The Zbb̄ Coupling Anomaly Revisited in MSSM and NMSSM (Postprint)

Authors: Cao,J, Yang,JM

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Full Text

Anomaly of $Zb\bar{b}$ Coupling Revisited in MSSM and NMSSM

Junjie Cao[1], Jin Min Yang[2]

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[1]Ottawa-Carleton Institute for Physics, Department of Physics, Carleton University, Ottawa, Canada K1S 5B6
[2]Institute of Theoretical Physics and Kavli Institute for Theoretical Physics China, Academia Sinica, Beijing 100190, China

Abstract

The $Zb\bar{b}$ coupling determined from the Z-pole measurements at LEP/SLD shows an approximately 3σ deviation from the SM prediction, which would signal the presence of new physics associated with the $Zb\bar{b}$ coupling. In this work, we present a comprehensive study of the full one-loop supersymmetric effects on the $Zb\bar{b}$ coupling in both the MSSM and NMSSM, considering all current constraints from precision electroweak measurements, direct searches for sparticles and Higgs bosons, Higgs potential stability, dark matter relic density, and the muon $g-2$ measurement. We analyze the characteristics of each type of correction and search for SUSY parameter regions where the corrections could be sizable. We find that sizable corrections may arise from the Higgs sector with light m_A and large $\tan\beta$, reaching -2% for ρ_b and -6% for $\sin^2\theta_b^{\rm eff}$, respectively.

However, such sizable negative corrections are opposite to what is needed to solve the anomaly. We also scan over the allowed parameter space to investigate the extent to which supersymmetry can narrow the discrepancy. We find that under all current constraints, supersymmetric effects are quite restrained and cannot significantly ameliorate the $Zb\bar{b}$ coupling anomaly. Compared with $\chi^2/\text{dof} = 9.62/2$ in the SM, the MSSM and NMSSM can only improve it to $\chi^2/\text{dof} = 8.77/2$ in the allowed parameter space. Therefore, if the $Zb\bar{b}$ coupling anomaly is not a statistical or systematic problem, it would suggest new physics beyond the MSSM or NMSSM.

Introduction

Although most electroweak data are consistent with the Standard Model (SM) to remarkable precision, some experimental results remain difficult to accommodate within the SM framework. A well-known example is that the effective electroweak mixing angle $\sin^2\theta_{\text{eff}}$ determined from leptonic asymmetry measurements is much lower than the value determined from hadronic asymmetry measurements [?, ?], and the average over all these asymmetries yields a χ^2/dof of 11.8/5, corresponding to only a 3.7% probability for the asymmetry data to be consistent with the SM hypothesis. This large discrepancy mainly stems from the two most precise determinations of $\sin^2\theta_{\text{eff}}$: the A_{LR} measurement by SLD and the bottom forward-backward asymmetry A_b^{FB} measurement at LEP, which give values on opposite sides of the average and differ by 3.2 standard deviations.

It is interesting to note that if this discrepancy is attributed to experimental origin and hadronic asymmetry measurements are excluded from the global fit, a rather light Higgs boson around 50 GeV is indicated [?, ?], which sharply contrasts with the LEP II direct search limit of 114 GeV [?] and results in an equally low compatibility probability of 3%. If we resort to new physics to resolve this discrepancy, the new physics effects must significantly modify the $Zb\bar{b}$ coupling while leaving the Z-boson couplings to other fermions essentially unchanged. In this work, we focus on the $Zb\bar{b}$ coupling and scrutinize supersymmetric effects.

In our analysis, we parameterize the $Zf\bar{f}$ interaction at the Z-pole in terms of the parameter ρ_f and effective electroweak mixing angle $\sin^2\theta_f^{\text{eff}}$ [?, ?]:

$$Zf\bar{f} = \frac{\sqrt{2}G_\mu \rho_f m_Z \gamma^\mu}{2}\left[I_3^f - 2Q_f \sin^2\theta_f^{\text{eff}} + I_3^f\gamma_5\right]$$

This parametrization is preferred from the experimental perspective because all measured asymmetries depend only on $\sin^2\theta_f^{\text{eff}}$, and their precise measurements can directly determine $\sin^2\theta_f^{\text{eff}}$. From the combined LEP and SLD data analysis, the fitted values of ρ_f and $\sin^2\theta_f^{\text{eff}}$ agree well with SM predictions for leptons and light quarks, but for the bottom quark their fitted values are 1.059 ± 0.021 and 0.281 ± 0.016 (with correlation coefficient 0.99), respectively, which significantly deviate from the SM predictions of 0.994 and 0.233 (for $m_t = 174$ GeV and $m_h = 115$ GeV) and lead to $\chi^2/\mathrm{dof} = 9.62/2$ (corresponding to a compatibility probability of 0.8%). To best fit the experimental data, ρ_b and $\sin^2\theta_b^{\mathrm{eff}}$ should be enhanced by about 6.5% and 20%, respectively.

While we can envisage that supersymmetric effects are not usually so large, we want to determine the extent to which supersymmetry can improve the situation. For this purpose, we examine two popular supersymmetric models: the Minimal Supersymmetric Standard Model (MSSM) [?] and the Next-to-Minimal Supersymmetric Standard Model (NMSSM) [?].

For the NMSSM effects on the $Zb\bar{b}$ coupling, which have not been studied in the literature, we perform the calculation to one-loop level. For the MSSM effects, which have been studied by many authors [?], we renew the study in the parametrization of ρ_b and $\sin^2\theta_b^{\mathrm{eff}}$ (previous studies usually examined effects on the Z-width, the ratio R_b, and the asymmetry A_b^{FB}). For both the MSSM and NMSSM, we consider various current experimental constraints on the parameter space from precision electroweak measurements, direct searches for sparticles and Higgs bosons, Higgs potential stability, cosmic dark matter relic density, and the muon $g-2$ measurement.

This paper is organized as follows. In Sec. II we introduce the general formula for calculating ρ_f and $\sin^2\theta_f^{\mathrm{eff}}$ and apply them to the MSSM and NMSSM. In Sec. III we summarize the constraints considered in this work and briefly discuss their characteristics. In Sec. IV and Sec. V we perform numerical studies of the corrections to ρ_b and $\sin^2\theta_b^{\mathrm{eff}}$ in the MSSM and NMSSM, respectively. We first show the characteristics of different types of corrections, then scan the whole SUSY parameter space to investigate the compatibility of supersymmetric predictions for ρ_b and $\sin^2\theta_b^{\mathrm{eff}}$ with experimental results. Finally, in Sec. VI we conclude with an outlook on the possibility of solving the $Zb\bar{b}$ anomaly.

II. General Formula to Calculate ρ_f and $\sin^2\theta_f^{\mathrm{eff}}$

In the SM with input parameters the Fermi constant G_F, the fine-structure constant α, Z-boson mass m_Z, and fermion masses m_f, the electroweak mixing angle $s_W = \sin\theta_W$ is determined at loop level by [?]:

$$s_W^2 = \frac{\sqrt{2}G_\mu m_Z^2}{1+\Delta r}$$

where Δr is given by:

$$\Delta r = \frac{\hat{\Sigma}_W(0)}{m_W^2} + 2\delta_v + \delta_b$$

with $\hat{\Sigma}_W$ denoting the renormalized W-boson self-energy, and δ_v and δ_b being the vertex correction and box diagram correction to muon decay $\mu \to \nu_\mu e \bar{\nu}_e$, respectively. To obtain a more precise numerical result for s_W^2, one can iterate Eqs. (2) and (3) a few times.

With s_W defined above, the effective $Zf\bar{f}$ coupling at the Z-pole takes the following form [**?**, **?**]:

$$Zf\bar{f} = \frac{\sqrt{2}G_\mu(1+\delta_Z)}{2} m_Z \gamma^\mu \left[(v_f + \delta v_f) - (a_f + \delta a_f)\gamma_5\right]$$

where $v_f = I_3^f - 2Q_f s_W^2$ and $a_f = I_3^f$ are respectively the vector and axial-vector coupling coefficients of the $Zf\bar{f}$ interaction at tree level, and δv_f and δa_f are their corresponding corrections. Σ'_Z is the derivative of the unrenormalized Z-boson self-energy Σ_Z with respect to the squared momentum p^2, and $\delta_Z = -\Sigma'_Z(m_Z^2)$ is the field renormalization constant of the Z-boson given by:

$$\Sigma'_Z(m_Z^2) = \frac{\Sigma_Z(m_Z^2) - \Sigma_Z(0)}{m_Z^2}$$

The factor $(1+\delta_Z)$ in Eq. (4) arises because the residue of the renormalized Z propagator differs from 1, while the last term enters due to Z-γ mixing at the Z-pole.

