

## Low-Scale $SU(4)_W$ Unification Postprint

**Authors:** Balazs,C, Li,T, Wang,F, Yang,JM

**Date:** 2016-12-28T00:00:00+00:00

### Abstract

We embed the minimal left-right model  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  into an  $SU(4)_W$  gauge group, and break the unified group via five-dimensional  $S^1/(Z_2 \times Z_2)$  orbifolding. Leptons are fitted into  $SU(4)_W$  multiplets and located on a symmetry

### Full Text

#### Preamble

Preprint typeset in JHEP style. - HYPER VERSION

#### Low-Scale $SU(4)_W$ Unification

Csaba Balazs<sup>1</sup>, Tianjun Li<sup>2,3</sup>, Fei Wang<sup>1</sup>, Jin Min Yang<sup>2</sup>

<sup>1</sup> *School of Physics, Monash University, Melbourne, Victoria 3800, Australia*

<sup>2</sup> *Key Laboratory of Frontiers in Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, P. R. China*

<sup>3</sup> *George P. and Cynthia W. Mitchell Institute for Fundamental Physics, Texas A&M University, College Station, TX 77843, USA*

**Abstract:** We embed the minimal left-right model  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  into an  $SU(4)_W$  gauge group, and break the unified group via five-dimensional  $S^1/(Z_2 \times Z_2)$  orbifolding. Leptons are fitted into  $SU(4)_W$  multiplets and located on a symmetry-preserving O brane, while quarks are placed onto an O' brane where the symmetry is broken. This approach predicts  $\sin^2\theta_W = 0.25$  for the weak mixing angle at tree level and leads to a rather low weakly (strongly) coupled unification scale of order  $3 \times 10^2$  TeV (several TeV) with supersymmetry, or as low as several TeV in the non-supersymmetric case. Another symmetry breaking chain with the low-energy gauge group  $SU(2)_L \times U(1)_{3R} \times U(1)_{B-L}$  can also give rise to a weak mixing angle  $\sin^2\theta_W = 0.25$  at

tree level after gauge symmetry breaking by orbifolding. Such theories with low-scale unification have interesting phenomenological consequences.

---

## 1. Introduction

The Standard Model (SM) of electroweak interactions, based on the spontaneously broken gauge symmetry  $SU(2)_L \times U(1)_Y$ , has been extremely successful in describing phenomena below the weak scale. However, the SM leaves some theoretical and aesthetical questions unanswered, two of which are the origin of parity violation and the smallness of neutrino masses. Both of these questions can be addressed in the left-right model based on  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  [?]. The supersymmetric extension of this model [?, ?, ?] is especially intriguing since it automatically preserves R-parity. This can lead to a low-energy theory without baryon number violating interactions after R-parity is spontaneously broken. However, in such left-right models parity invariance and the equality of the  $SU(2)_L$  and  $SU(2)_R$  gauge couplings is ad hoc and has to be put in by hand. Only in grand unified theories, based on  $SO(10)$  [?], can the equality of the two  $SU(2)$  gauge couplings be naturally guaranteed through gauge coupling unification. But in these grand unified theories the unification scale is usually much higher than the weak scale. For example, in the supersymmetric  $SU(5)$  [?, ?] model the weak mixing angle is predicted to be  $3/8$  at tree level while the measured value is 0.23 at the weak scale. The difference can only be bridged via a long renormalization evolution, which in turn requires a rather high unification scale at about  $10^{16}$  GeV. This high-scale unification has the unsatisfactory feature that a large energy desert lies between the weak scale and the unification scale. Therefore, it is interesting to explore the unification of the left-right symmetries at low energy scales.

Novel attempts for the unification of the left-right symmetries have been proposed in the literature, such as the  $SU(4)_{PS} \times SU(4)_W$  or the  $SU(4)_W \times U(1)_{B-L}$  models [?, ?, ?]. However, in these new unification models the weak mixing angle either cannot be predicted (in  $SU(4)_W \times U(1)_{B-L}$  the weak mixing angle is arbitrary) or is predicted as  $3/8$  at tree level, implying a relatively high unification scale. Besides, in order to accommodate matter unification, mirror fermions are necessarily introduced in order to fill each  $SU(4)_W$  multiplet. The problems of these models are similar to the difficulty in the  $SU(3)_W \times U(1)$  extension [?, ?], which uses  $SU(3)_W$  to unify the SM groups  $SU(2)_L \times U(1)_Y$ .

With orbifold gauge symmetry breaking (OGSB) [?, ?, ?, ?, ?], we can achieve gauge interaction unification while leaving matter fields partially unified or un-unified. The problem of the  $SU(3)_W$  unification can be nicely tackled in this approach [?, ?, ?].

In this work, we propose the use of an  $SU(4)_W$  group to unify the left-right gauge couplings on a  $S^1/(Z_2 \times Z'_2)$  orbifold, in which leptons are fitted into  $SU(4)_W$  multiplets and located on the symmetry-preserving O brane while quarks are

placed onto an  $O'$  brane with broken symmetry. This model predicts the weak mixing angle to be  $\sin^2\theta_W = 0.25$  at tree level and achieves gauge coupling unification at the order of  $10^2$  TeV in supersymmetric cases and several TeV in non-supersymmetric cases.

The content of this work is organized as follows. In Sec. 2 we briefly review orbifold gauge symmetry breaking. In Sec. 3 we discuss  $SU(4)_W$  left-right unification in the supersymmetric (SUSY) context, focusing on gauge symmetry breaking on the five-dimensional orbifold. In Sec. 4 we examine the gauge coupling running and unification, especially the compactification scale from the weak mixing angle. In Sec. 5 we discuss another  $SU(4)_W$  symmetry breaking chain into  $SU(2)_L \times U(1)_{3R} \times U(1)_{B-L}$ . Sec. 6 contains our conclusions.

## 2. Brief Review of Orbifold Gauge Symmetry Breaking

We consider a five-dimensional space-time  $M_4 \times S^1/(Z_2 \times Z'_2)$  comprising Minkowski space  $M_4$  with coordinates  $x^\mu$  and the orbifold  $S^1/(Z_2 \times Z'_2)$  with the coordinate  $y \equiv x^5$ . The orbifold  $S^1/(Z_2 \times Z'_2)$  is obtained from  $S^1$  by modding out the equivalent classes:

$$P : y \sim -y, \quad P' : y' \sim -y',$$

with  $y' \equiv y + \pi R/2$ . There are two inequivalent 3-branes located at  $y = 0$  and  $y = \pi R/2$  which are denoted by  $O$  and  $O'$ , respectively.

