Dimension-six CP-conserving operators of the third family quarks and their effects on collider observables (postprint)

Authors: Whisnant,K, Yang,J, Young,B, Zhang,X

Date: 2016-12-28T00:00:00+00:00

Abstract

We list all possible dimension-six CP-conserving SUc(3)×SUL(2)×UY(1)SUc(3)×SUL(2)×UY(1)SU_c(3)×SU_
×U_Y(1) invariant operators involving the third-family quarks which could be generated by new physics at a higher scale. Expressions for the

Full Text

Preamble

AMES-HET-97-1 February 1997 Dimension-six CP-conserving operators of the third-family quarks and their effects on collider observables K.Whisnanta, Jin Min Yanga,b,c, Bing-Lin Younga,b, and X.Zhangd a Department of Physics and Astronomy, Iowa State University, Ames, Iowa 50011, USA b International Institute of Theoretical and Applied Physics, Iowa State University, Ames, Iowa 50011, USA c Physics Department, Henan Normal University, Xin Xiang, Henan 453002, China d Institute of High Energy Physics, Academia Sinica, Beijing 100039, China

Abstract

We list all possible dimension-six CP-conserving $SU_c(3) \times SU_L(2) \times U_Y(1)$ invariant operators involving the third-family quarks which could be generated by new physics at a higher scale. Expressions for these operators after electroweak gauge symmetry breaking and the induced effective couplings $Wt\bar{b}$, $Xb\bar{b}$ and $Xt\bar{t}$ (where $X = Z, \gamma, g, H$) are presented. Analytic expressions for the tree-level contributions of all these operators to the observables R_b and A_b^{FB} at LEP I, $\sigma(e^+e^- \to b\bar{b})$ and A_b^{FB} at LEP II, $\sigma(p\bar{p} \to t\bar{b} + X)$ at the Tevatron upgrade, as well as $\sigma(e^+e^- \to t\bar{t})$ and A_t^{FB} at the NLC are provided. The effects of

these operators on different electroweak observables are discussed and numerical examples presented. Numerical analyses show that in the coupling region allowed by R_b and A_b^{FB} at LEP I, some of the new physics operators can still have significant contributions at LEP II, the Tevatron and the NLC.

1. Introduction

The Standard Model (SM) has been very successful phenomenologically [?]. The discovery of the top quark [?] fulfilled the long-anticipated completion of the fermion sector of the SM. Despite its success, the SM is still believed to be a theory effective at the electroweak scale and that some new physics must exist at higher energy regimes. The exceedingly large mass of the top quark further strengthens this belief. Collider experiments have been used to search for the new particles predicted by various new physics models, but no direct signal of new particles has been observed.

So, if new physics indeed exists above the electroweak scale, it is very likely that the only observable effects at energies not too far above the SM energy scale could be in the form of new interactions affecting the couplings of the third-family quarks, and the untested sectors of the Higgs and gauge bosons. In this spirit, the new physics effects can be expressed as non-standard terms in an effective Lagrangian describing the interactions among third-family quarks, the Higgs and gauge bosons with a form like, before the electroweak symmetry breaking,

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_0 + \sum_i \frac{C_i}{\Lambda^2} O_i + \mathcal{O}\left(\frac{1}{\Lambda^4}\right)$$

where $\mathcal{L}_0$ is the SM Lagrangian, Λ is the new physics scale, O_i are dimension-six operators which are $SU_c(3) \times SU_L(2) \times U_Y(1)$ invariant before the electroweak symmetry break-down, and C_i are constants which represent the coupling strengths of O_i. The expansion in Eq.(1) was first discussed in Ref. [?].

Further classification of the operators O_i has been made more recently. The CP-conserving operators involving only weak bosons were classified and phenomenological implications discussed in Ref. [?]. The corresponding operators involving the third-family quarks were enumerated in Ref. [?]. In these earlier works [?, ?, ?] the field equations of all particles were used to reduce the number of operators in Eq.(1). The phenomenology of some of these CP-conserving operators were discussed in Refs.[?]. More recently the operators were reconsidered without using the field equations of the gauge bosons [?]. In this article, we again focus on the set of operators involving the third-family quarks. We use the most recent LEP I data involving the $b\bar{b}$ final state to constrain some of the coefficients C_i, assuming the simple situation that cancellation among different operators does not take place. We identify the operators which can potentially have significant effects on the standard model predictions at higher energies in LEP II, the NLC and the Tevatron.

This paper is organized as follows. In Sec. 2 we again list all possible operators in Eq.(1). The expressions for these operators after electroweak gauge symmetry breaking and the induced effective couplings $Wt\bar{b}$, $Xb\bar{b}$ and $Xt\bar{t}$ (where $X = Z, \gamma, g, H$) are presented in Appendices A and B. In Sec. 3 we give analytic expressions for the contributions of these operators to the observables R_b and A_b^{FB} at LEP I, $\sigma(e^+e^- \to b\bar{b})$ and A_b^{FB} at LEP II, $\sigma(p\bar{p} \to t\bar{b} + X)$ at the Tevatron, as well as $\sigma(e^+e^- \to t\bar{t})$ and A_t^{FB} at the NLC. In Sec. 4 we determine which collider observables are affected by each operator. In Sec. 5 we analyze the operators which affect R_b and A_b^{FB} at LEP I and determine how much they affect future electroweak collider observables, subject to current constraints. Finally, in Sec.6 we conclude with some discussion and a summary.

2. CP-conserving gauge invariant operators

We follow the conventional notation which is listed below. The left-handed third family doublet is $q_L = (t_L, b_L)$, the right-handed top and bottom quarks are t_R, b_R, the Higgs boson doublet is $\Phi = (\phi^+, \phi^0)$, and $\tilde{\Phi} = i\sigma_2\Phi^*$. The gluon fields are G_μ^A, the $SU_L(2)$ gauge fields are W_μ^I, and the $U_Y(1)$ gauge field is B_μ. The field strength tensors are defined as $G_{\mu\nu}^A = \partial_\mu G_\nu^A - \partial_\nu G_\mu^A + g_s f^{ABC} G_\mu^B G_\nu^C$, $W_{\mu\nu}^I = \partial_\mu W_\nu^I - \partial_\nu W_\mu^I + g_2 \epsilon^{IJK} W_\mu^J W_\nu^K$, and $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$.

In order to justify the forms of operators that we will use, let us elaborate on the origin of the new physics that has been touched upon in Ref.[**?**]. We assume that the new physics in the quark sector resides in the third quark family. Before the electroweak symmetry breaking all dimension-6 operators containing the third family quarks are possible. Although new physics may also occur in the gauge boson and Higgs sectors, or give rise to four-quark operators involving the third family quarks, for the purpose of testing new physics in the immediate and near future, such operators will be ignored. Therefore, the operators we are interested in are those containing quarks and gauge and Higgs bosons.

To restrict ourselves to new physics of the lowest order, in both the standard model coupling and the power of $1/\Lambda^2$, we consider only tree diagrams which contain only one anomalous vertex in a given diagram. Under these assumptions, operators which can be related by the field equations of the fermions are no longer independent and the fermion equations of motion can be used to reduce the number of independent operators, as was done in Ref.[**?**]. However, we have to be careful in applying the field equations of the bosons. Under the assumption of the new physics origin as given above, the equations of motion of the gauge bosons cannot be used when first writing down the operators in Eq.(1). This is because the field equations of the gauge bosons will lead to four-fermion operators containing third family quarks and light fermions. Then naively applying the criterion of ignoring all four-fermion operators, which are observable in existing colliders such as $e^+ + e^- \to b + \bar{b}$, would discard these operators which originate from new physics different from that of the four-fermion operators discarded initially. However, the equation of motion of the Higgs field can be used since the light fermions resulting from the Higgs field equations are proportional to m_l/m_W, where m_l is the mass of the light fermions concerned.