If we re-express $\Gamma^\mu_{Zf\bar{f}}$ in Eq. (4) in terms of ρ_f and $\sin^2\theta_f^{\text{eff}}$ as in Eq. (1), we obtain:

$$\rho_f = 1 + \delta\rho_{\text{se}} + \delta\rho_{f,v}, \quad \sin^2\theta_f^{\text{eff}} = (1 + \delta\kappa_{\text{se}} + \delta\kappa_{f,v}) s_W^2$$

with:

$$\delta\rho_{\text{se}} = \frac{\Sigma_Z(0)}{m_Z^2} - \frac{\Sigma_W(0)}{m_W^2}, \quad \delta\kappa_{\text{se}} = \Delta\kappa$$

and:

$$\delta\rho_{f,v} = \frac{2}{a_f}\delta v_f, \quad \delta\kappa_{f,v} = \frac{1}{2Q_f a_f s_W^2}(a_f \delta v_f - v_f \delta a_f)$$

In the above equations, the subscript 'se' denotes contributions from gauge boson self-energies which are flavor-independent, while 'f,v' denotes contributions from vertex corrections to the $Zf\bar{f}$ interaction. In practice, it is convenient to express $\delta\rho_{f,v}$ and $\delta\kappa_{f,v}$ in terms of δg_f^L and δg_f^R:

$$\delta\rho_{f,v} = \frac{(a_f + v_f)\delta g_f^L + (a_f - v_f)\delta g_f^R}{4Q_f a_f s_W^2}, \quad \delta\kappa_{f,v} = \frac{\delta g_f^L - \delta g_f^R}{2Q_f}$$

where $\delta g_f^{L,R} = \delta v_f \mp \delta a_f$ are the corrections to $Z f_L \bar{f}_L$ and $Z f_R \bar{f}_R$ interactions, respectively.

From these equations, one can see that the correction to $\delta\rho_{f,v}$ is determined by the competition between δg_f^L and δg_f^R, while $\delta\kappa_{f,v}$ is mainly determined by δg_f^R due to the factor $(a_f + v_f)/(a_f - v_f) \approx 1/(4s_W^2) \gg 1$ for the bottom quark. Noting that Feynman rules for Z-boson couplings in SUSY models usually differ from their SM counterparts by a minus sign [**?**, **?**], $\Sigma_{\gamma Z}$ and $\delta\kappa_{f,v}$ in the above formulas should change sign if one uses the Feynman rules in SUSY models. The self-energies and vertex corrections in SUSY models then include both SM-particle loop contributions and SUSY-particle loop contributions. Since the SM-particle contributions are well known, in Appendices A and B we only list the one-loop expressions for the SUSY contributions.

One subtlety concerns avoiding double-counting of Higgs contributions. This problem arises for the following reason. On one hand, the SM values of ρ_b and $\sin^2\theta_b^{\text{eff}}$ are known to higher orders, and one usually incorporates such high-order SM effects when performing numerical calculations in SUSY models. On the other hand, because the SUSY Higgs sector is quite different from the SM, one cannot obtain the SUSY Higgs contributions simply by adding additional terms to the SM Higgs contributions. In our SUSY model calculations, to avoid double-counting of Higgs contributions, we first subtract the SM Higgs contributions from their SM values (calculated by the codes TOPAZ0 [**?**] and ZFITTER [**?**]), and then add the full one-loop contributions from SUSY Higgs bosons and sparticles.

III. Constraints on SUSY Parameters

Before discussing SUSY corrections to the $Zb\bar{b}$ coupling in the MSSM and NMSSM, we examine the SUSY parameters involved in our calculations. From the expressions for the $Zf\bar{f}$ vertex corrections listed in Appendix B, one can see that the SUSY-EW correction depends on the masses and mixings of top squarks, bottom squarks, charginos, and neutralinos; the SUSY-QCD vertex correction depends on the gluino mass and the masses and chiral mixing of bottom squarks; and the Higgs-mediated vertex correction depends on the masses and mixings of Higgs bosons. The expressions for gauge boson self-energies listed in Appendix A indicate that SUSY corrections also depend on slepton masses and first-two-generation squark masses. Regarding these SUSY parameters, we consider the following constraints:

1. **Constraints from direct sparticle searches at LEP and Tevatron** [**?**]: $m_{\tilde{\chi}_1^0} > 41$ GeV, $m_{\tilde{\chi}_2^0} > 62.4$ GeV, $m_{\tilde{\chi}_3^0} > 99.9$ GeV, $m_{\tilde{\chi}^\pm} > 94$ GeV, $m_{\tilde{e}} > 73$ GeV, $m_{\tilde{\mu}} > 94$ GeV, $m_{\tilde{\tau}} > 81.9$ GeV, $m_{\tilde{q}} > 250$ GeV, $m_{\tilde{t}} > 89$ GeV, $m_{\tilde{b}} > 95.7$ GeV, $m_{\tilde{g}} > 195$ GeV, where $m_{\tilde{\chi}^0}$ denotes neutralino masses and $m_{\tilde{q}}$ denotes first-two-generation squark masses.

2. **Constraint from direct Higgs boson searches at LEP** [**?**]: This

constraint limits the values of m_A, $\tan\beta$, and the masses and chiral mixing of top squarks. In the case of large $\tan\beta$, it can also constrain the masses and mixing of bottom squarks. Generally, this constraint requires the product of two top squark masses, $m_{\tilde{t}_1} m_{\tilde{t}_2}$, to be much larger than m_t^2 [?].

3. **Constraint from theoretical requirements**: There must be no Landau pole for the running Yukawa couplings Y_b and Y_t below the GUT scale, and the physical minimum of the Higgs potential with non-vanishing vacuum expectation values must be lower than local minima with vanishing values.

4. **Constraints from precision electroweak observables** such as $\rho_{\rm lept}$, $\sin^2\theta^{\rm eff}_{\rm lept}$, ρ_c, $\sin^2\theta^{\rm eff}_c$, and M_W. These constraints are equivalent to those from the well-known $\epsilon_i (i = 1, 2, 3)$ parameters [?] or S, T, and U parameters [?]. The measured values are [?]: $\rho_{\rm lept} = 1.0050 \pm 0.0010$, $\sin^2\theta^{\rm eff}_{\rm lept} = 0.23153 \pm 0.00016$, $\rho_c = 1.013 \pm 0.021$, $\sin^2\theta^{\rm eff}_c = 0.2355 \pm 0.0059$, $M_W = 80.403 \pm 0.029$ GeV, and their SM fitted values are $\rho^{\rm SM}_{\rm lept} = 1.0051$, $\sin^2\theta^{\rm eff}_{\rm lept,SM} = 0.2314$, $\rho^{\rm SM}_c = 1.0058$, $\sin^2\theta^{\rm eff}_c = 0.23149$, and $M_W = 80.36$ GeV for $m_t = 173$ GeV and $m_h = 111$ GeV. In our calculations, we require theoretical predictions to agree with experimental values at the 2σ level.

5. **Constraint from $R_b = \Gamma(Z \to b\bar{b})/\Gamma(Z \to$ hadrons)**: The measured value is $R_b = 0.21629 \pm 0.00066$ and the SM prediction is $R^{\rm SM}_b = 0.21578$ for $m_t = 173$ GeV. In our analysis, we require $R^{\rm SUSY}_b$ to be within the 2σ range of its experimental value.

6. **Constraint from cosmic dark matter relic density**: $0.0945 < \Omega h^2 < 0.1287$ [?]. This constraint can rule out broad parameter regions for gaugino masses $M_{1,2}$, the μ parameter, m_A, and $\tan\beta$ [?].

7. **Constraint from muon anomalous magnetic moment a_μ**: Both theoretical predictions and experimental measurements of a_μ have reached remarkable precision, showing a significant deviation $a^{\rm exp}_\mu - a^{\rm SM}_\mu = (29.5 \pm 8.8) \times 10^{-10}$ [?]. In our analysis, we require SUSY effects to account for this difference at the 2σ level.

