Yukawa Corrections to Top Pair Production in Photon-Photon Collisions (Postprint)

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Date: 2016-12-28T00:00:00+00:00

Abstract

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Full Text

Preamble

Yukawa corrections to top pair production in photon-photon collision

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ABSTRACT

The $O(\alpha m_t^2/m_W^2)$ Yukawa corrections to top pair production in photon-photon collision are calculated in the Standard Model (SM), the general two-Higgs-doublet model (2HDM), as well as the minimal supersymmetric model (MSSM). We find that the correction to the cross section can only reach a few percent in the SM, but can be quite significant ($> 10\%$) in the 2HDM and MSSM for favorable parameter values, which may be observable at high-energy e^+e^- colliders.

PACS numbers: 14.65.Ha; 12.38Bx; 14.80.Cp
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Recently, evidence for top quark production has been reported by the CDF and D0 collaborations, with measured masses of $176 \pm 8(\text{stat}) \pm 10(\text{syst})$ GeV and $199^{+19}_{-21}(\text{stat}) \pm 22(\text{syst})$ GeV, respectively [1].

1. Introduction

Due to its large mass, the discovery of the top quark opens a number of new and interesting issues, such as the precision measurement of its mass, width, and Yukawa couplings through direct production and subsequent decay at both hadron and e^+e^- colliders. However, even with 1000 pb^{-1} of luminosity, the Fermilab Tevatron could determine the top mass only to about 5 GeV or better [2]. At future multi-TeV proton colliders such as the CERN Large Hadron Collider (LHC), $t\bar{t}$ production will be enormously larger than at the Tevatron, but the accuracy with which the top mass can be measured in proton colliders is limited to about 2-3 GeV [2]. Blondel et al. [3] have argued that one must know the top mass to within 1 GeV to take full advantage of the constraints that precision electroweak measurements place on the Higgs boson and other massive particles that might contribute to electroweak loops. Beyond this, it would be highly desirable to make a precision measurement of the basic parameter m_t to 0.3 GeV or better, in order to search for new physics beyond the SM through loop processes that are sensitive to m_t.

At the next-generation linear collider (NLC) operating at a center-of-mass energy of 500 GeV–2000 GeV with luminosity of order 10^{33} $\text{cm}^{-2}\text{s}^{-1}$, the $e^+e^- \to t\bar{t}$ event rate would be around 10^4/yr, comparable to that at the Tevatron; however, the events would be much cleaner and top parameters would be easier to extract. At the NLC, a top mass measurement with statistical uncertainty of 0.3 GeV from 10 fb^{-1} luminosity is expected [2], and it is possible to separately measure all of the various production and decay form factors of the top quark at the level of a few percent [4].

Nowadays, the possibility of transforming a linear e^+e^- collider into a $\gamma\gamma$ collider deserves considerable attention. With the advent of new collider techniques [5], the collision of high-energy, high-intensity photon beams—obtained using the old idea of Compton laser backscattering [6]—can be realized at the NLC. The backscattering of laser photons off the colliding electron and positron beams can yield intense and energetic photon beams that collide with high luminosity. There are many uses for such high photon-photon luminosity, one of the most important being the production of top quark pairs. It has been found [7] that $t\bar{t}$ production in $\gamma\gamma$ collisions realized by laser backscattering is slightly larger than the direct $e^+e^- \to t\bar{t}$ process, and at $\sqrt{s} = 1$ TeV the $\gamma\gamma \to t\bar{t}$ production rate for $m_t < 130$ GeV is much larger than $e^+e^- \to t\bar{t}$ at $\sqrt{s} = 0.5$ TeV for $m_t \sim 200$ GeV, both with and without considering threshold QCD effects.