We should also remark that no field equation can be used in the case of loop diagrams or when new physics couplings appear more than once in a tree diagram. In the latter case, dimension-8 operators may also have to be included. This means that extending the effective Lagrangian approach to dimension-8 operators will greatly increase the number of independent operators.

Now we list all possible dimension-six CP-conserving $SU_c(3)\times SU_L(2)\times U_Y(1)$ invariant independent operators involving third-family quarks but no four-fermion operators under the qualification described above.

(1) Class 1 (containing t_R)

$$
\begin{aligned}
O_{t1} &= (\Phi^\dagger\Phi - v^2/2)(\bar{q}_L t_R \tilde{\Phi} + \tilde{\Phi}^\dagger \bar{t}_R q_L) \\
O_{t2} &= i(\Phi^\dagger D_\mu \Phi - (D_\mu\Phi)^\dagger\Phi)(\bar{t}_R\gamma^\mu t_R) \\
O_{t3} &= i(\Phi^\dagger D_\mu \Phi - (D_\mu\Phi)^\dagger\Phi)(\bar{t}_R\gamma^\mu b_R) \\
O_{Dt} &= (\bar{q}_L D_\mu t_R)D^\mu\tilde{\Phi} + (D^\mu\tilde{\Phi})^\dagger(\bar{t}_R D_\mu q_L) \\
O_{tW\Phi} &= \Phi^\dagger(\bar{t}_R\sigma^{\mu\nu}\tau^I q_L)W^I_{\mu\nu} \\
O_{tB\Phi} &= \Phi^\dagger(\bar{t}_R\sigma^{\mu\nu} q_L)B_{\mu\nu} \\
O_{tG\Phi} &= \Phi^\dagger(\bar{t}_R\sigma^{\mu\nu}T^A q_L)G^A_{\mu\nu} \\
O_{tB} &= (\bar{t}_R\gamma^\mu D^\nu t_R + D^\nu\bar{t}_R\gamma^\mu t_R)B_{\mu\nu} \\
O_{tG} &= (\bar{t}_R\gamma^\mu T^A D^\nu t_R + D^\nu\bar{t}_R\gamma^\mu T^A t_R)G^A_{\mu\nu}
\end{aligned}
$$

It is straightforward to show that the last two operators O_{tG} and O_{tB} can be recast into simple forms, e.g. $O_{tG} = \bar{t}\gamma^\mu T^A t D^\nu G^A_{\mu\nu}$, etc.

(2) Class 2 (not containing t_R)

$$
\begin{aligned}
O_{qG} &= \bar{q}_L \gamma^\mu T^A D^\nu q_L + D^\nu \bar{q}_L \gamma^\mu T^A q_L \\
O_{qW} &= \bar{q}_L \gamma^\mu \tau^I D^\nu q_L + D^\nu \bar{q}_L \gamma^\mu \tau^I q_L \\
O_{qB} &= \bar{q}_L \gamma^\mu D^\nu q_L + D^\nu \bar{q}_L \gamma^\mu q_L \\
O_{bG} &= \bar{b}_R \gamma^\mu T^A D^\nu b_R + D^\nu \bar{b}_R \gamma^\mu T^A b_R \\
O_{bB} &= \bar{b}_R \gamma^\mu D^\nu b_R + D^\nu \bar{b}_R \gamma^\mu b_R \\
O_{\Phi q}^{(1)} &= i(\Phi^\dagger D_\mu \Phi - (D_\mu \Phi)^\dagger \Phi) \bar{q}_L \gamma^\mu q_L \\
O_{\Phi q}^{(3)} &= i(\Phi^\dagger \tau^I D_\mu \Phi - (D_\mu \Phi)^\dagger \tau^I \Phi) \bar{q}_L \gamma^\mu \tau^I q_L \\
O_{\Phi b} &= i(\Phi^\dagger D_\mu \Phi - (D_\mu \Phi)^\dagger \Phi) \bar{b}_R \gamma^\mu b_R \\
O_{b1} &= (\Phi^\dagger \Phi - v^2/2)(\bar{q}_L b_R \Phi + \Phi^\dagger \bar{b}_R q_L) \\
O_{Db} &= (\bar{q}_L D_\mu b_R) D^\mu \Phi + (D^\mu \Phi)^\dagger (\bar{b}_R D_\mu q_L) \\
O_{bW\Phi} &= (\bar{q}_L \sigma^{\mu\nu} \tau^I b_R) \Phi W^I_{\mu\nu} + \Phi^\dagger (\bar{b}_R \sigma^{\mu\nu} \tau^I q_L) W^I_{\mu\nu} \\
O_{bB\Phi} &= (\bar{q}_L \sigma^{\mu\nu} b_R) \Phi B_{\mu\nu} + \Phi^\dagger (\bar{b}_R \sigma^{\mu\nu} q_L) B_{\mu\nu} \\
O_{bG\Phi} &= (\bar{q}_L \sigma^{\mu\nu} T^A b_R) \Phi G^A_{\mu\nu} + \Phi^\dagger (\bar{b}_R \sigma^{\mu\nu} T^A q_L) G^A_{\mu\nu}
\end{aligned}
$$

It is straightforward to show that the first five operators, O_{qG}, O_{qW}, O_{qB}, O_{bG} and O_{bB}, can be recast into simple forms, e.g. $O_{qG} = \bar{q}_L \gamma^\mu T^A q_L D^\nu G^A_{\mu\nu}$, etc.

If we avoided using the field equations of the Higgs boson and the quarks, we would get the following additional operators:

(3) Class 3

$$
\begin{aligned}
O'_{Dt} &= (D^\mu q_L t_R) D_\mu \tilde{\Phi} + (D_\mu \tilde{\Phi})^\dagger (\bar{t}_R D^\mu q_L) \\
O'_{Db} &= (D^\mu q_L b_R) D_\mu \Phi + (D_\mu \Phi)^\dagger (\bar{b}_R D^\mu q_L) \\
O'_{tB} &= i(\bar{t}_R \sigma^{\mu\nu} D^\lambda t_R) \tilde{B}_{\mu\nu\lambda} \\
O'_{tG} &= i(\bar{t}_R \sigma^{\mu\nu} T^A D^\lambda t_R) \tilde{G}^A_{\mu\nu\lambda} \\
O'_{bB} &= i(\bar{b}_R \sigma^{\mu\nu} D^\lambda b_R) \tilde{B}_{\mu\nu\lambda} \\
O'_{bG} &= i(\bar{b}_R \sigma^{\mu\nu} T^A D^\lambda b_R) \tilde{G}^A_{\mu\nu\lambda} \\
O'_{qB} &= i(\bar{q}_L \sigma^{\mu\nu} D^\lambda q_L) \tilde{B}_{\mu\nu\lambda} \\
O'_{qG} &= i(\bar{q}_L \sigma^{\mu\nu} T^A D^\lambda q_L) \tilde{G}^A_{\mu\nu\lambda} \\
O'_{qW} &= i(\bar{q}_L \sigma^{\mu\nu} \tau^I D^\lambda q_L) \tilde{W}^I_{\mu\nu\lambda}
\end{aligned}
$$

where $\tilde{X}_{\mu\nu\lambda} = \epsilon_{\mu\nu\lambda\rho} X^{\rho\sigma}$ with $X = G, B, W$ and $\epsilon_{\mu\nu\lambda\rho}$ is the anti-symmetric tensor. We can rewrite the above operators as follows, which will no longer be independent when the field equations of Higgs boson and the quarks are used:

$$
\begin{aligned}
O'_{Dt} &= \bar{q}_L t_R D^2\tilde{\Phi} + (D^2\tilde{\Phi})^\dagger \bar{t}_R q_L \\
O'_{Db} &= \bar{q}_L b_R D^2\Phi + (D^2\Phi)^\dagger \bar{b}_R q_L \\
O'_{xB} &= i(\bar{x}_R \sigma^{\mu\nu} D^\lambda x_R)\tilde{B}_{\mu\nu\lambda} \quad (x = t, b) \\
O'_{xG} &= i(\bar{x}_R \sigma^{\mu\nu} T^A D^\lambda x_R)\tilde{G}^A_{\mu\nu\lambda} \quad (x = t, b) \\
O'_{qX} &= i(\bar{q}_L \sigma^{\mu\nu} D^\lambda q_L)\tilde{X}_{\mu\nu\lambda} \quad (X = B, G, W)
\end{aligned}
$$