Note that we do not include constraints from B physics, such as $b \to s\gamma$ and B_s-$\bar{B}_s$ mixing [?], because these constraints are sensitive to squark flavor mixings [?] which are irrelevant to our discussion.

Among the constraints listed above, constraints (4) and (5), especially the observables M_W, $\rho_{\rm lept}$, $\sin^2\theta^{\rm eff}_{\rm lept}$, and R_b, are most relevant to our study of ρ_b and $\sin^2\theta^{\rm eff}_b$. Let us examine these constraints in more detail.

First, precise measurements of M_W, $\rho_{\rm lept}$, and $\sin^2\theta^{\rm eff}_{\rm lept}$ stringently constrain $\delta\rho_{\rm se}$, $\delta\kappa_{\rm se}$, and the gaugino loop contributions to $\delta\rho_{b,v}$ and $\delta\kappa_{b,v}$. The approximate forms of SUSY corrections to M_W, $\delta\rho_{\rm se}$, and $\delta\kappa_{\rm se}$ [?] in the heavy sparticle limit are:

$$\delta(\Delta r) \approx -\frac{\Sigma_Z(0)}{m_Z^2} + \frac{\Sigma_W(0)}{m_W^2} + \frac{\Sigma_{\gamma Z}(0)}{m_Z^2}$$

where $\Delta\rho \equiv \Sigma_Z(0)/m_Z^2 - \Sigma_W(0)/m_W^2$ is the correction to the classical ρ parameter [?] and is only sensitive to the mass spectrum of third-generation squarks. Through these relations, the precisely measured M_W stringently restricts $\Delta\rho$ (of order 10^{-4}) and subsequently restricts $\delta\rho_{\rm se}$ and $\delta\kappa_{\rm se}$. This restriction, together with precisely determined $\rho_{\rm lept}$ and $\sin^2\theta^{\rm eff}_{\rm lept}$, stringently constrains the magnitude of $\delta\rho_{l,v}$ and $\delta\kappa_{l,v}$ defined in Eq. (9) to be below 10^{-4}. Since gaugino loop effects in $\delta\rho_{b,v}$ and $\delta\kappa_{b,v}$ are strongly correlated with $\delta\rho_{l,v}$ and $\delta\kappa_{l,v}$ (the main difference arises from the mass difference between sleptons and squarks), the gaugino loop contributions to $\delta\rho_{b,v}$ and $\delta\kappa_{b,v}$ are also suppressed, which we find to be below 5×10^{-4} from numerical calculations.

For the constraint from precision observable R_b, an interesting feature is that it does not stringently constrain the magnitude of δv_b and δa_b, but favors the relation $\delta v_b \approx 1.44\delta a_b$, as seen from the expression for the radiative correction to R_b [?]:

$$\delta R_b \approx R_b^{\rm SM}\frac{2a_b^2\beta^2}{(v_b^2+a_b^2\beta^2)^2}\left[(v_b^2-a_b^2\beta^2)\delta v_b + 2a_b^2\beta^2\delta a_b\right] \propto (\delta v_b + 1.44\delta a_b)$$

with $\beta = \sqrt{1-4m_b^2/m_Z^2}$ being the velocity of the bottom quark in Z decay.

Now we turn to the constraint from muon anomalous magnetic moment. For an intuitive understanding, we consider a simple MSSM case where all gaugino masses and soft-breaking masses in the smuon sector have a common scale M. In this case, $a_\mu^{\rm SUSY}$ is approximated by [?]:

$$a_\mu^{\rm SUSY} \approx \frac{\alpha(M_Z)}{8\pi}\frac{m_\mu^2}{M^2}\tan\beta\,{\rm sign}(\mu)$$

The gap between $a_\mu^{\rm SM}$ and $a_\mu^{\rm exp}$ then prefers positive μ and constrains the product $(\tan\beta/M)$ in the range $[1.0, 3.6]\times10^{-3}$ GeV^{-1} at 2σ level. Thus, the SUSY scale can be higher for larger $\tan\beta$.

In our calculations, we use the code NMSSMTools [?] to generate masses and mixings for all sparticles and Higgs bosons in the NMSSM framework with all known radiative corrections included. There are two advantages to using this code: first, all masses and mixings in the MSSM can be easily recovered by setting $\lambda = \kappa = 0$ and A_κ to be negatively small; second, it incorporates the code MicrOMEGAs [?] which calculates the cosmic dark matter relic density.

Note that the current version of NMSSMTools only includes constraints (1), (2), (3), and (6), which we extend by including constraints (4), (5), and (7). We note that the muon anomalous magnetic moment was recently calculated in the NMSSM [**?**] and our calculations agree with theirs.

IV. One-Loop Corrections to ρ_b and $\sin^2\theta_b^{\text{eff}}$ in the MSSM

In this section, we investigate ρ_b and $\sin^2\theta_b^{\text{eff}}$ at one-loop level in the MSSM. As discussed above, self-energy corrections to these observables are generally small, so we mainly scrutinize vertex corrections, which include SUSY-EW corrections, SUSY-QCD corrections, and Higgs-mediated vertex corrections. We pay special attention to cases where correction magnitudes are large and demonstrate that $\tan\beta$ is crucial for enhancing vertex corrections. Our analysis proceeds as follows: we first investigate the characteristics of vertex corrections to gain intuitive understanding, then scan the MSSM parameter space to study the compatibility of MSSM predictions for ρ_b and $\sin^2\theta_b^{\text{eff}}$ with experimental results.

The SM input parameters in our calculations are taken from [**?**]: $\alpha = 1/128.93$, $G_F = 1.16637\times10^{-5}$ GeV^{-2}, $\alpha_s(m_Z) = 0.1172$, $m_Z = 91.1876$ GeV, $m_t = 172.5$ GeV, and $m_b(m_b) = 4.2$ GeV.

A. Characteristics of Vertex Corrections in the MSSM

For the SUSY-EW contribution to $\delta\rho_{b,v}$ and $\delta\kappa_{b,v}$, the relevant parameters are gaugino masses $M_{1,2}$, Higgsino mass μ, $\tan\beta = v_2/v_1$ (with $v_{1,2}$ being the vacuum expectation values of the Higgs fields), soft-breaking masses M_{Q3}, M_{U3}, M_{D3}, and trilinear term coefficients A_t and A_b. The first four parameters enter the mass matrices of neutralinos and charginos, while the last seven parameters affect the masses and chiral mixings of third-generation squarks [**?**].

As discussed in the preceding section, the gaugino loop contribution is small, so we focus on the Higgsino loop contribution. The magnitude of this Higgsino loop contribution is sensitive to $\tan\beta$, the Higgsino mass μ, and the masses and chiral mixings of third-generation squarks. This contribution has two key features: first, because the bottom Yukawa coupling Y_b is proportional to $1/\cos\beta$, the contribution can be potentially large for large $\tan\beta$ and small μ; second, the contribution is moderately sensitive to the chiral mixings of third-generation squarks, with potentially large contributions arising when the mixing is small and the lighter squark component is dominated by the left-handed squark [**?**]. To illustrate these features, we consider three cases in the squark sector: - **Case I**: $M_S = M_{Q3} = M_{U3} = M_{D3} = 400$ GeV, $A_t = A_b = 800$ GeV - **Case II**: $M_{Q3} = 200$ GeV, $M_{U3} = M_{D3} = 600$ GeV, $A_t = A_b = 800$ GeV - **Case III**: $M_{Q3} = 600$ GeV, $M_{U3} = M_{D3} = 200$ GeV, $A_t = A_b = 800$ GeV

We fix other SUSY parameters as $M_1 = 75$ GeV, $M_2 = 150$ GeV, $m_A = 500$ GeV, and $M_{\text{SUSY}} = 1$ TeV, where M_{SUSY} denotes soft-breaking parameters for sleptons and first-two-generation squarks. Case I corresponds to maximal chiral mixing, Case II is the small mixing case with the lighter squark component dominated by the left-handed squark, and Case III is also a small mixing case but with the lighter squark component dominated by the right-handed squark.