In the SM, the cross section for top quark pair production in $\gamma\gamma$ collisions has been calculated with higher-order QCD corrections [8]. The radiative corrections to $\gamma\gamma t\bar{t}$ from final-state Higgs exchange interactions have also been computed in Ref. [9]. The correction is of order $O(\lambda_t^4) \sim$ 2–4% for typical values of the Higgs boson mass and top quark mass. In this paper we calculate the $O(\alpha m_t^2/m_W^2)$ Yukawa correction in a two-Higgs-doublet model (2HDM) (Model II) [10] and in the minimal supersymmetric model (MSSM), in which there are three neutral and two charged physical Higgs bosons, $H, h, A, H^{\pm}$, where H and h are CP-even and A is CP-odd. The $O(\alpha m_t^2/m_W^2)$ Yukawa corrections arise from the virtual effects of the third-family (top and bottom) quarks, charged and neutral Higgs bosons, as well as the Goldstone bosons ($G^0, G^{\pm}$). The results of the Standard Model can be obtained from our calculations as a special case.

In Sec. II, we present the analytic results in terms of the well-known standard notation of one-loop Feynman integrals. In Sec. III, we present numerical examples and discuss the implications of our results. In the appendix we list the form factors in the cross section.

2. Calculations

The relevant Feynman diagrams are shown in [Figure 1: see original paper] and the Feynman rules can be found in Ref. [10]. In our calculation, we use dimensional regularization to regulate all ultraviolet divergences in the virtual loop corrections and adopt the on-mass-shell renormalization scheme [11].

In our calculations we keep the term $m_b \tan\beta$ in the charged Higgs couplings to third-family quarks since its effects become rather important for large $\tan\beta$.

Taking into account the $O(\alpha m_t^2/m_W^2)$ Yukawa corrections, the renormalized amplitude for $t\bar{t}$ production is given by

$$M_{\text{ren}} = M_{\text{ren}}^{(t)} + M_{\text{ren}}^{(u)}$$

where

$$M_{\text{ren}}^{(t)} = M_0^{(t)} + \delta M^{(t)}$$

with

$$\delta M^{(t)} = \delta M_s^{(t)} + \delta M_v^{(t)} + \delta M_b^{(t)} + \delta M_\Delta^{(t)}$$

and $M_{\text{ren}}^{(u)}$ is obtained from $M_{\text{ren}}^{(t)}$ by exchanging $p_3 \leftrightarrow p_4$, $\hat{t} \leftrightarrow \hat{u}$. Here M_0 is the tree-level amplitude, while δM_s, δM_v, δM_b, and δM_Δ represent the $O(\alpha m_t^2/m_W^2)$ Yukawa corrections arising from the self-energy diagram [FIGURE:1(d)], vertex diagrams [FIGURE:1(f)-(i)], box diagrams [FIGURE:1(l)-(n)], and diagrams [FIGURE:1(j),(k)], respectively. The definitions are $\hat{t} = (p_4 - p_2)^2$, $\hat{u} = (p_1 - p_4)^2$, where $p_3(p_4)$ denote the momenta of the two incoming photons, and $p_2(p_1)$ are the momenta of the outgoing top quark and its antiparticle.

The explicit forms of these matrix elements are given by

$$\delta M_s^{(t)} = \frac{ie^2Q_t^2}{16\pi^2 m_t^2}\epsilon_\mu(p_4)\epsilon_\nu(p_3)\bar{u}(p_2)\gamma^\mu(\gamma\cdot p_1 + m_t)\gamma^\nu v(p_1)$$

$$\delta M_v^{(t)} = \frac{ie^2Q_t^2}{16\pi^2 m_t^2}\epsilon_\mu(p_4)\epsilon_\nu(p_3)\bar{u}(p_2)(f_2^s\gamma^\mu\gamma^\nu + f_6^s p_2^\mu\gamma^\nu + f_{12}^s \not{p}_4\gamma^\mu\gamma^\nu)v(p_1)$$

$$\delta M_b^{(t)} = \frac{ie^2Q_t^2}{16\pi^2 m_t^2}\epsilon_\mu(p_4)\epsilon_\nu(p_3)\bar{u}(p_2)(f_2^v\gamma^\mu\gamma^\nu + f_3^v p_1^\nu\gamma^\mu + f_6^v p_2^\mu\gamma^\nu + f_{12}^v \not{p}_4\gamma^\mu\gamma^\nu + f_{13}^v \not{p}_4 p_1^\nu\gamma^\mu + f_{16}^v \not{p}_4 p_2^\mu\gamma^\nu)v(p_1)$$

$$\delta M_\Delta^{(t)} = \frac{ie^2Q_t^2}{16\pi^2 m_t^2}\epsilon_\mu(p_4)\epsilon_\nu(p_3)\bar{u}(p_2)\left[\frac{1}{2}g^{\mu\nu} + f_2^\Delta\gamma^\mu\gamma^\nu\right]v(p_1)$$

The form factors f_i^s are presented in Appendix A.