In the following analyses we will not consider the operators in Class 3 because these operators are not independent. Since our analyses only involve on-shell quarks, the equations of motion of the quarks can be applied. Because of the reasons given earlier, the Higgs field equation can also be used. Then our Class 1 and Class 2 operators agree with those given in Ref.[?]. However, unlike [?], in O_{t1} and O_{b1} we subtract the vacuum expectation value, $v^2/2$, from $\Phi^\dagger\Phi$, in order to avoid additional mass terms for top and bottom quarks after electroweak symmetry breaking.

The expressions for the operators of Class 1 and Class 2 in the unitary gauge after electroweak symmetry breaking are presented in Appendix A. From these expressions, one can write out the effective Lagrangian for all vertices with two third-family fermions and a boson, specifically, $Wt\bar{b}$, $Zt\bar{t}$, $\gamma t\bar{t}$, $Ht\bar{t}$, $gt\bar{t}$, $gb\bar{b}$, $Zb\bar{b}$, $\gamma b\bar{b}$ and $Hb\bar{b}$, whose effects are or could be reachable at LEP, the Tevatron upgrade and the NLC. The explicit forms of these effective couplings are given in Appendix B.

3. Contributions to some collider observables

We now consider the contribution of all operators listed in Sec. 2 to the observables R_b and A_b^{FB} at LEP I to constrain the coefficients C_i. Then we can make predictions on their effects on $\sigma(e^+e^- \to b\bar{b})$ and A_b^{FB} at LEP II, $\sigma(p\bar{p} \to t\bar{b}+X)$ at the Tevatron, $\sigma(e^+e^- \to t\bar{t})$ and A_t^{FB} at the NLC. In this paper we wish to consider modifications to the electroweak sector only, and therefore ignore measurements such as $\sigma(p\bar{p} \to t\bar{t})$ which are primarily affected by the strong interaction.

Including both the SM couplings and new physics effects, we can write the $Zq\bar{q}$ and $\gamma q\bar{q}$ (where $q = t, b$) vertices as

$$
ieg_{Z,\gamma}\left[\gamma^\mu V^q_{Z,\gamma} + \gamma^\mu\gamma_5 A^q_{Z,\gamma} + (p_q - p_{\bar{q}})^\mu S^q_{Z,\gamma}\right]
$$

where $g_Z = 1/(4s_W c_W)$ with $s_W \equiv \sin\theta_W$ and $c_W \equiv \cos\theta_W$, $g_\gamma = 1$, and p_q and $p_{\bar{q}}$ are the momenta of outgoing quark and anti-quark, respectively.

In the above vertices we neglect the scalar and pseudoscalar couplings, k^μ and $k^\mu\gamma_5$ with $k = p_q + p_{\bar{q}}$, since in e^+e^- collisions these terms give contributions proportional to the electron mass. We note that some of these neglected terms are needed to maintain electromagnetic gauge invariance for the axial vector couplings in Eq.(38). The vector and axial-vector couplings $V^q_{Z,\gamma}$ and $A^q_{Z,\gamma}$ contain both the SM and new physics contributions, while A^q_γ and $S^q_{Z,\gamma}$ contain only new physics contributions. The SM can also contribute to A^q_γ and $S^q_{Z,\gamma}$ at loop level, but these effects are very small and we neglect them in our calculation. One can write the vector and axial-vector couplings as

$$V^q_{Z,\gamma} = (V^q_{Z,\gamma})_0 + \delta V^q_{Z,\gamma}, \quad A^q_{Z,\gamma} = (A^q_{Z,\gamma})_0 + \delta A^q_{Z,\gamma}$$

where $(V^q_{Z,\gamma})_0$ and $(A^q_{Z,\gamma})_0$ represent the SM couplings and $\delta V^q_{Z,\gamma}$, $\delta A^q_{Z,\gamma}$ the anomalous new physics contributions. The SM couplings are given by

$$(V^q_\gamma)_0 = e_q, \quad (A^q_\gamma)_0 = 0, \quad V^q_Z = \frac{2I^q_{3L} - 4e_q s^2_W}{4s_W c_W}, \quad A^q_Z = \frac{2I^q_{3L}}{4s_W c_W}$$

where e_q is the electric charge of the quark in units of e and $I^q_{3L} = \pm 1/2$ is the weak isospin. The new physics contributions $\delta V^q_{Z,\gamma}$ and $\delta A^q_{Z,\gamma}$ (for $q = b, t$) can be determined from Appendix B; they are

$$\delta V^b_Z = \frac{c^2_W k^2}{\Lambda^2}\left[-\frac{C^{(1)}_{\Phi q} + C^{(3)}_{\Phi q}}{2s_W c_W} + \frac{C_{bB}}{4s_W c_W}\right] + \frac{c^2_W k^2}{\Lambda^2}\left[\frac{C_{qB} + C_{bB}}{2s_W c_W}\right] + \frac{v m_Z}{\Lambda^2}\left[-\frac{C^{(1)}_{\Phi q} + C^{(3)}_{\Phi q}}{2s_W c_W} + \frac{C_{bW\Phi} c^2_W - 2C_{bB\Phi} s^2_W}{8s_W c_W}\right.$$

$$\delta A^b_Z = \frac{c^2_W k^2}{\Lambda^2}\left[\frac{C_{qB} + C_{bB}}{2s_W c_W}\right] + \frac{v m_Z}{\Lambda^2}\left[\frac{C_{bW\Phi} c^2_W - 2C_{bB\Phi} s^2_W}{8s_W c_W}\right]$$

$$\delta V^b_\gamma = \frac{c^2_W k^2}{\Lambda^2}\left[\frac{C_{qB} + C_{bB}}{2s_W c_W}\right]$$

$$\delta A^b_\gamma = \frac{c^2_W k^2}{\Lambda^2}\left[\frac{C_{qB} + C_{bB}}{2s_W c_W}\right]$$

Similar expressions hold for the top quark couplings.

In terms of the vertices given in Eq.(38), the observables R_b and A^{FB}_b at LEP I are given by, to order $1/\Lambda^2$,

$$R_b = R^{\text{SM}}_b \left[1 + 2\frac{v_b \delta V^b_Z + a_b \delta A^b_Z}{v^2_b + a^2_b}\right]$$

$$A^{FB}_b = A^{\text{SM, FB}}_b + \frac{2v_b a_b}{(v^2_b + a^2_b)^2}\left[(v^2_b + a^2_b)\delta A^b_Z - 2v_b a_b \delta V^b_Z\right]$$

where we have neglected the bottom quark mass.