In Fig. 1 [Figure 1: see original paper], we show the dependence of the SUSY-EW contribution to $\delta\rho_{b,v}$ and $\delta\kappa_{b,v}$ on $\tan\beta$ for the three cases. Both $\delta\rho_{b,v}$ and $\delta\kappa_{b,v}$ are sensitive to $\tan\beta$. As $\tan\beta$ increases, $\delta\rho_{b,v}$ and $\delta\kappa_{b,v}$ receive more negative contributions and, for small μ, become negative with sizable magnitudes. This behavior can be understood as follows: as $\tan\beta$ grows large, the bottom Yukawa coupling increases, the correction to the right-handed $Zb\bar{b}$ coupling δg_b^R increases positively, and consequently $\delta\rho_{b,v}$ and $\delta\kappa_{b,v}$ receive more negative contributions from the increasing δg_b^R (see Eq. (10) and Appendix B). The figures also show that the magnitude of $\delta\kappa_{b,v}$ is usually larger than $\delta\rho_{b,v}$. The factor $1/(4Q_f a_f s_W^2)$ in the denominator of $\delta\kappa_{b,v}$ (see Eq. (9)) can largely account for this.

Note that in these figures we only plot results within the $\tan\beta$ range that survives constraints (1)-(5). Constraint (7), the muon anomalous magnetic moment, could in principle also limit $\tan\beta$, but this constraint depends on the smuon mass scale M_{SUSY} in Eq. (15), to which ρ_b and $\sin^2\theta_b^{\text{eff}}$ are not sensitive, so we do not apply it in these figures. Our numerical results indicate that the muon anomalous magnetic moment allows a vast region of M_{SUSY} and μ where $\tan\beta$ can be as large as 60, making sizable SUSY-EW corrections to ρ_b and $\sin^2\theta_b^{\text{eff}}$ possible. For example, with the parameters in Eq. (15), the $\tan\beta$ ranges allowed by the muon $g-2$ are: $\tan\beta \lesssim 25$ for $\mu = 200$ GeV, $\tan\beta \lesssim 33$ for $\mu = 500$ GeV, and $\tan\beta \lesssim 44$ for $\mu = 800$ GeV. If we choose $M_{\text{SUSY}} = 0.5$ TeV, these allowed ranges become $7 \lesssim \tan\beta \lesssim 57$, $12 \lesssim \tan\beta \lesssim 60$, and $\tan\beta \lesssim 60$, respectively.

Next, we discuss SUSY-QCD corrections. The relevant parameters are the gluino mass and M_{Q3}, M_{D3}, and $X_b = (A_b - \mu\tan\beta)$, which enter the bottom squark mass matrix. From the large strength of the strong coupling $g_s(m_Z)$, one might naively expect SUSY-QCD contributions to $\delta\rho_{b,v}$ and $\delta\kappa_{b,v}$ to be much larger than Higgsino loop contributions when $m_{\tilde{g}} \sim \mu$ and $\tan\beta \sim 50$. However, our numerical results show that for small sbottom chiral mixing, SUSY-QCD contributions to $\delta\rho_{b,v}$ and $\delta\kappa_{b,v}$ are negligibly small. The underlying reason is that for SUSY-QCD corrections, there is strong cancellation between different diagrams in the case of small sbottom chiral mixing, as seen from the expressions for $\delta g_b^{L,R}$ listed in Appendix B. This cancellation can be alleviated for large sbottom mixing, or equivalently, for a large $\mu\tan\beta$ term appearing in the off-diagonal elements of the sbottom mass matrix (we verified this numerically). Thus, the contribution may be sizable for large $\mu\tan\beta$, as shown in Fig. 2 [Figure 2: see original paper].

Compared with Higgsino loop corrections, the SUSY-QCD contributions in Fig. 2 exhibit similar behavior with respect to $\tan\beta$. The difference is that the most sizable effects come from Case I (maximal sbottom mixing) with large μ, rather than Case II with small μ as for Higgsino loop corrections.

Finally, we consider Higgs loop contributions to $\delta\rho_{b,v}$ and $\delta\kappa_{b,v}$ [?]. To calculate this contribution, we need the masses and mixings of Higgs bosons, which are determined by m_A and $\tan\beta$ at tree level, and also by soft-breaking masses for third-generation squarks if important loop corrections to Higgs boson masses are included. As shown in Fig. 3 [Figure 3: see original paper], the contributions exhibit similar dependence on $\tan\beta$, with significant contributions arising for small m_A and large $\tan\beta$. We verified that the results in Fig. 3 are not sensitive to μ or M_S, nor to the choice among Cases I-III.

From these figures, one can infer that among the three types of corrections, the potentially largest correction comes from Higgs loops, reaching -2% for ρ_b and -6% for $\sin^2\theta_b^{\text{eff}}$. Such large corrections reach current experimental sensitivity since the experimental measurements are $\rho_b^{\text{exp}} = 1.059 \pm 0.021$ and $\sin^2\theta_b^{\text{exp,eff}} = 0.281 \pm 0.016$.

Before concluding this section, we point out that in the large $\tan\beta$ limit, the cosmic dark matter relic density allows either small μ or small m_A (but not both simultaneously). This can be seen from Fig. 4 [Figure 4: see original paper], where we show allowed regions in the $\tan\beta$ versus μ plane for different m_A values. In plotting this figure, we choose Case I and fix other relevant parameters as in Eq. (15). Fig. 4 implies that SUSY-EW and Higgs-loop contributions to $\delta\rho_{b,v}$ and $\delta\kappa_{b,v}$ cannot simultaneously reach their maximal values.

B. MSSM Predictions for ρ_b and $\sin^2\theta_b^{\text{eff}}$

As mentioned above, the values extracted for ρ_b and $\sin^2\theta_b^{\text{eff}}$ from combined LEP and SLD data analysis are 1.059 ± 0.021 and 0.281 ± 0.016, respectively, with correlation coefficient 0.99 [?]. This result is shown in Fig. 5 [Figure 5: see original paper] with three ellipses corresponding to 68%, 95.5%, and 99.5% confidence levels. Noting that SM predictions are $\rho_b^{\text{SM}} = 0.994$ and $\sin^2\theta_b^{\text{SM,eff}} = 0.233$ (for $m_t = 174$ GeV and $m_h = 115$ GeV), one might infer that large positive corrections to ρ_b and $\sin^2\theta_b^{\text{eff}}$ are needed to narrow the gap between experimental data and SM predictions. As discussed in the preceding section, MSSM corrections can be sizable for large $\tan\beta$, but they are negative and thus cannot narrow the gap. To determine the extent to which MSSM predictions can agree with experiment, we consider all constraints discussed in Sec. III and scan the SUSY parameter space:

$$0 < M_1, M_2, M_3, \mu, M_{Q3}, M_{U3}, M_{D3}, M_A, M_{\text{SUSY}} < 3 \text{ TeV}, \quad |A_t|, |A_b| < 3 \text{ TeV}, \quad 1 < \tan\beta < 60$$

Based on a sample of twenty billion points, we find the best MSSM predictions are $\rho_b = 0.9960$ and $\sin^2\theta_b^{\text{eff}} = 0.2328$, yielding $\chi^2/\text{dof} = 9.07/2$ when compared with experimental data. If we do not consider the dark matter constraint, the best MSSM predictions are $\rho_b = 0.99737$ and $\sin^2\theta_b^{\text{eff}} = 0.2336$, giving $\chi^2/\text{dof} = 8.77/2$. Moreover, we find such optimal cases occur when μ, m_A, and $m_{\tilde{g}} \gtrsim 1$ TeV, suppressing all three types of vertex corrections.

V. One-Loop Predictions for ρ_b and $\sin^2\theta_b^{\text{eff}}$ in the NMSSM

A. Introduction to the NMSSM

As a popular extension of the MSSM, the NMSSM provides an elegant solution to the μ-problem by introducing a singlet Higgs superfield $\hat{S}$, which naturally develops a vacuum expectation value of order the SUSY breaking scale and generates the required μ term. Another virtue of the NMSSM is that it can alleviate the little hierarchy problem since the theoretical upper bound on the SM-like Higgs boson mass is increased and the LEP II lower bound on the Higgs boson mass is relaxed due to the suppressed ZZh coupling or suppressed decay $h \to b\bar{b}$ [?]. Given these motivations, NMSSM phenomenology has been intensively studied recently, including its effects in Higgs physics [?], neutralino physics [?], B-physics [?], and squark physics [?]. Below we recapitulate NMSSM basics with emphasis on differences from the MSSM.