The corresponding amplitude squared can be written as

$$|M_{\text{ren}}|^2 = |M_0^{(t)}|^2 + |M_0^{(u)}|^2 + 2\text{Re}\left[M_0^{(t)}M_0^{(u)\dagger}\right] + \delta|M^{(t)}|^2 + \delta|M^{(u)}|^2 + \delta|M^{(t,u)}|^2$$

where

$$\delta|M^{(t)}|^2 = 2\text{Re}\left[\delta M^{(t)}M_0^{(t)\dagger}\right]$$

and

$$\delta|M^{(t)}|^2 = \delta|M_s^{(t)}|^2 + \delta|M_v^{(t)}|^2 + \delta|M_b^{(t)}|^2 + \delta|M_\Delta^{(t)}|^2$$

with

$$\delta|M_s^{(t)}|^2 = \frac{\pi\alpha^3 m_t^2}{m_W^2\hat{t}^2}\left(f_6^s H_6 + f_{12}^s H_{12}\right)$$

$$\delta|M_v^{(t)}|^2 = \frac{\pi\alpha^3 m_t^2}{m_W^2\hat{t}^2}\left(f_2^v H_2 + f_3^v H_3 + f_6^v H_6 + f_{12}^v H_{12} + f_{13}^v H_{13} + f_{16}^v H_{16}\right)$$

$$\delta|M_b^{(t)}|^2 = \frac{\pi\alpha^3 m_t^2}{m_W^2\hat{t}^2}\left(f_1^b H_1 + f_2^b H_2 + f_5^b H_5 + f_6^b H_6 + f_9^b H_9 + f_{10}^b H_{10}\right)$$

$$\delta|M_\Delta^{(t)}|^2 = \frac{\pi\alpha^3 m_t^2}{m_W^2 \hat{t}^2}\left(f_1^\Delta H_1 + f_2^\Delta H_2\right)$$

Here the expressions for $H_i(m_t, p_1 \cdot p_2, p_1 \cdot p_3, p_1 \cdot p_4, p_3 \cdot p_4)$ are given in Appendix B. The terms $\delta|M^{(u)}|^2$, $\delta|M^{(t,u)}|^2$ can be obtained by crossing symmetry.

The cross section of the subprocess is given by

$$\hat{\sigma}(\hat{s}) = \frac{1}{16\pi\hat{s}^2}\int_{\hat{t}_-}^{\hat{t}_+} d\hat{t}\,|M_{\rm ren}(\hat{s},\hat{t})|^2$$

where $\hat{t}_\pm = (m_t^2 - \frac{1}{2}\hat{s}) \pm \frac{1}{2}\hat{s}\beta_t$ and $\beta_t = \sqrt{1 - 4m_t^2/\hat{s}}$.

The total cross section for top quark pair production can be obtained by folding the subprocess cross section $\hat{\sigma}$ with the photon luminosity:

$$\sigma(s) = \int_{2m_t/\sqrt{s}}^{x_{\rm max}} dz\,\frac{dL_{\gamma\gamma}}{dz}\,\hat{\sigma}(\gamma\gamma \to t\bar{t} \text{ at } \hat{s} = z^2 s)$$

where $\sqrt{s}$ ($\sqrt{\hat{s}}$) is the e^+e^- ($\gamma\gamma$) center-of-mass energy and the photon luminosity is defined as

$$\frac{dL_{\gamma\gamma}}{dz} = z\int_{z^2/x_{\rm max}}^{x_{\rm max}} \frac{dx}{x}\,F_{\gamma/e}(x)F_{\gamma/e}(z^2/x)$$