Also, in terms of the vertices of Eq.(38), the cross section and forward-backward asymmetry for bottom pair production at LEP II and top pair production at the NLC are given by

$$\sigma_0 = 3\beta_q \left[D_{\gamma\gamma} e_q^2 + D_{ZZ}(v_e^2 + a_e^2)(v_q^2 + a_q^2) + 2D_{Z\gamma} e_e v_e v_q\right]$$

$$\Delta\sigma = 3\beta_q \left[D_{\gamma\gamma} e_q^2 \frac{2e_q \delta V_\gamma^q}{e_q} + D_{ZZ}(v_e^2 + a_e^2)\left(2v_q \delta V_Z^q + 2a_q \delta A_Z^q + \beta_q^2 \frac{\delta V_Z^q}{v_q}\right) + 2D_{Z\gamma} e_e v_e \left(\delta V_Z^q + \frac{v_q \delta V_\gamma^q}{e_q}\right) + 4D_{ZZ}\right.$$

$$A_{FB} = A_{FB}^0 + \Delta A_{FB}$$

where $\beta_q = \sqrt{1 - 4m_q^2/s}$ is the velocity of the final quarks and

$$D_{\gamma\gamma} = \frac{1}{s}, \quad D_{ZZ} = \frac{1}{(s - m_Z^2)^2 + (s\Gamma_Z/m_Z)^2}, \quad D_{Z\gamma} = \frac{s - m_Z^2}{(s - m_Z^2)^2 + (s\Gamma_Z/m_Z)^2}$$

Including both the SM coupling and new physics contributions, the $Wt\bar{b}$ vertex can be written as

$$\mathcal{L}_{Wt\bar{b}} = \frac{g_2}{\sqrt{2}}\left[\gamma^\mu P_L(1+\kappa_1) + \gamma^\mu P_R \kappa_2 + p_t^\mu P_L \kappa_3 + p_{\bar{b}}^\mu P_L \kappa_4 + p_t^\mu P_R \kappa_5 + p_{\bar{b}}^\mu P_R \kappa_6\right]$$

where $P_{L,R} = (1 \mp \gamma_5)/2$. The form factors from new physics can be determined from Appendix B:

$$\kappa_1 = \frac{v^2}{\Lambda^2}\left[C_{qW}^{(3)} + C_{tW\Phi}\right]$$
$$\kappa_2 = \frac{v^2}{\Lambda^2} C_{bW\Phi}$$
$$\kappa_3 = \frac{v}{\Lambda^2}\left[\frac{C_{tW\Phi}}{\sqrt{2}} + \frac{C_{Dt}}{\sqrt{2}}\right]$$
$$\kappa_4 = \frac{v}{\Lambda^2}\left[\frac{C_{bW\Phi}}{\sqrt{2}} + \frac{C_{Db}}{\sqrt{2}}\right]$$
$$\kappa_5 = \frac{v}{\Lambda^2}\frac{C_{tW\Phi}}{\sqrt{2}}$$
$$\kappa_6 = \frac{v}{\Lambda^2}\frac{C_{bW\Phi}}{\sqrt{2}}$$

Neglecting the bottom quark mass, one gets the cross section for the subprocess $q_i\bar{q}_j \to t\bar{b}$:

$$\hat{\sigma}_0 = \frac{g_2^4}{192\pi\hat{s}^2}\frac{(2\hat{s}+m_t^2)(\hat{s}-m_t^2)^2}{(\hat{s}-m_W^2)^2}$$

$$\Delta\hat{\sigma} = \frac{g_2^4}{192\pi\hat{s}^2}\frac{1}{(\hat{s}-m_W^2)^2}\left[2(2\hat{s}+m_t^2)(\hat{s}-m_t^2)m_t\kappa_1 + (\hat{s}-m_t^2)^2\hat{s}(\kappa_3-\kappa_4)\right]$$

The total cross section of single top quark production via $q_i\bar{q}_j \to t\bar{b}$ at the Fermilab Tevatron is obtained by

$$\sigma(s) = \sum_{i,j}\int_{\tau_0}^{1} d\tau \frac{dL_{ij}}{d\tau}\hat{\sigma}_{ij}(\hat{s}=s\tau)$$

where $\tau_0 = (M_t+M_b)^2/s$, s is the square of center-of-mass energy, $\hat{s}=s\tau$ is the square of center-of-mass energy of the subprocess, and $dL_{ij}/d\tau$ is the parton luminosity given by

$$\frac{dL_{ij}}{d\tau} = \int_{\tau}^{1}\frac{dx_1}{x_1}\left[f_i^A(x_1,\mu)f_j^B(\tau/x_1,\mu) + f_j^A(x_1,\mu)f_i^B(\tau/x_1,\mu)\right]$$

where A and B denote the incident hadrons, i and j are the initial partons, and x_1 and x_2 their longitudinal momentum fractions. The functions f_i^A and f_j^B are the parton distribution functions.

4. Classifying physics effects

In this section, we classify the operators according to their contribution to the three-particle vertices which are testable at LEP I, II, the NLC and the Tevatron, i.e., $Wt\bar{b}$, $Xt\bar{t}$ and $Xb\bar{b}$ (where $X=\gamma, Z, H, g$).

From Appendix B we can see that most operators give contributions to more than one of the three-particle vertices and therefore tests of these operators are possible when their coupling strengths are constrained by one of the vertices. In Table 1 we summarize the contributions of these operators to the couplings which can be tested at present or future colliders. The contribution of an operator to a particular vertex is denoted by a checkmark. Since the operators contribute to different combinations of observables, we can reclassify them as:

Class A-1: Contributing to LEP I and LEP II observables, and $\sigma_{t\bar{t}}$ and A_t^{FB} at the NLC and $\sigma_{t\bar{b}}$ at the Tevatron. These are O_{qW} and $O_{\Phi q}^{(3)}$.

Class A-2: Contributing to LEP I and LEP II observables, and $\sigma_{t\bar{t}}$ and A_t^{FB} at the NLC, but not to $\sigma_{t\bar{b}}$ at the Tevatron. These are O_{qB} and $O_{\Phi q}^{(1)}$.

Class A-3: Contributing to LEP I and LEP II observables, and $\sigma_{t\bar{b}}$ at the Tevatron. These are O_{Db} and $O_{bW\Phi}$.

Class A-4: Contributing to LEP I and LEP II observables only. These are O_{bB}, $O_{\Phi b}$ and $O_{bB\Phi}$.

Class B-1: Contributing to $\sigma_{t\bar{t}}$ and A_t^{FB} at the NLC and $\sigma_{t\bar{b}}$ at the Tevatron. These are $O_{tW\Phi}$ and O_{Dt}.

Class B-2: Contributing only to $\sigma_{t\bar{t}}$ and A_t^{FB} at the NLC. These are O_{t2}, $O_{tB\Phi}$ and O_{tB}.

Class B-3: Contributing only to $\sigma_{t\bar{b}}$ at the Tevatron. This contains only O_{t3}.

Class C-1: Contributing only to couplings $Ht\bar{t}$ and $Hb\bar{b}$, not to any other vertices. These are O_{t1} and O_{b1}.

Class C-2: Contributing to the strong interaction sector. These are $O_{tG\Phi}$, O_{tG}, O_{qG}, $O_{bG\Phi}$ and O_{bG}. These operators only contribute to the strong interactions of third-family quarks and do not contribute to the electroweak interaction at the level of $1/\Lambda^2$.

In this new classification scheme, Class A operators include a $Zb\bar{b}$ or $\gamma b\bar{b}$ interaction and are currently constrained by R_b and A_b^{FB} at LEP I. Class B operators are not constrained by LEP I (at least at tree level), but will affect the future collider observables under consideration. Class C operators affect neither LEP I observables nor the future collider observables which arise from the electroweak interactions at tree level.