The five-dimensional  $\mathcal{N} = 1$  supersymmetric gauge theory has 8 real supercharges, corresponding to  $\mathcal{N} = 2$  supersymmetry in four dimensions. The vector multiplet physically contains a vector boson  $A_M$  where  $M = 0, 1, 2, 3, 5$ , two Weyl gauginos  $\lambda_{1,2}$ , and a real scalar  $\sigma$ . In the four-dimensional  $\mathcal{N} = 1$  language, it contains a vector multiplet  $V(A_\mu, \lambda_1)$  and a chiral multiplet  $\Sigma((\sigma + iA_5)/\sqrt{2}, \lambda_2)$  which transform in the adjoint representation of the gauge group. The five-dimensional hypermultiplet has two physical complex scalars  $\phi$  and  $\phi^c$ , a Dirac fermion  $\Psi$ , and can be decomposed into two 4-dimensional chiral multiplets  $\Phi(\phi, \psi \equiv \Psi_R)$  and  $\Phi^c(\phi^c, \psi^c \equiv \Psi_L)$ , which transform as each other's conjugates under gauge transformations.

The general action [?] for the gauge fields and their couplings to the bulk hypermultiplet  $\Phi$  is

$$S = \int d^4x \int_0^{2\pi R} dy \left[ \frac{1}{kg^2} \text{Tr} \int d^2\theta (W^\alpha W_\alpha + \text{h.c.}) + \int d^4\theta (\Phi^c e^V (-2\partial_5 + \bar{\Sigma}) e^{-V} \Phi + \bar{\Phi} e^{-V} (2\partial_5 + \Sigma) e^V \bar{\Phi}^c + \partial_5 \Phi + \partial_5 \bar{\Phi}^c) \right]$$

where  $\text{Tr}(T^a T^b) = k\delta^{ab}$ .

Because the action is invariant under the parity operation  $P$ , under this operation the vector multiplet transforms as

$$V(x^\mu, y) \rightarrow V(x^\mu, -y) = PV(x^\mu, y)P^{-1},$$

$$\Sigma(x^\mu, y) \rightarrow \Sigma(x^\mu, -y) = -P\Sigma(x^\mu, y)P^{-1}.$$

If the hypermultiplet belongs to the fundamental or anti-fundamental representations, since  $P = P^{-1}$ , we have

$$\Phi(x^\mu, y) \rightarrow \Phi(x^\mu, -y) = \eta_\Phi P\Phi(x^\mu, y),$$

$$\Phi^c(x^\mu, y) \rightarrow \Phi^c(x^\mu, -y) = -\eta_\Phi P\Phi^c(x^\mu, y).$$

Alternatively, if the hypermultiplet belongs to the symmetric, anti-symmetric or adjoint representations, we have

$$\Phi(x^\mu, y) \rightarrow \Phi(x^\mu, -y) = \eta_\Phi P\Phi(x^\mu, y)P,$$

$$\Phi^c(x^\mu, y) \rightarrow \Phi^c(x^\mu, -y) = -\eta_\Phi P\Phi^c(x^\mu, y)P,$$

where  $\eta_\Phi = \pm 1$ .

Similar results hold for the parity operation  $P'$ , we just need to make the following replacements in the above equations:

$$P \rightarrow P', \quad \eta_\Phi \rightarrow \eta'_\Phi.$$

The gauge symmetry and supersymmetry can be broken by choosing suitable representations for  $P$  and  $P'$ . For a field  $\phi$ , in the representation of unbroken gauge symmetry, we obtain the following transformation

$$\phi(x^\mu, y) \rightarrow \phi(x^\mu, -y) = p_\phi \phi(x^\mu, y),$$

$$\phi(x^\mu, y') \rightarrow \phi(x^\mu, -y') = p'_\phi \phi(x^\mu, y'),$$

where  $p_\phi = \pm 1$  and  $p'_\phi = \pm 1$ . Introducing the notation  $\phi_{p_\phi p'_\phi}$ , we obtain the Kaluza-Klein (KK) mode expansions for such  $\phi$  fields as follows

$$\phi_{+++}(x^\mu, y) = \sqrt{\frac{2\delta_{n,0}}{\pi R}} \sum_{n=0}^{\infty} \phi_{+++}^{(2n)}(x^\mu) \cos \frac{2ny}{R},$$

$$\begin{aligned}\phi_{+-}(x^\mu, y) &= \sqrt{\frac{2}{\pi R}} \sum_{n=0}^{\infty} \phi_{+-}^{(2n+1)}(x^\mu) \cos \frac{(2n+1)y}{R}, \\ \phi_{-+}(x^\mu, y) &= \sqrt{\frac{2}{\pi R}} \sum_{n=0}^{\infty} \phi_{-+}^{(2n+1)}(x^\mu) \sin \frac{(2n+1)y}{R}, \\ \phi_{--}(x^\mu, y) &= \sqrt{\frac{2}{\pi R}} \sum_{n=0}^{\infty} \phi_{--}^{(2n+2)}(x^\mu) \sin \frac{(2n+2)y}{R}.\end{aligned}$$

Here  $n$  is an integer and the fields  $\phi_{++}^{(2n)}$ ,  $\phi_{+-}^{(2n+1)}$ ,  $\phi_{-+}^{(2n+1)}$  and  $\phi_{--}^{(2n+2)}$  acquire masses of  $2n/R$ ,  $(2n+1)/R$ ,  $(2n+1)/R$  and  $(2n+2)/R$  upon compactification, respectively. Only  $\phi_{++}(x^\mu, y)$  possesses a four-dimensional massless zero mode.

It is easy to see that  $\phi_{++}$  and  $\phi_{+-}$  are non-vanishing at  $y = 0$ , while  $\phi_{++}$  and  $\phi_{-+}$  are non-vanishing at  $y = \pi R/2$ .

### 3. $SU(4)_W$ Unification of $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$

In the minimal left-right model based on  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ , the discrete symmetry ensuring identical  $SU(2)_L$  and  $SU(2)_R$  gauge couplings is put in by hand. In this work we use  $SU(4)_W$  to unify the left-right symmetries and interpret the origin of the parity invariance as the residual discrete symmetry from the symmetry breaking of the unification group. Since we intend to truly unify the left-right gauge groups, it is not possible to fill all the matter fields into  $SU(4)_W$  multiplets without introducing mirror fermions because of the different  $U(1)_{B-L}$  charge assignments for quarks and leptons. So we opt for the OGSB mechanism with partial unification for matter fields.