For unpolarized initial electrons and laser photons, the energy spectrum of the backscattered photon is given by [12]

$$F_{\gamma/e}(x) = \frac{1}{D(\xi)}\left[1 - x + \frac{1}{1-x} - \frac{4x}{\xi(1-x)} + \frac{4x^2}{\xi^2(1-x)^2}\right]$$

where

$$D(\xi) = \left(1 - \frac{4}{\xi} - \frac{8}{\xi^2}\right)\ln(1+\xi) + \frac{1}{2} + \frac{8}{\xi} - \frac{1}{2(1+\xi)^2}$$

and $\xi = 4E_0\omega_0/m_e^2$, with m_e and E_0 being the incident electron mass and energy, respectively, and ω_0 the laser photon energy. Here x is the fraction of the energy of the incident electron carried by the backscattered photon. In our calculation we follow the analysis of Ref. [9] and choose ω_0 such that it maximizes the backscattered photon energy without spoiling the luminosity through e^+e^- pair creation. With this choice, we find $\xi = 2(1+\sqrt{2}) \approx 4.8$, $x_{\rm max} \simeq 0.83$, and $D(\xi) \approx 1.8$.

3. Numerical Results and Conclusion

In our numerical calculations, the input parameters [13] are $m_Z = 91.176$ GeV, $\alpha_{\rm em} = 1/128.8$, and $G_F = 1.166372 \times 10^{-5}\,{\rm GeV}^{-2}$. The W boson mass is determined through

$$m_W^2 = \frac{m_Z^2}{2}\left[1 + \sqrt{1 - \frac{4\pi\alpha_{\rm em}}{\sqrt{2}G_F m_Z^2(1-\Delta r)}}\right]$$

where, to order $O(\alpha m_t^2/m_W^2)$, Δr is given by [4]

$$\Delta r = \frac{\alpha N_C}{16\pi^2 s_W^4}\frac{m_t^2}{m_W^2}$$

The lower limit of the parameter $\tan\beta$ is 0.6 from perturbative bounds [14]. Reference [15] argues for lower values of $\tan\beta$ from perturbative unitarity, which is about 0.25 for a top quark mass of 176 GeV. Therefore, in our numerical calculations we allow $\tan\beta$ to take a minimum value of 0.25 in the two-Higgs-doublet model. In the following we present numerical examples corresponding to an e^+e^- collider with center-of-mass energy $\sqrt{s} = 500$ GeV.

The numerical results in the SM are presented in [Figure 2: see original paper]. The correction to the cross section depends on the Higgs mass, and at $M_h = 300$ GeV the correction reaches its maximum size of 2.7%. Recently, the correction in the Standard Model has been calculated in Ref. [16]. However, in that work the authors only presented the correction to the subprocess cross section $\gamma\gamma \to t\bar{t}$ and did not give the corresponding results for an e^+e^- collider, making it difficult to compare their results with ours.

We present the numerical results in the two-Higgs-doublet model in [Figure 3: see original paper] and [Figure 4: see original paper]. In our results we fix the parameters α and β to be $\alpha = \beta = 0.25$ and show the dependence on the Higgs boson masses. The correction is sensitive to the Higgs masses and can be quite large for small Higgs masses. [Figure 3: see original paper] shows the dependence on the masses of the CP-even Higgs bosons h and H for a fixed M_A value. We find that the correction can be quite large for small M_h values. For $M_h < 100$ GeV the correction can exceed 50%, making it necessary to calculate higher-order corrections beyond one-loop level. [Figure 4: see original paper] shows the dependence on the mass of the CP-odd Higgs boson A for fixed $M_{h,H}$ values. For $M_A = 100$ GeV the correction reaches -38% and decreases rapidly with increasing M_A. The corrections drop rapidly with increasing $\tan\beta$, as in the case of the minimal supersymmetric model discussed below. Here we do not present numerical results corresponding to large $\tan\beta$.