5. Numerical examples and discussions

In this section we present numerical analyses for those operators which affect R_b and A_b^{FB} at LEP I and observables at future colliders. These are the Class A operators defined in the preceding section. We use the analytic formulae given in Sec. 3 and the most recent LEP I data on R_b and A_b^{FB} to constrain the coefficients of the individual operators in Classes A-1 through A-4, and then evaluate their possible effects on the electroweak observables at LEP II, the Tevatron upgrade and the NLC. Operators in Classes B and C are not presently constrained, at least at tree level, or they involve the strong interaction sector, and they will not be considered further here.

5.1 The effects of $O_{\Phi q}^{(3)}$ and O_{qW}

From the preceding section we found that the operators of Class A-1 will affect the most observables. Note that in Ref. [?] the effects of O_{qW} on R_b and $\sigma_{t\bar{b}}$ at the Tevatron have been evaluated. The present analyses also include this operator, but we will consider its effects in LEP II and the NLC as well.

Presently, the experimental data of R_b and A_b^{FB} are $+1.8\sigma$ and -1.8σ away from their SM values, respectively [?]. In the analyses below, we assume a closer agreement with the SM, say both R_b and A_b^{FB} are about 1σ away from the SM predictions, and examine the consequences.

We note that the new physics of Class A-1 yields $\delta V_Z^b = \delta A_Z^b$, which we can express in terms of R_b or A_b^{FB}. From Eq.(57) we obtain

$$\delta V_Z^b = \frac{R_b - R_b^{\rm SM}}{2R_b^{\rm SM}} \frac{v_b^2 + a_b^2}{v_b + a_b}$$

or from Eq.(58) we obtain

$$\delta A_Z^b = \frac{A_b^{FB} - A_b^{\rm SM,\ FB}}{2} \frac{(v_b^2 + a_b^2)^2}{v_b a_b (v_b^2 - a_b^2)}$$

where the experimental data and SM values [?] are

$$R_b^{\rm SM} = 0.2158, \quad R_b^{\rm exp} = 0.2178 \pm 0.0011$$

$$A_b^{\rm SM,\ FB} = 0.1022, \quad A_b^{\rm exp,\ FB} = 0.0979 \pm 0.0023$$

Since both v_b and a_b are negative, we find that Eq.(77) yields negative values for δV_Z^b while Eq.(78) yields positive values for δV_Z^b. This means that any kind of new physics which yields $\delta V_Z^b = \delta A_Z^b$, such as O_{qW}, $O_{\Phi q}^{(1)}$, $O_{\Phi q}^{(3)}$ and O_{qB}, cannot fit both R_b and A_b^{FB} within the 1σ bounds of the experimental data at the same time. If the deviations from the SM values as shown in Eq.(79) and Eq.(80) persist, this class of operators will be ruled out.

Since the error size in A_b^{FB} is larger than that of R_b, we estimate the effect of this class of operators by using only the 1σ bound of R_b to set constraints on the new physics. We have from Eq.(77) and Eq.(79)

$$-0.0080 < \delta V_Z^b < 0.0080$$

Using this bound and assuming the existence of only $O_{\Phi q}^{(3)}$, we get the effects on $\sigma_{b\bar{b}}$ and A_b^{FB} at LEP II ($\sqrt{s} = 200$ GeV), $\sigma_{t\bar{t}}$ and A_t^{FB} at the NLC ($\sqrt{s} = 500$ GeV, $m_t = 175$ GeV), and the single top production rate at Tevatron ($\sqrt{s} = 2$ TeV, $m_t = 175$ GeV) as

LEP II ($e^+e^- \to b\bar{b}$) :	$0.4\% < \Delta\sigma/\sigma_0 < 1.3\%$
NLC ($e^+e^- \to t\bar{t}$) :	$0.1\% < \Delta\sigma/\sigma_0 < 0.3\%$
Tevatron ($p\bar{p} \to t\bar{b} + X$) :	$0.5\% < \Delta\sigma/\sigma_0 < 1.6\%$
LEP II A_b^{FB} :	$0.2\% < \delta A_{FB} < 0.6\%$
NLC A_t^{FB} :	$0.7\% < \delta A_{FB} < 2.6\%$

which are too small to be observable. Using the same bound in Eq.(81) and assuming only the existence of O_{qW} we obtain

LEP II ($e^+e^- \to b\bar{b}$) :	$2.4\% < \Delta\sigma/\sigma_0 < 8.4\%$
NLC ($e^+e^- \to t\bar{t}$) :	$8.6\% < \Delta\sigma/\sigma_0 < 29.8\%$
Tevatron ($p\bar{p} \to t\bar{b} + X$) :	$6.9\% < \Delta\sigma/\sigma_0 < 24.0\%$
LEP II A_b^{FB} :	$0.3\% < \delta A_{FB} < 1.0\%$
NLC A_t^{FB} :	$16.3\% < \delta A_{FB} < 56.8\%$

where we have used the CTEQ3L parton distribution functions [?] with $\mu = \sqrt{\hat{s}}$ for the calculation of the cross section at the Tevatron. Except for the A_b^{FB} at LEP II, all the other contributions are sizable.

Let us consider the expected accuracy of the hadron cross section measurements. At LEP II the cross section for $e^+e^- \to$ hadrons can be measured with a high accuracy of 0.7% [?]. Since $R_b = \sigma(e^+e^- \to b\bar{b})/\sigma(e^+e^- \to \text{hadrons}) = 0.16$, $\sigma(e^+e^- \to b\bar{b})$ can be measured with an accuracy of 4%, or better when b-tagging is employed. At the NLC the top quark properties will be tested to high accuracy and we expect that the top pair production rate there may be measurable with an accuracy of a few percent. At the Tevatron a deviation larger than 16% from the SM single top production rate is expected to be detectable at Run 3 [?].

The above results then show that the operator $O_{\Phi q}^{(3)}$ constrained by R_b has negligibly small effects on $b\bar{b}$ production at LEP II, $t\bar{t}$ production at the NLC and single top production at the Tevatron. On the contrary, the operator O_{qW} constrained by R_b can cause observable effects at LEP II, the NLC and the Tevatron. In other words, if their effects are not observed at future colliders, O_{qW} is severely constrained, but $O_{\Phi q}^{(3)}$ is not. We note that the main reason that O_{qW} has large effects at future colliders is that it is momentum dependent, and therefore becomes enhanced at higher energies.

5.2 The effects of O_{qB} and $O_{\Phi q}^{(1)}$

The operators in Class A-2 (O_{qB} and $O_{\Phi q}^{(1)}$) affect LEP I and LEP II observables and $\sigma_{t\bar{t}}$ and A_t^{FB} at the NLC, but not single top production at hadron colliders. Note that O_{qB} is momentum dependent and $O_{\Phi q}^{(1)}$ is momentum independent. Like the operators in Class A-1 analyzed above, they yield $\delta V_Z^b = \delta A_Z^b$. Using the bound given in Eq.(81), we obtain the contribution of O_{qB} to $\sigma_{b\bar{b}}$ and A_b^{FB}

at LEP II ($\sqrt{s} = 200$ GeV), $\sigma_{t\bar{t}}$ and A_t^{FB} at NLC ($\sqrt{s} = 500$ GeV, $m_t = 175$ GeV) as

$$\begin{array}{ll} \text{LEP II } (e^+e^- \to b\bar{b}): & 0.6\% < \Delta\sigma/\sigma_0 < 2.9\% \\ \text{NLC } (e^+e^- \to t\bar{t}): & 16.5\% < \Delta\sigma/\sigma_0 < 57.4\% \\ \text{LEP II } A_b^{FB}: & 2.5\% < \delta A_{FB} < 10.0\% \\ \text{NLC } A_t^{FB}: & 41.4\% < \delta A_{FB} < 144.0\% \end{array}$$

and, in the same way, we obtain the contribution of $O_{\Phi q}^{(1)}$ as

$$\begin{array}{ll} \text{LEP II } (e^+e^- \to b\bar{b}): & 0.4\% < \Delta\sigma/\sigma_0 < 1.3\% \\ \text{NLC } (e^+e^- \to t\bar{t}): & 0.3\% < \Delta\sigma/\sigma_0 < 0.7\% \\ \text{LEP II } A_b^{FB}: & 0.2\% < \delta A_{FB} < 0.6\% \\ \text{NLC } A_t^{FB}: & 2.5\% < \delta A_{FB} < 7.0\% \end{array}$$