Starting from the five-dimensional  $SU(3)_C \times SU(4)_W$  gauge theory, we can choose the following  $Z_2$  matrix representations for  $P$  and  $P'$  in the adjoint representation of  $SU(3)_C \times SU(4)_W$ :

$$P = \text{diag}(+1, +1, +1) \otimes \text{diag}(+1, +1, +1, +1),$$

$$P' = \text{diag}(+1, +1, +1) \otimes \text{diag}(+1, +1, -1, -1).$$

The gauge symmetry  $SU(4)_W$  is broken by boundary conditions to  $SU(2)_L \times SU(2)_R \times U(1)_X$  on the boundary  $O'$  brane while it is preserved in the bulk and on the  $O$  brane. Consequently, the parity assignments for  $V$  and  $\Sigma$  are

$$V^{(15)} = (3, 1)_{(+,+)} \oplus (1, 3)_{(+,+)} \oplus (2, \bar{2})_{(+,-)} \oplus (\bar{2}, 2)_{(+,-)} \oplus (1, 1)_{(+,+)},$$

$$\Sigma^{(15)} = (3, 1)_{(-,-)} \oplus (1, 3)_{(-,-)} \oplus (2, \bar{2})_{(-,+)} \oplus (\bar{2}, 2)_{(-,+)} \oplus (1, 1)_{(-,-)}.$$

We place the lepton sector on the O brane while keeping the quark sector on the O' brane. This means that only the leptons are filled into  $SU(4)_W$  multiplets (4):

$$L = \text{diag}(\nu_L, e_L, e_L^c, -\nu_L^c),$$

where  $L \equiv (\phi^c)_L$  and the minus sign conforms to our choice of  $Q_a = (\nu_L^c, -\nu_L^c, e_L^c)$  in  $SU(2)_R$  representations  $\bar{2}$  and being related to its conjugate by  $Q_a = \epsilon_{ab} Q^b$ . From the  $SU(4)_W$  fundamental representation and its proper normalization follows that the  $U(1)_X$  charge assignment of the fundamental representation can be written as

$$Y_X = \text{diag}(-1, -1, 1, 1).$$

From this we can identify  $U(1)_X$  as  $U(1)_{B-L}$ . The normalization of the gauge group  $U(1)_{B-L}$  reads

$$T_{B-L} = \frac{1}{2\sqrt{2}} \text{diag}(-1, -1, 1, 1).$$

From the normalization condition follows the relation between the gauge couplings of  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  and  $SU(4)_W$ :

$$\frac{1}{g_{B-L}^2} = \frac{2}{g_4^2}, \quad \frac{1}{g_L^2} = \frac{1}{g_R^2} = \frac{1}{g_4^2},$$

which holds at the  $SU(4)_W$  unification scale. Hence we can predict the tree-level weak mixing angle as

$$\sin^2 \theta_W = \frac{g_Y^2}{g_2^2 + g_Y^2} = \frac{g_R^2}{g_L^2 + 2g_R^2} = \frac{1}{4}.$$

To induce Yukawa couplings for the  $SU(4)_W$  multiplet leptons, we can introduce bulk Higgs fields in the  $SU(4)_W$  antisymmetric representation  $\Phi_{ab}(\bar{6})$  and symmetric representations  $\Delta_{ab}^i(10)$ . In four dimensions there are eight  $\mathcal{N} = 1$  chiral multiplets  $\Phi, (\Phi^c), \Delta^i, (\Delta^c)^i$  ( $i = 1, 2, 3$ ). We assign the boundary conditions for the Higgs fields as

$$\eta_\Phi = 1, \quad \eta'_\Phi = -1, \quad \eta_{\Delta_1} = 1, \quad \eta'_{\Delta_1} = -1, \quad \eta_{\Delta_i} = 1, \quad \eta'_{\Delta_i} = 1 \quad (i = 2, 3).$$

The parities of the Higgs fields in terms of the  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  representation are given by

$$\Phi(\bar{6}) = (1, 1)_{(+,-)} \oplus (1, 1)_{(+,-)} \oplus (\bar{2}, \bar{2})_{(+,+)} \oplus (2, 2)_{(+,-)},$$

$$\Delta_1(\bar{10}) = (\bar{3}, 1)_{(+,-)} \oplus (1, \bar{3})_{(+,-)} \oplus (\bar{2}, \bar{2})_{(+,+)} \oplus (1, 1)_{(+,+)},$$

$$\Delta_2(\bar{10}) = (\bar{3}, 1)_{(+,+)} \oplus (1, \bar{3})_{(+,+)} \oplus (\bar{2}, \bar{2})_{(+,+)} \oplus (1, 1)_{(+,+)},$$

$$\Delta_3(10) = (3, 1)_{(+,+)} \oplus (1, 3)_{(+,+)} \oplus (\bar{2}, \bar{2})_{(+,+)} \oplus (1, 1)_{(+,+)}.$$

Under these parity assignments the conjugate chiral fields  $(\Phi^c), (\Delta^c)^i$  ( $i = 1, 2, 3$ ) have no zero modes and are irrelevant to the low-energy phenomenology. The zero modes form two  $SU(2)_L$  and  $SU(2)_R$  triplets with opposite  $U(1)_{B-L}$  quantum numbers and two bi-doublets  $(2, 2)$  with vanishing  $U(1)_{B-L}$  quantum numbers which give exactly the Higgs field contents of the supersymmetric left-right model.