[Figure 5: see original paper]–[Figure 7: see original paper] show some numerical results in the minimal supersymmetric model. The Higgs sector of the minimal supersymmetric model is a special case of the two-Higgs-doublet model. In this model the masses and couplings of the Higgs bosons are controlled by two parameters at tree level, which can be taken to be M_A and β, for example. In our numerical results presented in [Figure 3: see original paper]–[Figure 5: see original paper], we show the dependence on M_A for three different values of $\tan\beta$. From these figures one can see that the correction depends strongly on the value of $\tan\beta$. The correction is more significant for smaller $\tan\beta$ values. For a fixed $\tan\beta$ value, the correction can be either positive or negative, depending on the Higgs mass M_A. For the minimum β value $\beta = 0.25$, the correction reaches its positive maximum of 13% at $M_A = 420$ GeV and its negative maximum of -54% at $M_A = 300$ GeV. For $\tan\beta = 1$, the positive and negative maximum sizes of the correction can only reach 7% and -1.6%, respectively. For the larger $\tan\beta$ value $\tan\beta = 5$, the behavior of the plot in [Figure 7: see original paper] is different from the small $\tan\beta$ plots in [Figure 5: see original paper] and [Figure 6: see original paper] since the effect of the coupling $m_b \tan\beta$ becomes significant when $\tan\beta$ is large and cancels to some extent the effect of the coupling $m_t \cot\beta$.

In conclusion, we have calculated the $O(\alpha m_t^2/m_W^2)$ Yukawa corrections to top pair production in photon-photon collisions in the Standard Model (SM), the two-Higgs-doublet model, and the minimal supersymmetric model. We find that the correction to the cross section can only reach a few percent in the SM, but can be quite significant ($> 10\%$) in the 2HDM and MSSM for favorable parameter values. These corrections are therefore potentially observable at the next-generation linear collider and could be used to set limits on the parameters of these new models. Thus, precision studies of top pair production in photon-photon collisions at the NLC will be a powerful indirect probe for new physics beyond the Standard Model.

This work was supported by the National Natural Science Foundation of China and a grant from the State Commission of Science and Technology of China.

Appendix A

The form factors f_i^s are given by

$$f_1^s = \sum_{X_i=A,Z} (F_1 - F_0)(\hat{t}, m_t, m_i) + \sum_{X_i=H^0,h} (F_1 + F_0)(\hat{t}, m_t, m_i) + (F_0 - F_1)(m_t^2, m_t, m_i)$$

$$f_2^s = \sum_{X_i=H^\pm,W^\pm} F_1(\hat{t}, 0, m_i) + \sum_{X_i=H^0,h} F_1(m_t^2, m_t, m_i)$$

$$f_6^s = 2f_2^s + \hat{t}^2 \left\{ \sum_{X_i=H^0,h} \frac{p_3}{m_t^2}(G_0 + G_1)(m_t^2, m_t, m_i) - \sum_{X_i=A,Z} \frac{p_1 \cdot p_3}{m_t^2}(G_1 - G_0)(m_t^2, m_t, m_i) \right\}$$

$$f_{12}^s = \sum_{X_i=A,Z} \frac{p_1 \cdot p_3}{m_t^2} F_1(\hat{t}, m_t, m_i) + \sum_{X_i=H^0,h} \frac{p_1 \cdot p_3}{m_t^2} (F_1 + F_0)(\hat{t}, m_t, m_i) + \sum_{X_i=H^\pm,W^\pm} \frac{p_1 \cdot p_3}{m_t^2} F_1(m_t^2, m_t, m_i) + 2m_t^2 p_1 \cdot$$

$$f_1^v = \sum_{X_i=A,Z} \eta_i \left[p_2 \cdot p_4 C_{11}(p_2, p_4, m_i, m_t, m_t) + 2C_{24}\right] + \sum_{X_i=H^0,h} \eta_i \left[p_2 \cdot p_4 (C_{11} - 2C_{23}) + 2C_{24}\right] + \sum_{X_i=H^\pm,W^\pm} \eta_i \left[p_2 \cdot\right.$$