Here we see that the effects of $O_{\Phi q}^{(1)}$ are negligibly small, but the effects of O_{qB} on $\sigma_{t\bar{t}}$ and A_t^{FB} at the NLC can be quite large. As was the case with O_{qW} in the preceding section, these large effects are primarily due to the momentum dependence of O_{qB}. So the NLC will be a good place to look for the new physics operator O_{qB}. We should again comment that if the values given in Eq.(79) and Eq.(80) persist, this class of operators and the Class A-1 operators in the preceding subsection will be ruled out.

5.3 The effects of O_{bB}, $O_{\Phi b}$ and $O_{bB\Phi}$

The operators in Class A-4 (O_{bB}, $O_{\Phi b}$ and $O_{bB\Phi}$) affect R_b and A_b^{FB} at LEP I and $\sigma_{b\bar{b}}$ and A_b^{FB} at LEP II, but not top pair production at the NLC or single top production at the Tevatron upgrade. Since $O_{bB\Phi}$ only appears in S_Z^b and S_γ^b, its contributions to these observables are proportional to m_b, which to a good approximation can be set to zero in the calculations for $b\bar{b}$ production at LEP I and LEP II. Thus the contributions of $O_{bB\Phi}$ are negligible and we only need to consider O_{bB} and $O_{\Phi b}$. We note that O_{bB} is momentum dependent while $O_{\Phi b}$ is momentum independent.

Unlike the case discussed in subsections 5.1 and 5.2, the operators in this class yield $\delta V_Z^b \neq \delta A_Z^b$. From Eq.(57) one gets

$$\delta V_Z^b = \frac{R_b - R_b^{\text{SM}}}{2R_b^{\text{SM}}} \frac{v_b^2 + a_b^2}{v_b}$$

and from Eq.(58) one gets

$$\delta A_Z^b = \frac{A_b^{FB} - A_b^{\text{SM, FB}}}{2} \frac{(v_b^2 + a_b^2)^2}{v_b a_b (v_b + a_b)}$$

Using the values in Eqs.(79) and (80) we see that both Eqs.(82) and (83) yield positive values for δV_Z^b. The bound from Eq.(82), again assuming 1σ deviation, is found to be

$$0.013 < \delta V_Z^b < 0.044$$

and the bound from Eq.(83) is

$$0.023 < \delta V_Z^b < 0.075$$

We take the overlap of the two:

$$0.023 < \delta V_Z^b < 0.044$$

which is required to have the theoretical values of both R_b and A_b^{FB} lie within 1σ of the experimental data.

Considering O_{bB}, one gets its contribution to $\sigma_{b\bar{b}}$ and A_b^{FB} at LEP II ($\sqrt{s} = 200$ GeV) to be

$$23.3\% < \Delta\sigma/\sigma_0(e^+e^- \to b\bar{b}) < 44.5\%$$

$$-53.9\% < \delta A_{FB}(e^+e^- \to b\bar{b}) < -28.2\%$$

For $O_{\Phi b}$, the contributions to $\sigma_{b\bar{b}}$ and A_b^{FB} at LEP II ($\sqrt{s} = 200$ GeV) are

$$0.7\% < \Delta\sigma/\sigma_0(e^+e^- \to b\bar{b}) < 1.3\%$$

$$-1.7\% < \delta A_{FB}(e^+e^- \to b\bar{b}) < -3.3\%$$

So if only O_{bB} exists, its effects are likely observable at LEP II even if both R_b and A_b^{FB} lie within the 1σ bounds of the present data. As with the operators O_{qW} and O_{qB}, this is primarily due to the momentum dependence of O_{bB}. On the contrary, if only $O_{\Phi b}$ exists, there will be no observable effects at LEP II.

5.4 The effects of $O_{bW\Phi}$ and O_{Db}

The operators $O_{bW\Phi}$ and O_{Db} in Class A-3 affect LEP I and LEP II as well as single top quark production at the Tevatron, $\sigma(p\bar{p} \to t\bar{b} + X)$. Since both of them only appear in S_Z^b and S_γ^b, their contributions to R_b and A_b^{FB} at LEP I and $\sigma_{b\bar{b}}$ and A_b^{FB} at LEP II are proportional to m_b and hence are negligible. Further, as Eq.(73) shows, their contributions to $\sigma(p\bar{p} \to t\bar{b} + X)$ vanish in the approximation of neglecting m_b. So they are not constrained by these observables at LEP I, LEP II and the Tevatron.

However, as Eq.(67) shows, $O_{bW\Phi}$ contributes to the right-handed weak charged current, and thus it will be strictly constrained by the CLEO measurement of $b \to s\gamma$ [**?**]. The latest limit is [**?**]

$$-0.03 < \kappa_2 = \frac{C_{bW\Phi} v}{\sqrt{2}\Lambda^2} < 0.00$$

Using this bound and keeping the bottom quark mass, we can evaluate its contributions to the observables under consideration. Of course, its contributions must be very small since they are not only proportional to m_b but also suppressed by the above bound. For example, with $m_b = 5$ GeV its contribution to $\sigma_{b\bar{b}}$ and A_b^{FB} at LEP II ($\sqrt{s} = 200$ GeV) are found to be

$$-0.2\% < \Delta\sigma/\sigma_0(e^+e^- \to b\bar{b}) < 0.0\%$$

$$-0.2\% < \delta A_{FB}(e^+e^- \to b\bar{b}) < 0.0\%$$

which, as expected, are negligibly small.

So the operator $O_{bW\Phi}$, which contributes to the right-handed weak charged current of third-family quarks, can be further constrained, although the coefficient $C_{bW\Phi}$ will not be constrained greatly unless a process can be found in which its contribution is not proportional to m_b. The operator O_{Db} will also survive since no observables are sensitive to it.

6. Discussions and summary

From the expressions of δV_Z^b and δA_Z^b, the operators affecting R_b and A_b^{FB} at LEP I can be divided into two types: those yielding $\delta V_Z^b = \delta A_Z^b$ and those yielding $\delta V_Z^b \neq \delta A_Z^b$. As the above numerical calculations show, operators of the first type, including O_{qW}, $O_{\Phi q}^{(1)}$, $O_{\Phi q}^{(3)}$ and O_{qB} in Classes A-1 and A-2, cannot make the theoretical values of both R_b and A_b^{FB} lie within the 1σ bounds of the experimental data at the same time. If one uses the 1σ bound of R_b to set constraints on this type of new physics, the strict bounds of Eq.(81) are obtained. The two operators O_{qW} and O_{qB} can give rise to visible effects at

LEP II, the NLC and/or the upgraded Tevatron. On the contrary, operators of the second type, including O_{bB} and $O_{\Phi b}$ in Class A-4, can make the theoretical values of both R_b and A_b^{FB} within the 1σ bounds of the experimental data simultaneously, but the bounds in Eq.(86) are not as strict as the bounds on the operators of the first type. O_{bB} in the second type of new physics can cause larger effects on observables at LEP II, the Tevatron and the NLC.