As the leptons are fitted into the  $SU(4)_W$  multiplets, we can write down their Yukawa interactions with the bulk Higgs fields. Since the leptons are placed on the O brane, it is obvious that they are invariant under  $P$  transformation. The transformation property for the leptons under  $P'$  is determined by the requirement that the operators on the O brane must transform covariantly under  $P'$ , otherwise the gauge symmetry preserved at  $y = 0$  will not be preserved at the  $y = \pi R$  brane. From the kinetic terms of the leptons we can get the transformation property of the leptons under  $P'$  as

$$P' : (\nu_L, e_L, e_L^c, -\nu_L^c) \rightarrow \pm(+, +, -, -).$$

The transformation of the Yukawa interactions under  $P'$  is

$$P' : \sum_{ij} Y_{ij} L_i^a L_j^b \Phi_{[ab]}(\bar{6}) = -,$$

$$P' : \sum_{ij} Y_{ij} L_i^a L_j^b \Delta_{ab}(10) = -,$$

$$P' : \sum_{ij} Y_{ij} L_i^c L_j^c \Delta_{ab}(10) = +,$$

where  $i, j$  are the family indices. So we can write the Yukawa interactions as

$$\mathcal{L}_{\text{Yuk}} = \int d^2\theta \int_0^{2\pi R} dy \left[ \{\delta(y) - \delta(y - \pi R)\} \sum_{ij} \left( Y_1^{[ij]} L_i^a L_j^b \Phi_{[ab]} + Y_2^{ij} L_i^a L_j^c \Delta_{ab} \right) + \{\delta(y) + \delta(y - \pi R)\} \sum_{ij} Y_3^{ij} L_i^a L_j^b \Delta_{ab} \right]$$

After integrating out the fifth dimensional coordinate, we get the Yukawa couplings in four dimensions

$$\mathcal{L}_{\text{Yuk}}^{4D} = \int d^2\theta \left[ \sum_{ij} \left( y_1^{[ij]} L_i L_j \phi_1^{(2n)} + y_2^{ij} L_i L_j \phi_2^{(2n)} + y_3^{ij} L_i L_j \Delta_1^{(2n)} + y_4^{ij} (L_i^c) L_j^c \Delta_2^{(2n)} \right) \right] + \text{h.c.},$$

where the lepton  $SU(4)_W$  multiplets are decomposed as  $4 = (L, L^c)$ , the bi-doublet Higgs fields  $\phi_1$  and  $\phi_2$  belong to the  $(2, 2)_0$  representations of  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ , and the triplets  $\Delta_1$  and  $\Delta_2$  belong to  $(3, 1)_{-2}$  and  $(1, 3)_2$  representations.

The interactions for the zero modes of the Higgs fields are the Yukawa couplings in the supersymmetric left-right model. Similarly, we can write the couplings of the lepton multiplets with the vector multiplet  $V^a$  and the chiral multiplet  $\Sigma^a$ .

Supersymmetry breaking can be realized via the Scherk-Schwarz mechanism through the boundary conditions [?, ?, ?, ?, ?, ?]. It is well known that  $\mathcal{N} = 1$  supersymmetry in five dimensions possesses an  $SU(2)_R$  global R-symmetry under which the gauginos from the vector multiplets  $(\lambda_1, \lambda_2)$  and complex scalars  $(\phi, \phi^{c\dagger})$  from hypermultiplets form  $SU(2)_R$  doublets. The non-trivial twist  $T$  for translation with respect to  $SU(2)_R$  R-symmetry can be written as [?, ?]

$$T = \exp(-2\pi i \sigma_2 \alpha),$$

with orbifolding projection

$$P' = \sigma_3.$$

Besides, the symmetric Higgs bosons  $\Delta_i(10)$  ( $i = 1, 2, 3$ ) have an  $SU(3)$  flavor symmetry. Also, the consistent relation between the translation and the orbifolding is  $TP'T = P'$ , where  $P'$  is the reflection according to  $Z_2$  (or  $Z'_2$ ), and  $T$  is the translation

$$T\phi(x^\mu, y) = \phi(x^\mu, y + 2\pi R).$$

We denote the translation operator  $T$  corresponding to the global  $SU(3)$  flavor symmetry as follows

$$T = \exp \left( 2\pi i \sum_a T^a \theta^a \right),$$

where  $T^a$  are SU(3) generators. From formula  $TP'T = P'$ , we obtain  $\{T^a \theta^a, P'\} = 0$ .

For the following non-trivial  $3 \times 3$  matrix

$$P' = \begin{pmatrix} \pm 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

the most general form of  $T$  can be described by

$$T = \exp (2\pi i(\gamma_0 T^0 + \gamma_1 T^1 + \gamma_2 T^2)),$$

with

$$T^0 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad T^1 = \begin{pmatrix} 0 & -i & i \\ i & 0 & 0 \\ -i & 0 & 0 \end{pmatrix},$$

or

$$T^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad (P'_{11} = 1),$$

$$T^2 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad (P'_{11} = -1).$$

The parameter  $\gamma_0$  can be rotated away by the residual global symmetry, so the twist for flavor SU(3) compatible with non-trivial  $P'$  can be written as

$$T = \exp (2\pi i(\gamma_1 T^1 + \gamma_2 T^2)).$$

In the case of the SU(3) flavor symmetry for  $\Delta_1(\bar{10})$ ,  $\Delta_2(\bar{10})$ ,  $\Delta_3(10)$ , the relative parity assignments under  $P$  and  $P'$  are nontrivial with

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad P' = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

So the twist boundary condition  $T$  compatible with both is

$$T = \exp(2\pi i \gamma_1 T^1).$$

The most general boundary conditions for the fields are

$$A_M(x^\mu, y + 2\pi R) = A_M(x^\mu, y),$$

$$\sigma(x^\mu, y + 2\pi R) = \sigma(x^\mu, y),$$

$$\begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} (x^\mu, y + 2\pi R) = e^{-2\pi i \alpha \sigma_2} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} (x^\mu, y),$$

$$\begin{pmatrix} \phi \\ \phi^{c\dagger} \end{pmatrix} (x^\mu, y + 2\pi R) = e^{-2\pi i \alpha \sigma_2} \begin{pmatrix} \phi \\ \phi^{c\dagger} \end{pmatrix} (x^\mu, y),$$

$$\begin{pmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{pmatrix} (x^\mu, y + 2\pi R) = e^{2\pi i \gamma_1 T^1} \begin{pmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{pmatrix} (x^\mu, y),$$

$$\begin{pmatrix} \tilde{\delta}_1 \\ \tilde{\delta}_2 \\ \tilde{\delta}_3 \end{pmatrix} (x^\mu, y + 2\pi R) = e^{2\pi i \gamma_1 T^1} \begin{pmatrix} \tilde{\delta}_1 \\ \tilde{\delta}_2 \\ \tilde{\delta}_3 \end{pmatrix} (x^\mu, y).$$

Here we denote the components of chiral supermultiplets  $\Delta_i(10)$  as  $(\delta_i(10), \tilde{\delta}_i(10))$  with their conjugate chiral supermultiplets  $\Delta_i^c(\bar{10})$  as  $(\delta_i^c(10), \tilde{\delta}_i^c(10))$ . The complex scalar components for hypermultiplets  $(\Phi(6), \Phi^c(\bar{6}))$  are denoted as  $(\phi, \phi^{c\dagger})$ .