The NMSSM superpotential takes the form [?, ?]:

$$W = \lambda\epsilon_{ij}\hat{H}_u^i\hat{H}_d^j\hat{S} + \frac{\kappa}{3}\hat{S}^3 + h_u\epsilon_{ij}\hat{Q}^i\hat{U}\hat{H}_u^j + h_d\epsilon_{ij}\hat{Q}^i\hat{D}\hat{H}_d^j + h_e\epsilon_{ij}\hat{L}^i\hat{E}\hat{H}_d^j$$

where $\hat{S}$ is the singlet Higgs superfield and $\epsilon_{12} = \epsilon_{21} = 1$. For soft SUSY breaking terms, we take:

$$V_{\text{soft}} = M_2\tilde{\lambda}_a\tilde{\lambda}_a + (\lambda A_\lambda\epsilon_{ij}H_u^iH_d^jS + \frac{\kappa}{3}A_\kappa S^3 + \text{h.c.}) + (M_1\tilde{\lambda}'\tilde{\lambda}' + h_uA_U\epsilon_{ij}\tilde{Q}^i\tilde{U}H_u^j + h_dA_D\epsilon_{ij}\tilde{Q}^i\tilde{D}H_d^j + h_eA_E\epsilon_{ij}\tilde{L}^i\tilde{E}H_d^j$$

With this model configuration, the μ parameter is given by $\mu = \lambda\langle S\rangle$, where $\langle S\rangle$ is the vacuum expectation value of the S field, and the m_A parameter in the MSSM corresponds to $m_A^2 = \lambda\langle S\rangle(A_\lambda + \kappa\langle S\rangle)/\sin 2\beta$ (see Eq. (20)). Thus, compared with the MSSM, the NMSSM has three additional input parameters: λ, κ, and A_κ. These parameters are subject to the constraints listed in Sec. III, and the requirement that the NMSSM remain perturbative up to the Planck scale demands λ and κ be smaller than 0.7.

The differences between NMSSM and MSSM arise in the Higgs sector and the neutralino sector. In the Higgs sector, we now have three CP-even and two CP-odd Higgs bosons. In the basis $[\text{Re}(H_u^0), \text{Re}(H_d^0), \text{Re}(S)]$, the CP-even Higgs boson mass-squared matrix entries are [?, ?]:

$$\mathcal{M}_{S,11}^2 = m_A^2\cos^2\beta + m_Z^2\sin^2\beta$$

$$\mathcal{M}^2_{S,22} = m_A^2 \sin^2\beta + m_Z^2 \cos^2\beta$$
$$\mathcal{M}^2_{S,33} = \lambda^2 v^2 \sin^2 2\beta + \kappa^2 \langle S\rangle^2 + \mu(\lambda A_\kappa + 4\kappa\mu)/\lambda$$
$$\mathcal{M}^2_{S,12} = (2\lambda^2 v^2 - m_A^2 - m_Z^2)\sin\beta\cos\beta$$
$$\mathcal{M}^2_{S,13} = 2\lambda\mu v \sin\beta - \lambda\kappa\langle S\rangle v\cos\beta$$
$$\mathcal{M}^2_{S,23} = 2\lambda\mu v \cos\beta - \lambda\kappa\langle S\rangle v\sin\beta$$

and for CP-odd Higgs bosons, their mass-squared matrix entries in the basis $[\tilde{A}, \mathrm{Im}(S)]$ with $\tilde{A} = \cos\beta\,\mathrm{Im}(H_u^0) + \sin\beta\,\mathrm{Im}(H_d^0)$ are:

$$\mathcal{M}^2_{P,11} = m_A^2 + \lambda\kappa\langle S\rangle^2 \sin 2\beta + \frac{3\lambda\mu}{\kappa}\sin^2 2\beta$$
$$\mathcal{M}^2_{P,22} = \lambda\kappa v^2 \sin 2\beta + \frac{\lambda A_\kappa}{\kappa}\sin 2\beta + 3\kappa\mu\langle S\rangle$$
$$\mathcal{M}^2_{P,12} = \lambda(A_\lambda - 2\kappa\langle S\rangle)v$$

Equations (19) and (20) indicate that parameters λ and $\kappa\mu$ affect mixings of doublet fields with the singlet field, A_κ only affects the squared mass of the singlet field, and in the limit $\lambda, \kappa \to 0$, the NMSSM reduces to the MSSM. One can also see that for small λ and κ where mixings are small, the physical state dominated by the singlet component should couple weakly to bottom quarks, and thus its loop contribution to ρ_b and $\sin^2\theta_b^{\rm eff}$ should be small.

The NMSSM predicts five neutralinos, and in the basis $(\tilde{B}, \tilde{W}^0, \tilde{H}_u^0, \tilde{H}_d^0, \tilde{S})$ their mass matrix is [?, ?]:

$$M_{\tilde{\chi}^0} = \begin{pmatrix} M_1 & 0 & -m_Z\sin\theta_W\cos\beta & m_Z\sin\theta_W\sin\beta & 0 \\ 0 & M_2 & m_Z\cos\theta_W\cos\beta & -m_Z\cos\theta_W\sin\beta & 0 \\ -m_Z\sin\theta_W\cos\beta & m_Z\cos\theta_W\cos\beta & 0 & -\mu & -\lambda v\sin\beta \\ m_Z\sin\theta_W\sin\beta & -m_Z\cos\theta_W\sin\beta & -\mu & 0 & -\lambda v\cos\beta \\ 0 & 0 & -\lambda v\sin\beta & -\lambda v\cos\beta & 2\kappa\langle S\rangle \end{pmatrix}$$

This mass matrix is independent of A_κ, and the role of λ is to introduce mixings of $\tilde{S}$ with $\tilde{H}_u^0$ and $\tilde{H}_d^0$, while $\kappa\mu$ affects the mass of $\tilde{S}$. Similar to the discussion of Higgs bosons, for small λ the correction to ρ_b and $\sin^2\theta_b^{\rm eff}$ should be insensitive to the value of $\kappa\mu$.

B. NMSSM Corrections to ρ_b and $\sin^2\theta_b^{\text{eff}}$

We first examine SUSY-EW corrections in the NMSSM. Compared with corresponding MSSM corrections, NMSSM effects involve two additional parameters λ and κ. As discussed below Eq. (21), for small λ the corrections are insensitive to κ (verified by our numerical results), so we mainly study the λ dependence. We choose κ values that yield a wide allowed range for λ.

In Fig. 6 [Figure 6: see original paper], we show SUSY-EW contributions to $\delta\rho_{b,v}$ and $\delta\kappa_{b,v}$ as functions of λ for $\tan\beta = 40$, $\kappa = 0.4$, $A_\kappa = -100$ GeV, and other parameters as in Fig. 1. One feature is that both $\delta\rho_{b,v}$ and $\delta\kappa_{b,v}$ become more negative with increasing λ, enlarging the gap between theoretical values and experimental data. Another feature is that contributions become less sensitive to λ when μ is large. This can be explained by Eq. (21), which shows that mixings between $\tilde{S}$ and the doublets $(\tilde{H}_u^0, \tilde{H}_d^0)$ become negligibly small for sufficiently large μ, reducing the sensitivity of contributions to λ.

We now turn to Higgs loop contributions to $\delta\rho_{b,v}$ and $\delta\kappa_{b,v}$ in the NMSSM. Besides m_A and $\tan\beta$, parameters λ, κ, and A_κ are also involved. Since these contributions are more sensitive to λ and κ than to A_κ, we only study their dependence on λ and κ.

In Fig. 7 [Figure 7: see original paper], we show contributions versus λ for $\tan\beta = 40$, $\kappa = 0.4$, $A_\kappa = -100$ GeV, and other parameters as in Fig. 3. This figure shows the same behavior as Fig. 6, with the dependence on λ becoming rather weak for large m_A.

In Fig. 8 [Figure 8: see original paper], we show the dependence on κ, which exhibits similar behavior to Fig. 7. Comparing Figs. 7 and 8 reveals that contributions have stronger dependence on λ than on κ.

As in Fig. 5, we investigate the extent to which NMSSM predictions can agree with experiment by scanning the SUSY parameter space in the region of Eq. (16) with -1 TeV $< A_\kappa < 1$ TeV. Our result is shown in Fig. 9 [Figure 9: see original paper]. Compared with Fig. 5, one can see that the NMSSM cannot improve agreement and may instead exacerbate it over much of the allowed parameter space.

