current simulated by the flavor current. We have seen that the procedure under gauge/gravity duality to study DIS off holographic quark-gluon plasma is quite general and easily promoted to other gauge/gravity duality setups. The result obtained for the structure function under the Sakai-Sugimoto model is quite different from the well-studied $\mathcal{N}=4$ SYM plasma with(out) flavors. This should be regarded as the effect of non-conformality of the Sakai-Sugimoto model, which was the most important motivation for our study. To confirm this, we also performed a comparative study using the non-critical version of the Sakai-Sugimoto model. We found that the structure functions for the latter model and the $\mathcal{N}=4$ SYM plasma are much alike. The result of this note seems to contradict intuition from asymptotic freedom of pQCD, but we should keep in mind that holographic quark-gluon plasma considered here is a strongly-coupled system and we should not expect it to behave exactly as the weakly-interacting regime of realistic QCD. In fact, a more realistic holographic QCD model taking into account the running of the gauge coupling constant was proposed previously, and we expect to use such models to explore the effect of gauge coupling running on the structure of sQGP.

In realistic QCD, N_f and N_c are of order one, while the applicability of gravity dual of SU(N_c) gauge theory usually requires a large N_c limit. Therefore, we cannot directly compare our results with data from heavy-ion collisions. One prescription to overcome this obstacle is to consider flavor backreaction to the background geometry and then carry out similar calculations in the $N_f/N_c \sim 1$ limit. Although the hadronic matter produced in heavy-ion collisions is at high temperature and high density, we in this note only take into account the high temperature element as in the literature. So in this sense, our present models are not so realistic and need to be promoted to high-energy and high-density quark-gluon plasma cases. Fortunately, under the gauge/gravity duality setup, matter density also has a gravity realization—charge or flavor charge, which can be realized by rotating color branes along internal space or as the time component of the flavor gauge field respectively. Then the analysis of DIS off quark-gluon plasma at high density and high temperature can also be carried out by including this element in the model setup. We leave these problems for future investigations.

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Note: Figure translations are in progress. See original paper for figures.