We now consider the mode expansion of the fields with respect to the previous Scherk-Schwarz type boundary conditions. For simplicity, we write explicitly only the relative  $P$  and  $P'$  parity assignments under orbifolding projections:

$$\begin{pmatrix} \lambda^{(++)} \\ \lambda^{(--)} \end{pmatrix} (x^\mu, y) = e^{-i\alpha\sigma_2 y/R} \sum_{n=0}^{\infty} \begin{pmatrix} \lambda_{(2n)}^{(++)}(x^\mu) \cos \frac{2ny}{R} \\ \lambda_{(2n+2)}^{(--)}(x^\mu) \sin \frac{(2n+2)y}{R} \end{pmatrix},$$

$$\begin{pmatrix} \phi^{(++)} \\ \phi^{c\dagger(-,-)} \end{pmatrix} (x^\mu, y) = e^{-i\alpha\sigma_2 y/R} \sum_{n=0}^{\infty} \begin{pmatrix} \phi_{(2n)}^{(++)}(x^\mu) \cos \frac{2ny}{R} \\ \phi_{(2n+2)}^{c\dagger(-,-)}(x^\mu) \sin \frac{(2n+2)y}{R} \end{pmatrix},$$

$$\begin{pmatrix} \tilde{\delta}^{(+-)} \\ \tilde{\delta}^{c\dagger(-+)} \\ \tilde{\delta}^{c\dagger(--)} \\ \tilde{\delta}^{c\dagger(++)} \end{pmatrix} (x^\mu, y) = e^{-i\alpha\sigma_2 y/R} \sum_{n=0}^{\infty} \begin{pmatrix} (\tilde{\delta}_1)^{(+-)}_{(2n+2)} \\ (\tilde{\delta}_1^c)^{(-+)}_{(2n+2)} \\ (\tilde{\delta}_2)^{(++)}_{(2n)} \\ (\tilde{\delta}_2^c)^{(--)}_{(2n+2)} \\ (\tilde{\delta}_3)^{(-)}_{(2n+2)} \\ (\tilde{\delta}_3^c)^{(++)}_{(2n)} \end{pmatrix} e^{i\gamma_1 T^1 y/R},$$

$$\begin{pmatrix} \delta^{(+-)} \\ \delta^{c\dagger(-+)} \\ \delta^{c\dagger(--)} \\ \delta^{c\dagger(++)} \end{pmatrix} (x^\mu, y) = e^{-i\alpha\sigma_2 y/R} \sum_{n=0}^{\infty} \begin{pmatrix} (\delta_1)^{(+-)}_{(2n+2)} \\ (\delta_1^c)^{(-+)}_{(2n+2)} \\ (\delta_2)^{(++)}_{(2n)} \\ (\delta_2^c)^{(--)}_{(2n+2)} \\ (\delta_3)^{(-)}_{(2n+2)} \\ (\delta_3^c)^{(++)}_{(2n)} \end{pmatrix} e^{i\gamma_1 T^1 y/R},$$

in which we represent the symmetric  $\Delta_i(\bar{10})$  ( $i = 1, 2, 3$ ) by its  $(\bar{3}, 1)_{-2}$  modes.

The zero modes from the orbifold projection can get mass terms from the previous Scherk-Schwarz boundary conditions:

$$\mathcal{L} = -\frac{\alpha^2}{R^2} \text{Tr} [(\partial_5 \Phi)^\dagger (\partial_5 \Phi) + (\partial_5 \Phi^c)^\dagger (\partial_5 \Phi^c)] - \frac{\gamma^2}{R^2} \text{Tr} [(\partial_5 \Delta_i)^\dagger (\partial_5 \Delta_i) + (\partial_5 \Delta_i^c)^\dagger (\partial_5 \Delta_i^c)] + \text{h.c.}$$

$$- \left[ \int d^2\theta \left( \frac{\alpha}{R} \text{Tr}[\tilde{\Delta}_1 \Delta_1] + \frac{\gamma}{R} \text{Tr}[\tilde{\Delta}_2 \Delta_2] + \frac{\gamma}{R} \text{Tr}[\tilde{\Delta}_3 \Delta_3] \right) + \text{h.c.} \right].$$

Here triplets  $\Delta_1^{(1,3)(2)}$  and  $\Delta_1^{(1,\bar{3})(-2)}$  are zero modes from  $\Delta_2(\bar{10})$  while triplets  $\Delta_2^{(3,1)(2)}$  and  $\Delta_2^{(3,1)(-2)}$  are zero modes from  $\Delta_3(10)$ . The bi-doublets  $\Phi_1^{(2,2)_0}$  are zero modes from  $\Phi(\bar{6})$  while  $\Phi_2^{(2,2)_0}$  from  $\Delta_1(\bar{10})$ . The gauge index  $a$  runs over the left-right gauge group  $SU(3)_c$ ,  $SU(2)_L$ ,  $SU(2)_R$ , and  $U(1)_{B-L}$ . The continuous parameters  $\alpha$  and  $\gamma$  can be chosen to be  $\alpha, \gamma \ll 1$  [?] or  $\alpha, \gamma \sim \mathcal{O}(1)$  [?]. We choose the former case with  $\alpha, \gamma \ll 1$ . If the scale of the supersymmetry breaking soft mass terms  $\alpha/R$  and  $\gamma/R$  is chosen to be at the order of the electroweak scale, we can get the relation  $M_S < M_R$ . Otherwise, if the scale for the supersymmetry breaking soft mass terms is higher than  $M_R$  which will not explain the gauge hierarchy problem,  $M_R < M_S$  is also possible. In our case the matter contents are placed at the orbifold fixed point so that no tree-level mass terms are generated through orbifolding. However, the sfermion masses can be radiatively generated through renormalization group equations below the compactification scale. Since such interactions are almost flavor universal,

the supersymmetric flavor problems can be solved. There is a lot of freedom to tune the complicated Higgs potential to break the left-right symmetry down to  $U(1)_Q$  directly.