If we define a quantity $F(\lambda,\kappa) \equiv F(\lambda,\kappa) - F(0,0)$, where F denotes either $\delta\rho_{b,v}$ or $\delta\kappa_{b,v}$, with $F(\lambda,\kappa)$ being the NMSSM value for arbitrary λ and κ, and $F(0,0)$ being the MSSM limit value, then studying various cases we find this quantity is generally smaller than 5×10^{-3}, meaning that in the allowed region for λ and κ, the NMSSM only slightly modifies MSSM predictions for ρ_b and $\sin^2\theta_b^{\text{eff}}$.

VI. Conclusions

The $Zb\bar{b}$ coupling determined from Z-pole measurements at LEP/SLD deviates significantly from SM predictions. In terms of ρ_b and $\sin^2\theta_b^{\text{eff}}$, the SM prediction is about 3σ below experimental data. If this anomaly is not a statistical or systematic effect, it would signal new physics associated with the $Zb\bar{b}$ coupling. In this work, we scrutinized full one-loop supersymmetric effects on the $Zb\bar{b}$ coupling in both the MSSM and NMSSM, considering all current constraints from precision electroweak measurements, direct searches for sparticles and Higgs bosons, Higgs potential stability, dark matter relic density, and muon $g-2$ measurement. We analyzed the characteristics of each correction type and searched for SUSY parameter regions where corrections could be sizable. We found that potentially sizable corrections come from the Higgs sector with light m_A and large $\tan\beta$, reaching -2% for ρ_b and -6% for $\sin^2\theta_b^{\rm eff}$. However, such sizable negative corrections are opposite to what is needed to solve the anomaly. We also scanned the allowed parameter space to investigate the extent to which supersymmetry can narrow the discrepancy between theoretical predictions and experimental values. We found that under all current constraints, supersymmetric effects are quite restrained and cannot significantly ameliorate the $Zb\bar{b}$ coupling anomaly. Compared with $\chi^2/\text{dof} = 9.62/2$ in the SM, the MSSM and NMSSM can only improve it to $\chi^2/\text{dof} = 8.77/2$ in the allowed parameter space.

In the future, the GigaZ option at the proposed International Linear Collider (ILC) with an integrated luminosity of 30 fb^{-1} is expected to produce more than 10^9 Z bosons [?] and will enable more precise measurement of the $Zb\bar{b}$ coupling, allowing tests of new physics models. If the $Zb\bar{b}$ coupling anomaly persists, it would suggest new physics beyond the MSSM and NMSSM. One possible form of such new physics is models with additional right-handed gauge bosons that couple predominantly to third-generation quarks [?]. These new gauge bosons typically mix with Z and W, potentially greatly altering the $Zb_R\bar{b}_R$ and $Wb_R\bar{t}_R$ couplings in the SM. Careful investigation of top quark processes at the LHC, such as top quark decay to polarized W bosons [?], may test these models in the near future.

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Appendix A: Gauge Boson Self-Energy in the NMSSM

In the NMSSM, contributions to vector boson self-energies come from loops mediated by SM fermions, gauge bosons, Higgs bosons, sfermions, charginos, and neutralinos. Below we list expressions for pure new physics contributions, namely from loops of Higgs bosons, sfermions, charginos, and neutralinos. We adopt the convention of [?] for SUSY parameters.

1. Higgs Contribution

The NMSSM has an extended Higgs sector with a pair of charged Higgs bosons $H^{\pm}$, two CP-odd Higgs bosons a_i, and three CP-even Higgs bosons h_i. The Higgs contribution to gauge boson self-energy arises from VHH, VVH interactions, and $VVHH$ interactions. Since we choose ' t Hooft-Feynman gauge, the gauge boson and Higgs contributions are generally entangled. In our calculation, we are interested in the difference between the NMSSM Higgs sector contribution and the SM Higgs sector contribution (see discussion in the last paragraph of Sec. II). As the SM contribution is well known [?, ?], we only list the NMSSM contribution:

$$\Pi_{\gamma\gamma}(p^2) = \frac{e^2}{16\pi^2} B_5(p, m_{H^+}, m_{H^+})$$

$$\Pi_{\gamma Z}(p^2) = \frac{e^2 \cos 2\theta_W}{2\cos\theta_W} B_5(p, m_{H^+}, m_{H^+})$$

$$\Pi_{ZZ}(p^2) = \frac{e^2}{4\cos^2\theta_W}\left[2\cos^2 2\theta_W A(m_{H^+}) + \sum_i \left(2|\cos\beta S_{i1} - \sin\beta S_{i2}|^2 A(m_{h_i}) + \sum_j |P'_{i1}|^2 2B_{22}(p, m_{a_j}, m_{h_i}) \right.\right.$$

$$\Pi_{WW}(p^2) = \frac{e^2}{4\sin^2\theta_W}\left[\sum_i |\cos\beta S_{i1} - \sin\beta S_{i2}|^2 2B_{22}(p, m_{H^+}, m_{h_i}) + \sum_i |\cos\beta S_{i2} + \sin\beta S_{i1}|^2 2B_{22}(p, m_W, m\right.$$

In these equations, e is the electromagnetic coupling, and S and P' are rotation matrices defined in Appendix A of [?] to diagonalize CP-even and CP-odd Higgs mass matrices, respectively. A and B_{22} are standard one- and two-point loop functions first defined in [?]. B_5 is related to standard loop functions by [?]:

$$B_5(p, m_1, m_2) = A(m_1) + A(m_2) - 4B_{22}(p, m_1, m_2)$$

2. Sfermion Contribution

The sfermion contributions are:

$$\Pi_{WW}(p^2) = \frac{e^2}{16\pi^2\sin^2\theta_W}\sum_{\alpha,\beta} |R^{\tilde{u}*}_{\alpha 1} R^{\tilde{d}}_{\beta 1}|^2 B_5(p, m_{\tilde{u}_\alpha}, m_{\tilde{d}_\beta})$$

$$\Pi_{ZZ}(p^2) = \frac{e^2}{16\pi^2\sin^2\theta_W\cos^2\theta_W}\sum_{f,\alpha,\beta}\left[(I_{3f} - Q_f\sin^2\theta_W)^2 |R^{\tilde{f}}_{\alpha 1}|^2 |R^{\tilde{f}}_{\beta 1}|^2 + (Q_f\sin^2\theta_W)^2 |R^{\tilde{f}}_{\alpha 2}|^2 |R^{\tilde{f}}_{\beta 2}|^2\right] B_5(p, m_{\tilde{f}_\alpha}$$

$$\Pi_{\gamma\gamma}(p^2) = \frac{e^2}{16\pi^2} \sum_{f,\alpha} C_f Q_f^2 B_5(p, m_{\tilde{f}_\alpha}, m_{\tilde{f}_\alpha})$$

$$\Pi_{\gamma Z}(p^2) = \frac{e^2}{16\pi^2 \sin\theta_W \cos\theta_W} \sum_{f,\alpha} C_f I_{3f} Q_f |R^{\tilde{f}}_{\alpha 1}|^2 B_5(p, m_{\tilde{f}_\alpha}, m_{\tilde{f}_\alpha})$$

where C_f is the color factor (3 for squarks, 1 for sleptons), Q_f is the electric charge, and I_{3f} is the third component of weak isospin. $R^{\tilde{f}}$ is the rotation matrix that diagonalizes the sfermion mass matrix.

3. Chargino and Neutralino Contribution

For a generic interaction between a vector boson and two fermions, the contribution to vector boson self-energy is:

$$\Pi_V^{\psi_i\psi_j}(p^2) = \frac{1}{16\pi^2}\Big[(g^L_{\psi_j\psi_i V} g^{L*}_{\psi_i\psi_j V} + g^R_{\psi_j\psi_i V} g^{R*}_{\psi_i\psi_j V})(2p^2 B_3 - B_4)(p, m_{\psi_i}, m_{\psi_j}) + (g^L_{\psi_j\psi_i V} g^{R*}_{\psi_i\psi_j V} + g^R_{\psi_j\psi_i V} g^{L*}_{\psi_i\psi_j V}$$

where $g^{L,R}_{\psi_i\psi_j V}$ are left- and right-handed couplings. Functions B_3 and B_4 are related to standard two-point functions by [**?**]:

$$B_3(p, m_1, m_2) = B_1(p, m_1, m_2) - B_1(p, m_2, m_1) - B_{21}(p, m_1, m_2)$$

$$B_4(p, m_1, m_2) = 2B_1(p, m_1, m_2) - B_{21}(p, m_1, m_2)$$

For charginos and neutralinos, interaction coefficients with vector bosons are:

$$g_{\tilde{\chi}^0_i \tilde{\chi}^+_j W^-} = \frac{e}{\sqrt{2}\sin\theta_W}(N_{i3}V^*_{j2} + N_{i2}V^*_{j1})$$

$$g^R_{\tilde{\chi}^0_i \tilde{\chi}^+_j W^-} = \frac{e}{\sqrt{2}\sin\theta_W}(N^*_{i4}U_{j2} + N^*_{i2}U_{j1})$$

$$g_{\tilde{\chi}^0_i \tilde{\chi}^0_j Z} = \frac{e}{2\cos\theta_W \sin\theta_W}(N_{i4}N^*_{j4} - N_{i3}N^*_{j3})$$

$$g_{\tilde{\chi}^+_i \tilde{\chi}^+_j Z} = \frac{e}{\sin\theta_W \cos\theta_W}(V_{i2}V^*_{j2} - U^*_{i1}U_{j1} - \delta_{ij}\sin^2\theta_W)$$

Due to the Majorana nature of neutralinos, an additional factor of 1/2 should be multiplied when using these formulas to obtain the neutralino contribution to Z-boson self-energy.