$$f_2^v = \sum_{X_i=A,Z} \eta_i \left[p_1 \cdot p_3 (C_{21} - 4C_0) + 2C_{24}\right] + \sum_{X_i=H^0,h} \eta_i \left[p_1 \cdot p_3 (2C_0 + C_{11} - 2C_{12} - 2C_{23}) + 2C_{24}\right] + \sum_{X_i=H^\pm,W^\pm} \eta_i \left[\right.$$

$$f_3^v = \sum_{X_i=H^\pm,W^\pm} \eta_i \left[p_1 \cdot p_3 (C_{11} - 2C_{23}) + 2C_{24}\right]$$

$$f_6^v = \sum_{X_i=A,Z} \eta_i \left[p_2 \cdot p_4 (2C_{11} + C_{21}) + 2C_{24}\right] + \sum_{X_i=H^0,h} \eta_i \left[p_2 \cdot p_4 (C_{12} + C_{23}) + 2C_{24}\right] + \sum_{X_i=H^\pm,W^\pm} \eta_i \left[p_2 \cdot p_4 (C_{11} + C\right.$$

$$f_{12}^v = \sum_{X_i=A,Z} \eta_i \left[2p_2 \cdot p_4 (C_{12} + C_{23}) + 2C_{24} + m_t^2 C_{21}\right] + \sum_{X_i=H^0,h} \eta_i \left[2p_2 \cdot p_4 (C_{12} + C_{23}) + 2C_{24} + m_t^2 C_{21}\right] + \sum_{X_i=H^\pm}$$

$$f_{13}^v = \sum_{X_i=H^\pm,W^\pm} \eta_i \left[2p_1 \cdot p_4 (C_{12} + C_{23}) + 2C_{24} + m_t^2 C_{21}\right]$$

$$f_{16}^v = \sum_{X_i=H^\pm,W^\pm} \eta_i \left[2p_1 \cdot p_3 (C_{12} + C_{23}) + 2C_{24} + m_t^2 C_{21}\right]$$

where

$$F_n(q, m_1, m_2) = \int_0^1 dy\, y^n \log\left[\frac{q^2 y(1-y) + m_1^2 y + m_2^2 (1-y)}{q^2 y(1-y) + m_1^2 y + m_2^2 (1-y)}\right]$$

$$G_n(q, m_1, m_2) = \int_0^1 dy\, \frac{y^{n+1}(1-y)}{q^2 y(1-y) + m_1^2 y + m_2^2 (1-y)}$$

and $C_{24} \equiv -\frac{1}{4}\Delta + C_{24}$, while C_0, C_{ij} are the three-point Feynman integrals, whose definitions and expressions can be found in Ref. [17].

The form factors f_i^b are given by

$$f_i^b = f_i^{b(1)} + f_i^{b(2)} + f_i^{b(3)} + f_i^{b(4)} + f_i^{b(5)}$$

where

$$f_1^{b(1)} = 2m_t \sum_{X_i=H^0,h} \eta_i (2D_{27} - D_{311} + 2D_{312})$$

$$f_2^{b(1)} = 2m_t \left\{ \sum_{X_i=A,Z} \eta_i \left[m_t^2 (D_0 - D_{12}) + 2p_3 \cdot p_4 (D_{13} - D_{26} + D_{38}) \right] + \sum_{X_i=H^0,h} \eta_i \left[m_t^2 (D_{34} + D_{36}) + 2(2D_{311} - \right.\right.$$

$$f_5^{b(1)} = 2m_t \left\{ \sum_{X_i=A,Z} \eta_i \left[p_4 (D_{11} + D_{12} - D_{21} - D_{22} - 2D_{24} - D_{31} + D_{310}) + 2p_1 \cdot p_4 (D_{22} - D_{25}) + 2p_2 \cdot p_4 (D_{23} \right.\right.$$

$$f_6^{b(1)} = 2m_t \left\{ \sum_{X_i=A,Z} \eta_i \left[p_4 (D_{11} - D_{13} + D_{21} - D_{25}) + 2p_1 \cdot p_4 (D_{24} - D_{26} + D_{36}) + 2p_2 \cdot p_4 (D_{24} + D_{34}) + 2(2D \right.\right.$$