In supersymmetric left-right models, sneutrinos can couple to the Higgs sector which leads to spontaneously broken R-parity if such sneutrino doublets acquire vacuum expectation values. The couplings between the triplets and sneutrino which arise from the Yukawa superpotential are rather arbitrary. Detailed discussion on Higgs potential coupled to sneutrino doublets can be found in Ref. [?, ?] which will not be discussed here. In SUSY left-right cases, R-parity is automatically conserved.

The chiral anomaly cancellation in OGSB cases has been studied in [?, ?, ?, ?]. In our case with  $S^1/(Z_2 \times Z'_2)$  OGSB, if the gauge anomaly in four dimensions is cancelled, the five-dimensional fixed-point gauge anomaly can be cancelled by introducing appropriate bulk Chern-Simons terms with jumping coefficients. At the fixed point  $O$ , the gauge anomaly from the lepton 4 representation and the bulk Higgsinos in representations  $\bar{6}$ ,  $10$ ,  $\bar{10}$  are cancelled by the five-dimensional Chern-Simons terms. Such Chern-Simons terms also cancel the quark contribution on the  $O'$  brane. At the fixed point  $O'$  we can see that the four-dimensional anomaly associated with the bulk Higgsinos is cancelled automatically although the bulk fermion contributions to the anomaly associated with the unbroken gauge group add up.

**Alternative Models:** It is also possible to put the leptons into the bulk by introducing mirror leptons and placing quarks on the broken symmetry  $O'$  brane. We can introduce bulk hypermultiplets  $(F_L, F_R)$  in the  $(1, 4)$  representation and  $(F_L^c, F_R^c)$  in the  $(1, \bar{4})$  representation. These multiplets are filled as:

$$F_L = (L_L, X_L), \quad F_R = (X_R^c, L_R^c),$$

$$F_L^c = (\bar{L}_L, \bar{X}_L), \quad F_R^c = (\bar{X}_R^c, \bar{L}_R^c),$$

where  $(X_L, \bar{X}_L)$  and  $(X_R, \bar{X}_R)$  are left (right) handed mirror leptons.  $L_L$  and  $L_R^c$  are left and right handed leptons in minimal left-right model, respectively. Lepton doublets in the minimal left-right model can be obtained by introducing the following parity assignments:

$$\eta_{F_L} = 1, \quad \eta'_{F_L} = 1, \quad \eta_{F_R} = +1, \quad \eta'_{F_R} = -1.$$

Lepton  $SU(2)_L$  doublets survive projections from  $F_L$  while lepton  $SU(2)_R$  doublets from  $F_R$ . We can see from the charge assignments of the bulk hypermultiplets that the tree-level weak mixing angle  $\sin^2\theta_W = 0.25$  still holds in this scenario.

The Higgs sector can be placed in the bulk or localized on the broken symmetry  $O'$  brane. In the latter case, we need two bi-doublets  $(2, 2, 0)$ , two  $SU(2)_L$  triplets  $(3, 1, \pm 2)$  and two  $SU(2)_R$  triplets  $(1, 3, \pm 2)$  in left-right gauge group  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  representations. The case of the bulk Higgs is almost identical to that of the previous case, so we do not discuss it in detail.

It is also possible to put quarks in the bulk while locating leptons on the  $O'$  brane. Then we introduce mirror quarks  $\bar{Q}, \bar{Q}^c$  to fill  $SU(3)_c \times SU(4)_W$  representations as:

$$(3, 4) = \text{diag}(U_L, \bar{Q}_L^c), \quad (\bar{3}, 4) = \text{diag}(\bar{Q}_L, U_L^c),$$

where  $U_L^c = (d_L^c, -u_L^c)$  denote the 2 representations in  $SU(2)_R$ . In this case the  $U(1)_{B-L}$  charge for  $SU(4)_W$  fundamental representation reads

$$Y_{B-L} = \text{diag}\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, -1\right),$$

which is normalized with respect to the  $SU(4)_W$  generator  $T_{B-L}$  as:

$$T_{B-L} = \frac{1}{2\sqrt{6}} \text{diag}(1, 1, 1, -3).$$

From the gauge coupling relations  $g_{B-L} = \sqrt{6}g_4$  and  $g_L = g_R = g_4$ , we can get the tree-level weak mixing angle

$$\sin^2 \theta_W = \frac{g_R^2}{g_L^2 + 2g_R^2} = 0.45,$$

which is not acceptable as a low-energy unification model.

#### 4. Gauge Coupling Running and Unification Scale

In this section we discuss the renormalization group equation (RGE) running of the gauge couplings in the orbifold breaking case. We consider only the simplest scenario without mirror fermions. At the weak scale our inputs are [?]

$$M_Z = 91.1876 \pm 0.0021 \text{ GeV},$$

$$\sin^2 \theta_W(M_Z) = 0.2312 \pm 0.0002,$$

$$\alpha_{\text{em}}^{-1}(M_Z) = 127.906 \pm 0.019,$$

$$\alpha_3(M_Z) = 0.1187 \pm 0.0020,$$

which fix the numerical values of the standard  $U(1)_Y$  and  $SU(2)_L$  couplings at the weak scale

$$\alpha_1^{-1}(M_Z) = \alpha_{\text{em}}^{-1}(M_Z) \cos^2 \theta_W = (98.3341)^{-1},$$

$$\alpha_2^{-1}(M_Z) = \alpha_{\text{em}}^{-1}(M_Z) \sin^2 \theta_W = (29.5718)^{-1}.$$

The RGE running of the gauge couplings reads

$$\frac{d\alpha_i^{-1}}{d \ln E} = -\frac{b_i}{2\pi},$$

where  $E$  is the energy scale and  $b_i$  are the beta functions. At the scale of the  $SU(2)_R$  gauge boson mass  $M_R$ , the left-right  $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  symmetry breaks to the SM gauge group. From the symmetry breaking chain and the normalization of the gauge field  $(g_{B-L} Y_{B-L}/2) A_{B-L}$  in the kinetic term, we obtain the relation

$$\frac{1}{\alpha_Y(M_R)} = \frac{1}{\alpha_{B-L}(M_R)} + \frac{1}{\alpha_R(M_R)},$$

from which we can calculate the coupling  $g_{B-L}$  at the scale  $M_R$ .