Appendix B: Vertex Corrections to $Z\bar{f}f$ in the NMSSM

We present expressions for radiative corrections to the $Z\bar{f}f$ vertex in the NMSSM, namely δv_f and δa_f defined in Eq. (4). In our calculation, we neglect terms proportional to fermion mass except for $f = b$ (bottom quark), where we keep terms proportional to the bottom quark Yukawa coupling $Y_b \propto 1/\cos\beta$, as these may be enhanced by large $\tan\beta$. Throughout this section, all Z-boson coupling coefficients such as δv_f and δa_f are defined with the common factor $e/(2\sin\theta_W \cos\theta_W)$ extracted.

To present δv_f and δa_f compactly, we introduce quantities δg_f^λ ($\lambda = L, R$) denoting vertex corrections to $Z\bar{f}_\lambda f_\lambda$ interactions, related to δv_f and δa_f by:

$$\delta v_f = \frac{\delta g_f^L + \delta g_f^R}{2}, \quad \delta a_f = \frac{\delta g_f^L - \delta g_f^R}{2}$$

The general expression for δg_f^λ is [**?**]:

$$\delta g_f^\lambda = \Gamma_{f\lambda}(m_f^2) - \Sigma_{f\lambda}(m_f^2) + 2\delta_{\lambda L} a_f \frac{\cos\theta_W}{\sin\theta_W} \Sigma_{\gamma Z}(0)$$

where $\Gamma_{f\lambda}$ is the unrenormalized vertex correction to $Z\bar{f}_\lambda f_\lambda$ interaction, the second term is the counterterm from fermion f_λ self-energy, and the last term is the counterterm from vector boson self-energy.

Assuming the interaction between scalars ϕ_i and Z boson takes the form $\Gamma_{\phi_i^*\phi_j Z}(p_{\phi_i} + p_{\phi_j})_\mu$, we can write $\Sigma_{f\lambda}(p_f^2)$ and the vertex function $\Gamma_{f\lambda}(q^2)$ mediated by a fermion ψ and a scalar ϕ in compact generic notation:

$$(4\pi)^2 \Sigma_{f\lambda}(p_f^2) = C_g \sum_{i,j} |g^\lambda_{\bar{\psi}_j f \phi_i^*}|^2 B_1(p_f, m_{\phi_i}, m_{\psi_j})$$

$$(4\pi)^2 \Gamma_{f\lambda}(q^2) = C_g \sum_{i,j,k} \Big[g^\lambda_{\bar{\psi}_j f \phi_i^*} g^{\lambda *}_{\bar{\psi}_k f \phi_i^*} g^L_{\psi_j \psi_k Z} m_{\psi_j} m_{\psi_k} C_0 + \Big(g^\lambda_{\bar{\psi}_j f \phi_i^*} g^{\lambda *}_{\bar{\psi}_j f \phi_k^*} g_{\phi_i^* \phi_k Z} + g^\lambda_{\bar{\psi}_j f \phi_i^*} g^{\lambda *}_{\bar{\psi}_k f \phi_i^*} g^L_{\psi_j \psi_k Z} \Big) (q^2(C_{12} +$$

where C_g is 4/3 for gluino contributions ($\psi = \tilde{g}$) and 1 for others. The chirality index follows the rule: $L \leftrightarrow R$. If f is a lepton, the following combinations contribute to the vertex:

Chargino correction:

$$g^L_{\tilde{\chi}_j^- \tilde{\nu}_l^* l} = g U_{j1}^*, \quad g^R_{\tilde{\chi}_j^- \tilde{\nu}_l^* l} = 0$$

$$g_{\tilde{\nu}_l \tilde{\nu}_l^* Z} = \delta_{ij} \sin^2\theta_W, \quad g^L_{\tilde{\chi}_i^+ \tilde{\chi}_j^+ Z} = 2(U_{i1}^* U_{j1} + U_{i2}^* U_{j2}) - \delta_{ij} \sin^2\theta_W$$

Neutralino correction:

$$g^L_{\tilde{\chi}^0_j \tilde{l}^*_\alpha l} = \alpha_1 (N^*_{j2} + \tan\theta_W N^*_{j1}), \quad g^R_{\tilde{\chi}^0_j \tilde{l}^*_\alpha l} = \alpha_2 \tan\theta_W N_{j1}$$

$$g^L_{\tilde{\chi}^0_i \tilde{\chi}^0_j Z} = N_{j4} N^*_{i4} + N_{j3} N^*_{i3}, \quad g^R_{\tilde{\chi}^0_i \tilde{\chi}^0_j Z} = N^*_{j4} N_{i4} - N^*_{j3} N_{i3} - 2\sin^2\theta_W \delta_{ij}$$

If f is the bottom quark, the following combinations contribute:

Chargino correction:

$$g^L_{\tilde{\chi}^-_j \tilde{t}^*_\alpha b} = \alpha_1 V^*_{j1}, \quad g^R_{\tilde{\chi}^-_j \tilde{t}^*_\alpha b} = Y_t R^{\tilde{t}}_{\alpha 1} V^*_{j2}$$

$$g_{\tilde{t}_\alpha \tilde{t}^*_\beta Z} = \left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W\right) R^{\tilde{t}*}_{\alpha 1} R^{\tilde{t}}_{\beta 1} + \frac{2}{3}\sin^2\theta_W R^{\tilde{t}*}_{\alpha 2} R^{\tilde{t}}_{\beta 2}$$

Neutralino correction:

$$g^L_{\tilde{\chi}^0_j \tilde{b}^*_\alpha b} = \alpha_1 (N^*_{j2} - \frac{1}{3}\tan\theta_W N^*_{j1}), \quad g^R_{\tilde{\chi}^0_j \tilde{b}^*_\alpha b} = Y_b N_{j4} R^{\tilde{b}*}_{\alpha 1} + \frac{1}{3}\tan\theta_W N_{j1} R^{\tilde{b}*}_{\alpha 2}$$

Gluino correction:

$$g^L_{\tilde{g} \tilde{b}^*_\alpha b} = \sqrt{2} g_s R^{\tilde{b}*}_{\alpha 1}, \quad g^R_{\tilde{g} \tilde{b}^*_\alpha b} = \sqrt{2} g_s R^{\tilde{b}*}_{\alpha 2}$$

Charged Higgs contribution:

$$g^L_{\bar{t} b H^-} = Y_b \cot\beta, \quad g^R_{\bar{t} b H^-} = Y_t \tan\beta$$

$$g_{H^- H^- Z} = \frac{e}{\sin\theta_W \cos\theta_W}\left(\frac{1}{2} - \sin^2\theta_W\right)$$

Neutral Higgs contribution:

$$g_{\bar{b} b h_i} = \frac{e}{2\sin\theta_W \cos\theta_W}\frac{m_b}{m_W \cos\beta} S_{i2}, \quad g_{\bar{b} b a_i} = i\frac{e}{2\sin\theta_W \cos\theta_W}\frac{m_b}{m_W \cos\beta} P'_{i1}$$

Note that we did not include contributions to δg^λ_f from loops of t and G^-. Such contributions alone are UV-convergent and should be attributed to SM radiative effects. This situation differs for neutral Higgs contributions, where loops of b and G^0 are UV-divergent and must be included with other neutral Higgs contributions to obtain a finite result.