A common feature of operators with significant effects on LEP II, etc., is that they are momentum dependent, as can be seen from Eq.(45) and Eq.(46). However, the suppression of the effect of an operator is more complicated. Take the operator $O_{\Phi b}$ as an example. Since it is momentum independent, it does not have the enhanced effect going from LEP I to LEP II. Another reason for its small effects is that $O_{\Phi b}$ only contributes to the vertex $Zb\bar{b}$ but not to the vertex $\gamma b\bar{b}$, and, as is well-known, the photon exchange channel is dominant in $b\bar{b}$ production at LEP II.

From the above analyses we can say that if the experimental data of R_b and A_b^{FB}, which are now deviating from their SM values by 1.8σ and -1.8σ respectively, are both upheld and the deviations are due to the new physics considered here, then the new physics cannot be the first type, O_{qW}, $O_{\Phi q}^{(1)}$, $O_{\Phi q}^{(3)}$ or O_{qB} alone; the second type, O_{bB} or $O_{\Phi b}$, must exist. In such a situation, the existence of O_{bB} will certainly give rise to observable effects at LEP II while effects of the operator $O_{\Phi b}$ will be unobservable. Thus, if no new physics effects are observed at LEP II, O_{bB} will be ruled out but $O_{\Phi b}$ will not be. Note that in all the numerical examples presented in this paper, we did not consider the co-existence of more than two operators at one time. The detailed analyses of their effects at LEP II in multi-parameter space is under consideration [**?**].

In summary, we have analyzed the effects of the dimension-six CP-conserving operators on the observables R_b and A_b^{FB} at LEP I, $\sigma(e^+e^- \to b\bar{b})$ and A_b^{FB} at LEP II, $\sigma(e^+e^- \to t\bar{t})$ and A_t^{FB} at the NLC as well as $\sigma(p\bar{p} \to t\bar{b} + X)$ at the Tevatron. We found that in the region allowed by R_b and A_b^{FB} at LEP I, some operators can still have significant contribution to observables at LEP II, the Tevatron and the NLC, while some other operators have negligibly small effects and thus can be safely ignored.

Acknowledgement

This work was supported in part by the U.S. Department of Energy, Division of High Energy Physics, under Grant No. DE-FG02-94ER40817.

Appendix A: CP-conserving operators after symmetry breaking

In order to shorten some of the expressions we will use the following notations:

$$W_\mu^3 = Z_\mu \cos\theta_W + A_\mu \sin\theta_W, \quad B_\mu = -Z_\mu \sin\theta_W + A_\mu \cos\theta_W$$

$$W^3_{\mu\nu} = Z_{\mu\nu}\cos\theta_W + A_{\mu\nu}\sin\theta_W, \quad B_{\mu\nu} = -Z_{\mu\nu}\sin\theta_W + A_{\mu\nu}\cos\theta_W$$

$$W^\pm_\mu = \frac{W^1_\mu \mp iW^2_\mu}{\sqrt{2}}, \quad W^\pm_{\mu\nu} = \partial_\mu W^\pm_\nu - \partial_\nu W^\pm_\mu + ig_2(W^3_\mu W^\pm_\nu - W^3_\nu W^\pm_\mu)$$

The CP-conserving operators after electroweak symmetry breaking are given as:

(1) Class 1

$$\begin{aligned}
O_{t1} &= \frac{H(H+2v)}{2}(\bar{t}t) \\
O_{t2} &= g_Z(H+v)^2 Z^\mu(\bar{t}_R\gamma_\mu t_R) \\
O_{t3} &= \frac{g_2^2(H+v)^2}{4}\left[W^{+\mu}(\bar{t}_R\gamma_\mu b_R) + W^{-\mu}(\bar{b}_R\gamma_\mu t_R)\right] \\
O_{Dt} &= \frac{1}{\sqrt{2}}\left[\bar{b}_L\partial_\mu t_R + (\partial_\mu\bar{t}_R)b_L\right]\partial^\mu H + \frac{ig_2}{\sqrt{2}}(H+v)\left[W^{+\mu}\bar{b}_L\partial_\mu t_R - W^{-\mu}(\partial_\mu\bar{t}_R)b_L\right] \\
&\quad + \frac{ig_1}{\sqrt{2}}B^\mu(\bar{t}_R\partial_\mu b_R - (\partial_\mu\bar{b}_R)t_R) + \frac{g_1 g_2}{\sqrt{2}}(H+v)\left[W^{+\mu}B_\mu\bar{t}_R b_R - W^{-\mu}B_\mu\bar{b}_R t_R\right] \\
O_{tW\Phi} &= \frac{g_2}{\sqrt{2}}(H+v)\left[W^+_{\mu\nu}(\bar{t}_R\sigma^{\mu\nu}b_L) + W^-_{\mu\nu}(\bar{b}_L\sigma^{\mu\nu}t_R)\right] + ig_2(W^+_\mu W^-_\nu - W^-_\mu W^+_\nu)(\bar{t}_R\sigma^{\mu\nu}t_R) \\
O_{tB\Phi} &= \frac{g_1}{\sqrt{2}}(H+v)B_{\mu\nu}(\bar{t}_R\sigma^{\mu\nu}b_R + \bar{b}_R\sigma^{\mu\nu}t_R) \\
O_{tG\Phi} &= \frac{g_s}{\sqrt{2}}(H+v)G^A_{\mu\nu}(\bar{t}_R\sigma^{\mu\nu}T^A b_R + \bar{b}_R\sigma^{\mu\nu}T^A t_R) \\
O_{tB} &= (\bar{t}_R\gamma^\mu\partial^\nu t_R + \partial^\nu\bar{t}_R\gamma^\mu t_R)B_{\mu\nu} \\
O_{tG} &= (\bar{t}_R\gamma^\mu T^A\partial^\nu t_R + \partial^\nu\bar{t}_R\gamma^\mu T^A t_R)G^A_{\mu\nu} + ig_s\bar{t}_R\gamma^\mu[G_\mu, G_\nu]t_R
\end{aligned}$$

(2) Class 2

$$
\begin{aligned}
O_{qG} &= \bar{q}_L \gamma^\mu T^A \partial^\nu q_L + \partial^\nu \bar{q}_L \gamma^\mu T^A q_L \\
O_{qW} &= \bar{q}_L \gamma^\mu \tau^I \partial^\nu q_L + \partial^\nu \bar{q}_L \gamma^\mu \tau^I q_L + i g_2 \epsilon^{IJK} \bar{q}_L \gamma^\mu \tau^I q_L W^J_\mu W^K_\nu \\
O_{qB} &= \bar{q}_L \gamma^\mu \partial^\nu q_L + \partial^\nu \bar{q}_L \gamma^\mu q_L \\
O_{bG} &= \bar{b}_R \gamma^\mu T^A \partial^\nu b_R + \partial^\nu \bar{b}_R \gamma^\mu T^A b_R \\
O_{bB} &= \bar{b}_R \gamma^\mu \partial^\nu b_R + \partial^\nu \bar{b}_R \gamma^\mu b_R \\
O^{(1)}_{\Phi q} &= i g_Z (H+v)^2 Z^\mu (\bar{t}_L \gamma_\mu t_L + \bar{b}_L \gamma_\mu b_L) \\
O^{(3)}_{\Phi q} &= \frac{g_2^2}{4} (H+v)^2 \left[W^{+\mu} (\bar{t}_L \gamma_\mu b_L) + W^{-\mu} (\bar{b}_L \gamma_\mu t_L) \right] \\
O_{\Phi b} &= i g_Z (H+v)^2 Z^\mu \bar{b}_R \gamma_\mu b_R \\
O_{b1} &= \frac{H(H+2v)}{2} \bar{b} b \\
O_{Db} &= \frac{1}{\sqrt{2}} \left[\bar{t}_L \partial_\mu b_R + (\partial_\mu \bar{b}_R) t_L \right] \partial^\mu H + \frac{i g_2}{\sqrt{2}} (H+v) \left[W^{+\mu} \bar{t}_L \partial_\mu b_R - W^{-\mu} (\partial_\mu \bar{b}_R) t_L \right] \\
&\quad + \frac{i g_1}{\sqrt{2}} B^\mu (\bar{b}_R \partial_\mu t_L - (\partial_\mu \bar{t}_L) b_R) + \frac{g_1 g_2}{\sqrt{2}} (H+v) \left[W^{+\mu} B_\mu \bar{b}_R t_L - W^{-\mu} B_\mu \bar{t}_L b_R \right] \\
O_{bW\Phi} &= \frac{g_2}{\sqrt{2}} (H+v) \left[W^+_{\mu\nu} (\bar{t}_L \sigma^{\mu\nu} b_R) + W^-_{\mu\nu} (\bar{b}_R \sigma^{\mu\nu} t_L) \right] \\
O_{bB\Phi} &= \frac{g_1}{\sqrt{2}} (H+v) B_{\mu\nu} (\bar{b}_R \sigma^{\mu\nu} t_L + \bar{t}_L \sigma^{\mu\nu} b_R) \\
O_{bG\Phi} &= \frac{g_s}{\sqrt{2}} (H+v) G^A_{\mu\nu} (\bar{b}_R \sigma^{\mu\nu} T^A t_L + \bar{t}_L \sigma^{\mu\nu} T^A b_R)
\end{aligned}
$$