Note that in non-supersymmetric left-right models neutrino masses arise by a Type I or Type II see-saw mechanism. In this case an  $\mathcal{O}(\text{TeV})$  mass is unnatural for the  $W_R$  gauge boson due to the mixing term  $\text{Tr}(\Phi \Delta_L \Phi^\dagger \Delta_R^\dagger)$ . In the supersymmetric left-right model such a mixing term is not allowed by supersymmetry and thus a TeV-scale  $W_R$  mass is realistic [?]. We know that we need two bi-doublets to give tree-level Cabibbo-Kobayashi-Maskawa (CKM) mixings in supersymmetric left-right models. Thus, in the low-energy limit, the electroweak symmetry breaking Higgs sector is non-minimal, containing two bi-doublets. The corresponding supersymmetric extension (below  $M_R$ ) also contains two bi-doublets.

Assuming that the  $SU(2)_R$  gauge boson mass  $M_R$  is in the range  $1 \text{ TeV} < M_R < M_C$  (where  $M_C$  is the compactification scale) and the mass of its superpartner falls in the range  $200 \text{ GeV} < M_S < M_C$ , we have two possibilities:

(i) One possibility is that  $M_S < M_R$ . In this case  $\alpha, \gamma \ll 1$  with  $\alpha/R, \gamma/R$  at the order of the electroweak scale. Then the beta functions for the gauge couplings of  $U(1)_Y, SU(2)_L, SU(3)_c$  are given by

$$(b_1, b_2, b_3) = \left( \frac{22}{5}, 2, -3 \right) \text{ for } M_Z < E < M_S,$$

$$(b_1, b_2, b_3) = (12, 2, -3) \text{ for } M_S < E < M_R,$$

while for  $U(1)_{B-L}$ ,  $SU(2)_L$ ,  $SU(2)_R$ , and  $SU(3)_c$  they are given by

$$(b_1, b_2^L, b_2^R, b_3) = (8, 6, 6, -3) \text{ for } M_R < E < M_C.$$

(ii) The other possibility is  $M_R < M_S$ . In this case  $\alpha/R, \gamma/R$  are of order  $\mathcal{O}(10)$  TeV and the gauge hierarchy problem is not solved by the high-energy supersymmetry. Then we have

$$(b_1, b_2, b_3) = \left( \frac{22}{5}, 2, -3 \right) \text{ for } M_Z < E < M_R,$$

$$(b_1, b_2, b_3) = (12, 2, -3) \text{ for } M_R < E < M_S,$$

$$(b_1, b_2^L, b_2^R, b_3) = (8, 6, 6, -3) \text{ for } M_S < E < M_C.$$

Above the SUSY left-right scale the RGE running of the gauge couplings receives contributions from KK modes:

$$\alpha_i^{-1}(E) = \alpha_i^{-1}(M_R) + \frac{b_i}{2\pi} \ln \frac{E}{M_R} + \frac{b_i^e}{2\pi} \sum_{n=1}^{\infty} \Theta \left( E - \frac{2n}{R} \right) \ln \frac{E}{2n/R} + \frac{b_i^o}{2\pi} \sum_{n=0}^{\infty} \Theta \left( E - \frac{2n+1}{R} \right) \ln \frac{E}{(2n+1)/R},$$

where  $\Theta(x)$  is the step function defined as  $\Theta(x) = 1$  for  $x \geq 0$  and  $\Theta(x) = 0$  for  $x < 0$ .

The beta functions corresponding to the even and odd KK modes at 1-loop are

$$(b_{B-L}^e, b_{B-L}^o) = (12, 0),$$

$$(b_2^{L,e}, b_2^{L,o}) = (8, 4),$$

$$(b_2^{R,e}, b_2^{R,o}) = (8, 4),$$

$$(b_3^e, b_3^o) = (-6, 0),$$

that is, the RGE running of the  $SU(2)_L$  and  $SU(2)_R$  gauge couplings are identical.

The existence of the symmetry breaking  $O'$  brane allows localized kinetic terms for the unbroken gauge group  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ , which could spoil the  $SU(4)_W$  unification. The most general form of the gauge kinetic term is given by

$$\mathcal{S}_{\text{kin}} = \int d^4x \int_0^{2\pi R} dy \left[ \frac{1}{4g_5^2} F_{MN} F^{MN} + \delta(y) \frac{1}{4\bar{g}^2} F_{\mu\nu} F^{\mu\nu} + \delta(y - \pi R/2) \frac{1}{4\bar{g}'^2} F_{\mu\nu} F^{\mu\nu} \right].$$

After integrating out the higher modes, the gauge couplings of the zero modes are

$$\frac{1}{g_0^2} = \frac{2\pi R}{g_5^2} + \frac{1}{\bar{g}^2} + \frac{1}{\bar{g}'^2},$$

where  $g_0 = \sqrt{2}g_{B-L}$ , while  $g_1, g_2$  correspond to  $g_{2L}, g_{2R}$  coupling respectively. The term  $\bar{g}^2$  is irrelevant because it preserves  $SU(4)_W$  unification and will not affect the tree-level weak mixing angle. We can assume that the bulk and brane kinetic terms have comparable strength [?] at a cut-off scale  $\Lambda$  (higher than or equal to the unification scale  $M_U$ ). Since  $g_5^2$  has mass dimension  $-1$ , we can estimate its strength to be  $g_5^2 \sim \tilde{g}_i^2/\Lambda$  at the cut-off scale, which implies  $g_5^2 \Lambda \sim \tilde{g}_i^2$ . We can see that at tree level the  $SU(4)_W$  violating term is suppressed by  $M_C/\Lambda$  and hence the effects can be neglected if  $M_C \ll \Lambda$ . Besides, it is natural to set such localized gauge kinetic terms to zero at tree level in a fundamental theory. Then for a weakly coupled theory, such localized kinetic terms can only arise at loop level and thus are highly suppressed.