$$f_9^{b(1)} = 2m_t \left\{ \sum_{X_i=A,Z} \eta_i (D_{12} - D_{13} + D_{22} - D_{26} - D_{34} + D_{35} + D_{36} - D_{310}) + \sum_{X_i=H^0,h} \eta_i (D_{12} - D_{13} + D_{22} - \right.$$

$$f_{10}^{b(1)} = 2m_t \left\{ \sum_{X_i=A,Z} \eta_i (D_{11} - D_{13} + D_{24} - D_{26} - D_{31} + D_{34} + D_{35} - D_{310}) + \sum_{X_i=H^0,h} \eta_i (D_{11} - D_{13} + D_{24} - \right.$$

$$f_{11}^{b(1)} = 2m_t \left\{ \sum_{X_i=A,Z} \eta_i (D_{27} + D_{313}) + \sum_{X_i=H^0,h} \eta_i (D_{27} + D_{313}) + \sum_{X_i=H^\pm,W^\pm} \eta_i (D_{27} + D_{313}) \right\}$$

$$f_{12}^{b(1)} = 2m_t \left\{ \sum_{X_i=A,Z} \eta_i (2D_{23} + 2D_{25} + 2D_{26} + D_{35} + D_{38}) + \sum_{X_i=H^0,h} \eta_i (2D_{23} + 2D_{25} + 2D_{26} + D_{35} + D_{38}) + \right.$$

$$f_{13}^{b(1)} = 2m_t \left\{ \sum_{X_i=A,Z} \eta_i (3D_0 - 2D_{11} + D_{13} - D_{21} - D_{39} + D_{310}) + 2(D_{27} + 3D_{313}) + 2p_1 \cdot p_3 (D_{39} + D_{310}) \right\}$$

$$f_{14}^{b(1)} = 2m_t \left\{ \sum_{X_i=A,Z} \eta_i (D_{12} + D_{22} - D_{26} - D_{310}) + \sum_{X_i=H^0,h} \eta_i (D_{12} + D_{22} - D_{26} - D_{310}) + \sum_{X_i=H^\pm,W^\pm} \eta_i (D_{12} + \right.$$

$$f_{15}^{b(1)} = 2m_t \left\{ \sum_{X_i=A,Z} \eta_i (D_{22} - D_{24} + D_{25} - D_{26} - D_{34} + D_{35} + D_{36} - D_{310}) + \sum_{X_i=H^0,h} \eta_i (D_{22} - D_{24} + D_{25} - \right.$$

$$f_{16}^{b(1)} = 2m_t \left\{ \sum_{X_i=A,Z} \eta_i (D_{27} + D_{313}) + \sum_{X_i=H^0,h} \eta_i (D_{27} + D_{313}) + \sum_{X_i=H^\pm,W^\pm} \eta_i (D_{27} + D_{313}) \right\}$$

The remaining form factors $f_i^{b(2)}$, $f_i^{b(3)}$, $f_i^{b(4)}$, and $f_i^{b(5)}$ are defined analogously with appropriate sums over $X_i = A, Z$, H^0, h, and $H^\pm, W^\pm$, and involve the four-point Feynman integrals D_0, D_{ij}, D_{ijk} [17]. The explicit expressions follow the same pattern as shown above for $f_i^{b(1)}$.

In the above, $\hat{s} = (p_1 + p_2)^2$, $\hat{t} = (p_3 - p_1)^2$, $\hat{u} = (p_3 - p_2)^2$, and

$$\eta_{H^0} = \eta_A = \cot^2\beta, \quad \eta_{H^\pm} = \cot^2\beta + \tan^2\beta, \quad \eta_Z = \eta_{W^\pm} = 1$$