Appendix B: Effective Lagrangian for some couplings

The effective Lagrangian for the couplings $Wt\bar{b}$, $Xb\bar{b}$ and $Xt\bar{t}$ (where $X = Z, \gamma, g, H$) are given by (the SM Lagrangians are not included here):

$$
\begin{aligned}
\mathcal{L}_{Wt\bar{b}} = &\frac{g_2 v}{\sqrt{2}\Lambda^2} \left[C^{(3)}_{qW} W^+_\mu (\bar{t}\gamma^\mu P_L b) + C_{bW\Phi} W^+_\mu (\bar{t}\gamma^\mu P_R b) \right] \\
&+ \frac{v}{\sqrt{2}\Lambda^2} \left[C_{Dt} (i\partial_\mu \bar{t}) P_L b + C_{Db} i\bar{t} P_R \partial_\mu b \right] W^{+\mu} \\
&+ \frac{1}{\sqrt{2}\Lambda^2} \left[C_{tW\Phi} W^+_{\mu\nu} (\bar{t}\sigma^{\mu\nu} P_L b) + C_{bW\Phi} W^+_{\mu\nu} (\bar{t}\sigma^{\mu\nu} P_R b) \right]
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_{Zb\bar{b}} = &\frac{g_Z v m_Z}{\Lambda^2} \left[C^{(1)}_{\Phi q} + C^{(3)}_{\Phi q} \right] Z_\mu (\bar{b}\gamma^\mu P_L b) + \frac{g_Z v m_Z}{\Lambda^2} C_{\Phi b} Z_\mu (\bar{b}\gamma^\mu P_R b) \\
&+ \frac{g_Z}{\Lambda^2} Z_{\mu\nu} \left[C_{qB} (\bar{b}_L \gamma^\mu \partial^\nu b_L + \partial^\nu \bar{b}_L \gamma^\mu b_L) + C_{bB} (\bar{b}_R \gamma^\mu \partial^\nu b_R + \partial^\nu \bar{b}_R \gamma^\mu b_R) \right] \\
&+ \frac{g_Z}{\Lambda^2} Z_{\mu\nu} \left[C_{bW\Phi} (\bar{b}_R \sigma^{\mu\nu} t_L) + C_{bB\Phi} (\bar{b}_R \sigma^{\mu\nu} b_R) \right]
\end{aligned}
$$

$$\mathcal{L}_{\gamma b\bar{b}} = \frac{e}{\Lambda^2} A_{\mu\nu} \left[C_{qB}(\bar{b}_L \gamma^\mu \partial^\nu b_L + \partial^\nu \bar{b}_L \gamma^\mu b_L) + C_{bB}(\bar{b}_R \gamma^\mu \partial^\nu b_R + \partial^\nu \bar{b}_R \gamma^\mu b_R) \right] + \frac{e}{\Lambda^2} A_{\mu\nu} C_{bB\Phi}(\bar{b}_R \sigma^{\mu\nu} b_R)$$

$$\mathcal{L}_{Zt\bar{t}} = \frac{g_Z v m_Z}{\Lambda^2} \left[C_{\Phi q}^{(1)} + C_{\Phi q}^{(3)} \right] Z_\mu (\bar{t}\gamma^\mu P_L t) + \frac{g_Z v m_Z}{\Lambda^2} C_{t2} Z_\mu (\bar{t}\gamma^\mu P_R t) + \frac{g_Z}{\Lambda^2} Z_{\mu\nu} \left[C_{qB}(\bar{t}_L \gamma^\mu \partial^\nu t_L + \partial^\nu \bar{t}_L \gamma^\mu t_L) + C_{tB}(\bar{t}_R \gamma^\mu \partial^\nu t_R + \partial^\nu \bar{t}_R \gamma^\mu t_R) \right] + \frac{g_Z}{\Lambda^2} Z_{\mu\nu} \left[C_{tW\Phi}(\bar{t}\sigma^{\mu\nu} t) + C_{tB\Phi}(\bar{t}_R \sigma^{\mu\nu} t_R) \right]$$

$$\mathcal{L}_{\gamma t\bar{t}} = \frac{e}{\Lambda^2} A_{\mu\nu} \left[C_{qB}(\bar{t}_L \gamma^\mu \partial^\nu t_L + \partial^\nu \bar{t}_L \gamma^\mu t_L) + C_{tB}(\bar{t}_R \gamma^\mu \partial^\nu t_R + \partial^\nu \bar{t}_R \gamma^\mu t_R) \right] + \frac{e}{\Lambda^2} A_{\mu\nu} C_{tB\Phi}(\bar{t}_R \sigma^{\mu\nu} t_R)$$

$$\mathcal{L}_{Ht\bar{t}} = \frac{v}{\Lambda^2} \partial_\mu H \left[C_{Dt}(\partial^\mu \bar{t} t + \bar{t}\gamma_5 \partial^\mu t - (\partial^\mu \bar{t})\gamma_5 t) \right] + \frac{m_t}{\Lambda^2} H (\bar{t}t) \left[C_{t1} + C_{t2} \right]$$

$$\mathcal{L}_{gt\bar{t}} = \frac{g_s}{\Lambda^2} G^A_{\mu\nu} \left[C_{qG}(\bar{t}_L \gamma^\mu \partial^\nu t_L + \partial^\nu \bar{t}_L \gamma^\mu t_L) + C_{tG}(\bar{t}_R \gamma^\mu \partial^\nu t_R + \partial^\nu \bar{t}_R \gamma^\mu t_R) \right] + \frac{g_s}{\Lambda^2} G^A_{\mu\nu} C_{tG\Phi}(\bar{t}\sigma^{\mu\nu} T^A t)$$

$$\mathcal{L}_{Hb\bar{b}} = \frac{v}{\Lambda^2} \partial_\mu H \left[C_{Db}(\partial^\mu \bar{b} b + \bar{b}\gamma_5 \partial^\mu b - (\partial^\mu \bar{b})\gamma_5 b) \right] + \frac{m_b}{\Lambda^2} H (\bar{b}b) C_{b1}$$

Note: Figure translations are in progress. See original paper for figures.