If f is the charm quark, the following combinations contribute:

Chargino correction:

$$g^L_{\tilde{\chi}^+_j \tilde{s}^*_\alpha c} = \alpha_1 U^*_{j1}, \quad g^R_{\tilde{\chi}^+_j \tilde{s}^*_\alpha c} = 0$$

Neutralino correction:

$$g^L_{\tilde{\chi}^0_j \tilde{c}^*_\alpha c} = \alpha_1 (N^*_{j2} + \frac{2}{3}\tan\theta_W N^*_{j1}), \quad g^R_{\tilde{\chi}^0_j \tilde{c}^*_\alpha c} = \frac{2}{3}\tan\theta_W N_{j1} R^{\tilde{c}}_{\alpha 2}$$

Gluino correction:

$$g^L_{\tilde{g}\tilde{c}^*_\alpha c} = \sqrt{2} g_s R^{\tilde{c}*}_{\alpha 1}, \quad g^R_{\tilde{g}\tilde{c}^*_\alpha c} = \sqrt{2} g_s R^{\tilde{c}*}_{\alpha 2}$$

These expressions suffice to calculate all $Z\bar{f}_\alpha f_\alpha$ vertex corrections δg^λ_f. Summation should be performed over all non-vanishing coupling combinations, including indices of sfermions, charginos, neutralinos, scalar Higgs, and pseudoscalar Higgs.

Appendix C: NMSSM Contributions to Muon Decay

In the NMSSM, flavor-dependent corrections to the decay $\mu \to \nu_\mu e \bar{\nu}_e$ mainly come from gaugino loops, and the corrected amplitude can be written as [**?**]:

$$\mathcal{M} = \mathcal{M}_B \left[1 + 2\delta^{(v)} + \delta^{(b)}\right]$$

where $\mathcal{M}_B$ is the Born amplitude, $\delta^{(v)}$ is the vertex correction for either $\bar{e}\nu_e W$ or $\bar{\mu}\nu_\mu W$ interaction (since we assume mass degeneracy for the first two slepton generations, the two corrections are identical), and $\delta^{(b)}$ denotes the box diagram correction.

1. Vertex Corrections

Similar to Eq. (B1), the correction to $\bar{f}_1 f_2 W$ interaction can be expressed as:

$$\delta^{(v)} = \Gamma_{\bar{f}_1 f_2 W}(q^2) - \Sigma_{f_1}(m^2_{f_1}) - \Sigma_{f_2}(m^2_{f_2}) + \frac{c_W}{s_W}\Sigma_{\gamma Z}(0)$$

For the $\bar{e}\nu_e W$ interaction:

$$(4\pi)^2 \Sigma_{e_L}(m^2_e) = \sum_{i,j} |g^L_{\tilde{\chi}^0_i \tilde{e}^*_L e}|^2 B_1(m_e, m_{\tilde{e}_L}, m_{\tilde{\chi}^0_i})$$

$$(4\pi)^2 \Sigma_{\nu_e}(m^2_{\nu_e}) = \sum_{i,j} |g^L_{\tilde{\chi}^+_j \tilde{\nu}^*_e \nu_e}|^2 B_1(m_{\nu_e}, m_{\tilde{\nu}_e}, m_{\tilde{\chi}^+_j})$$

$$(4\pi)^2 \Gamma_{\bar{e}\nu_e W}(q^2) = \sum_{i,j} g^L_{\tilde{\chi}^+_j \tilde{\nu}^*_e e} g^{L*}_{\tilde{\chi}^+_j \tilde{e}^*_L \nu_e} g^L_{\tilde{\chi}^+_j \tilde{\chi}^+_j W} \left[2C_{24} + q^2(C_{12} + C_{23})\right]$$

The three-point loop functions are evaluated at zero external momentum, greatly simplifying their expressions:

$$C_0(m_1, m_2, m_3) = \frac{1}{2}\left[\frac{1+a}{a-b}\ln(1+a) + \frac{1+b}{b-a}\ln(1+b)\right]$$

$$C_{24}(m_1, m_2, m_3) = \frac{1}{4}\left[\frac{(1+a)^2}{a-b}\ln(1+a) + \frac{(1+b)^2}{b-a}\ln(1+b) - \frac{3}{2}\right]$$

with $a = m_2^2/m_1^2$ and $b = m_3^2/m_1^2$.

2. Box Corrections

Box diagram contributions to the $\mu \to \nu_\mu e \bar{\nu}_e$ amplitude can be expressed as:

$$i\mathcal{M}_{\text{box}} = i\frac{g^2}{2M_W^2}\left[\mathcal{M}^{(1)} + \mathcal{M}^{(2)} + \mathcal{M}^{(3)} + \mathcal{M}^{(4)}\right]\bar{u}_e\gamma^\mu P_L v_{\nu_e}\bar{u}_{\nu_\mu}\gamma_\mu P_L u_\mu$$

Taking into account the normalization of the tree-level amplitude, the box diagram contributions can be written as:

$$\delta^{(b)} = \sum_{i=1}^{4}\mathcal{M}^{(i)}$$

with each $\mathcal{M}^{(i)}$ given by:

$$16\pi^2\mathcal{M}^{(1)} = \sum_{i,j}|g^L_{\tilde{\chi}_j^+\tilde{\nu}_e^* e}|^2|g^L_{\tilde{\chi}_j^+\tilde{\mu}_L^*\mu}|^2 D_{27}(m_{\tilde{\mu}_L}, m_{\tilde{e}_L}, m_{\tilde{\chi}_j^+}, m_{\tilde{\chi}_i^0})$$

$$16\pi^2\mathcal{M}^{(2)} = \sum_{i,j}|g^L_{\tilde{\chi}_j^+\tilde{\nu}_e^* e}|^2|g^L_{\tilde{\chi}_j^+\tilde{\nu}_\mu^*\mu}|^2 D_{27}(m_{\tilde{\nu}_\mu}, m_{\tilde{\nu}_e}, m_{\tilde{\chi}_j^+}, m_{\tilde{\chi}_i^0})$$

$$16\pi^2\mathcal{M}^{(3)} = \sum_{i,j}(g^L_{\tilde{\chi}_j^+\tilde{\nu}_e^* e}g^{L*}_{\tilde{\chi}_j^+\tilde{e}_L^*\nu_e})(g^L_{\tilde{\chi}_j^+\tilde{\mu}_L^*\mu}g^{L*}_{\tilde{\chi}_j^+\tilde{\nu}_\mu^*\mu})D_0(m_{\tilde{\nu}_\mu}, m_{\tilde{e}_L}, m_{\tilde{\chi}_j^+}, m_{\tilde{\chi}_i^0})$$

$$16\pi^2\mathcal{M}^{(4)} = \sum_{i,j}(g^L_{\tilde{\chi}_j^+\tilde{\nu}_e^* e}g^{L*}_{\tilde{\chi}_j^+\tilde{\nu}_\mu^* e})(g^L_{\tilde{\chi}_j^+\tilde{\mu}_L^*\mu}g^{L*}_{\tilde{\chi}_j^+\tilde{e}_L^*\mu})D_0(m_{\tilde{\mu}_L}, m_{\tilde{\nu}_e}, m_{\tilde{\chi}_j^+}, m_{\tilde{\chi}_i^0})$$

All D-functions are evaluated at zero momentum transfer. Noting that $m_{\tilde{\mu}_L} \simeq m_{\tilde{\nu}_e}$ and $m_{\tilde{e}_L} \simeq m_{\tilde{\nu}_\mu}$, we can write:

$$D_0(m_1, m_1, m_2, m_3) = \frac{1}{2}\left[\frac{1+a}{a-b}\ln(1+a) + \frac{1+b}{b-a}\ln(1+b) - \frac{b(a+b) + ((a+b)(1+a+b)+b)\ln(1+a+b)}{b^2(a+b)^2}\right]$$

$$D_{27}(m_1, m_1, m_2, m_3) = \frac{1}{2}\left[\frac{(1+a+b)((1+a)^2\ln(1+a)-(1+b)^2\ln(1+b))}{b(a-b)} + \frac{((a+b)(1+a)+b)\ln(1+a+}{b^2(a+b)^2}\right.$$

with $a = m_2^2/m_1^2$ and $b = m_3^2/m_1^2$.

Note: Figure translations are in progress. See original paper for figures.

Source: ChinaXiv —Machine translation. Verify with original.