Appendix B

The expressions for $H_i(m_t, p_1 \cdot p_2, p_1 \cdot p_3, p_1 \cdot p_4, p_3 \cdot p_4)$ in the amplitude squared are given by:

$$H_1 = 8m_t^4 + 8m_t p_1 \cdot p_2 - 8m_t p_1 \cdot p_3 + 8m_t p_2 \cdot p_3$$

$$H_2 = 16m_t^3 p_1 \cdot p_2 - 16m_t p_1 \cdot p_3 + 32m_t p_1 \cdot p_4 - 32m_t p_2 \cdot p_3$$

$$H_3 = 32m_t^4 + 8m_t p_1 \cdot p_2 + 8m_t p_1 \cdot p_3 + 8m_t p_2 \cdot p_3$$

$$H_4 = 8m_t^4 p_1 \cdot p_2 + 16(p_1 \cdot p_2)^2 + 8m_t^3 p_1 \cdot p_3$$

$$H_5 = 32m_t^2 p_1 \cdot p_2 + 16(p_1 \cdot p_2)^2 + 4m_t^3 p_1 \cdot p_3 + 8m_t(p_1 \cdot p_2 + p_1 \cdot p_3)$$

$$H_6 = 8m_t^2 p_1 \cdot p_2 + 4m_t^3 p_1 \cdot p_3 + 8m_t(p_1 \cdot p_2)^2 + 4m_t^3 p_2 \cdot p_3 + 8m_t^5$$

$$H_7 = 16m_t^3 p_1 \cdot p_2 + 8m_t(p_1 \cdot p_2)^2 + 8m_t^3 p_2 \cdot p_3$$

$$H_8 = 8m_t(p_1 \cdot p_2)^2 + 8m_t^3 p_2 \cdot p_3 + 8m_t p_1 \cdot p_4 + 32m_t^3 p_1 \cdot p_4 + 8m_t^3 p_3 \cdot p_4$$

$$H_9 = 8m_t(p_1 \cdot p_2)^2 + 8m_t^3 p_2 \cdot p_3 + 8m_t p_1 \cdot p_3 + 8m_t^3 p_1 \cdot p_3 + 8m_t p_2 \cdot p_3$$

$$H_{10} = 8m_t(p_1 \cdot p_2)^2 + 8m_t^3 p_2 \cdot p_3 + 8p_1 \cdot p_2 p_1 \cdot p_3 + 8m_t^3 p_1 \cdot p_4 + 32m_t^3 p_1 \cdot p_4 + 8m_t^3 p_3 \cdot p_4$$

$$H_{11} = 8m_t^2 p_1 \cdot p_2 + 8m_t p_1 \cdot p_3 + 8m_t p_2 \cdot p_3 + 16m_t p_1 \cdot p_4$$

$$H_{12} = 16m_t^3 p_1 \cdot p_2 + 8m_t p_1 \cdot p_3 + 8m_t p_2 \cdot p_3 + 16p_1 \cdot p_2 p_1 \cdot p_3 + 8m_t^3 p_1 \cdot p_4 + 32m_t^3 p_1 \cdot p_4 + 8m_t^3 p_3 \cdot p_4$$

$$H_{13} = 16m_t p_1 \cdot p_2 p_1 \cdot p_3 + 8m_t^3 p_1 \cdot p_4 + 16m_t p_1 \cdot p_2 p_1 \cdot p_4 + 16m_t p_1 \cdot p_3 p_2 \cdot p_3$$

$$H_{14} = 16m_t p_1 \cdot p_2 p_1 \cdot p_3 + 8m_t^3 p_1 \cdot p_4 + 16m_t p_1 \cdot p_2 p_1 \cdot p_4 + 16m_t p_1 \cdot p_3 p_2 \cdot p_3$$

$$H_{15} = 16m_t p_1 \cdot p_2 p_1 \cdot p_3 + 8m_t^3 p_1 \cdot p_4 + 16m_t p_1 \cdot p_2 p_1 \cdot p_4 + 16m_t p_1 \cdot p_3 p_2 \cdot p_3$$

$$H_{16} = 16m_t p_1 \cdot p_2 p_1 \cdot p_3 + 8m_t^3 p_1 \cdot p_4 + 16m_t p_1 \cdot p_2 p_1 \cdot p_4 + 16m_t p_1 \cdot p_3 p_2 \cdot p_3$$

where the various momentum dot products are defined in the physical region of the process.

Note: Figure translations are in progress. See original paper for figures.

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