The one-loop corrections to the weak mixing angle come from the  $SU(4)_W$  violating effects but not from the  $SU(4)_W$  conserving effects. For the energy scale in the range  $2NM_C < E < (2N+1)M_C$  with  $N \gg 1$ , we can estimate the RGE running by summing over the contribution of the KK modes. Using Stirling's approximation

$$\ln(N!) \simeq N \ln N - N + \frac{1}{2} \ln(2\pi N),$$

$$\ln[1 \times 3 \times \dots \times (2N-1)] = \ln[(2N)!] - N \ln 2 - \ln(N!) \simeq N \ln N - N + \frac{1}{2} \ln(\pi N),$$

we can write

$$\alpha_i^{-1}(E) \simeq \alpha_i^{-1}(M_R) + \frac{b_i}{2\pi} \ln \frac{E}{M_R} + \frac{b_i^e + b_i^o}{2\pi} \left[ \frac{E}{2M_C} \ln \frac{E}{M'} - \frac{E}{2M_C} \ln 2 \right] + \frac{b_i^o}{2\pi} \ln \frac{\pi E}{2M_C},$$

where  $M' = 2M_C/\pi$ . Thus, after the KK mode contributions are included, the RGE running of the gauge couplings are proportional to  $N = E/(2M_C)$ , which is a power-law running (this agrees with the results of [?]). The relative running of  $SU(2)_L$  (identical to  $SU(2)_R$ ) and  $U(1)_{B-L}$  is not affected by the  $SU(4)_W$  conserving power-law running; instead this running is logarithmic due to  $SU(4)_W$  violating effects. In OGSB cases, it is general to have

$$b_{B-L}^e + b_{B-L}^o = b_2^{L,e} + b_2^{L,o} = b_2^{R,e} + b_2^{R,o},$$

which also holds in our case. Due to the universally occurring  $b^o + b^e$  term, we can replace  $b^e$  with  $-b^o$  in the relative running between the gauge couplings. The running of the minimal left-right gauge couplings is given by

$$\alpha_{2L}^{-1}(M_R) \simeq \alpha_{2L}^{-1}(M_U) + \frac{\tilde{b}_2}{2\pi} \ln \frac{M_U}{M_R} + \frac{a}{2\pi} \frac{M_U}{M'} - \frac{b_2^o}{2\pi} \ln \frac{\pi M_U}{2M_C},$$

$$\alpha_{B-L}^{-1}(M_R) \simeq \alpha_{B-L}^{-1}(M_U) + \frac{\tilde{b}_1}{2\pi} \ln \frac{M_U}{M_R} + \frac{a}{2\pi} \frac{M_U}{M'} - \frac{b_1^o}{2\pi} \ln \frac{\pi M_U}{2M_C},$$

where  $M' = 2M_C/\pi$ , the coefficient  $a = b_i^o + b_i^e$  is universal and  $\tilde{b}_i$  are given in our case by

$$a = b_i^o + b_i^e, \quad \tilde{b}_i = b_i - b_i^e, \quad \tilde{c}_i = b_i.$$

Then the Weinberg angle for non-SUSY cases is

$$\sin^2 \theta_W(M_Z) = \frac{1}{4} - \frac{\alpha_{\text{em}}(M_Z)}{4\pi} \left[ (\tilde{b}_1 - \tilde{b}_2) \ln \frac{M_R}{M_Z} + (\tilde{c}_1 - \tilde{c}_2) \ln \frac{M_U}{M_R} + (d_1 - 3d_2) \ln \frac{M_S}{M_Z} \right],$$

where  $(d_1, d_2)$  are the one-loop beta functions for  $U(1)_Y$ ,  $SU(2)_L$  in the energy range between  $M_R$  and  $M_Z$ . The Weinberg angle for SUSY cases is given by

$$\sin^2 \theta_W(M_Z) = \frac{1}{4} - \frac{\alpha_{\text{em}}(M_Z)}{4\pi} \left[ (\tilde{b}_1 - \tilde{b}_2) \ln \frac{M_R}{M_Z} + (\tilde{c}_1 - \tilde{c}_2) \ln \frac{M_U}{M_R} + (d_1 - 3d_2) \ln \frac{M_S}{M_Z} + (e_1 - 3e_2) \ln \frac{M_R}{M_S} \right],$$

where  $(d_1, d_2)$  and  $(e_1, e_2)$  denote the one-loop beta functions for  $U(1)_Y$  and  $SU(2)_L$  for  $M_R > E > M_S$  and  $M_S > E > M_Z$ , respectively.

From the above formulas, we can estimate the unification scale once the compactification scale  $M_C$  is specified. In fact, we can obtain the unification scale more precisely by taking into account each KK-mode contribution step by step. In SUSY left-right unification cases, we can see from the beta functions that  $\tilde{b}_i = 2$  is universal for  $SU(2)_L$ ,  $SU(2)_R$  and  $U(1)_{B-L}$ . It means that there is no relative running between the three gauge couplings. However, the approximation is not valid if the unification scale  $M_U \sim NM_C$  satisfies  $N \sim \mathcal{O}(1)$  for which the summation approximation is not valid. Thus, we anticipate the unification occurs at the order of the compactification scale if we require that the gauge coupling at unification scale be not strongly coupled (weakly coupled unification). We can also identify the unification scale  $M_U$  as the cut-off scale  $\Lambda$  if the gauge coupling would be strongly coupled at the unification scale. We know that  $M_S$  is fixed to be within several hundreds GeV in order to give an explanation of the gauge hierarchy problem by supersymmetry (we will not discuss the non-interesting case of high-scale supersymmetry with  $M_S > M_R$  here).

The detailed numerical calculations show that there is a fairly large parameter space for the values of  $M_R$  and  $M_C$  in which the weakly coupled unification is possible. We find that the compactification scale  $M_C$  is required to be larger than 150 TeV in order to get successful weakly coupled gauge coupling unification. While the larger the  $M_C$ , the lower the possible value of  $M_R$  that is allowed by the weakly coupled gauge coupling unification. For example, the parameter  $M_R$  is required to be larger than 70 TeV with  $M_C = 150$  TeV. While if  $M_C = 200$  TeV, the allowed  $M_R$  can be as low as 40 TeV. Fixing the left-right scale, which is identified as the  $SU(2)_R$  gauge boson masses, to  $M_R = 100$  TeV, the sfermion mass  $M_S = 600$  GeV, and the compactification scale  $M_C = 200$  TeV, we obtain

$$\alpha_{B-L}^{-1}(M_R) = 28.810705, \quad \alpha_{2L}^{-1}(M_R) = \alpha_{2R}^{-1}(M_R) = 28.742994,$$

and a weakly coupled unification scale

$$M_U = 323.5 \text{ TeV}.$$

Because the compactification scale is relatively high (higher than 150 TeV), the low-energy effective theory is the supersymmetric left-right model.

If the compactification scale  $M_C$  is lower than 150 TeV, strongly coupled unification can occur. For example, if we choose TeV-scale extra-dimension with  $M_C = 5.0$  TeV while  $M_R = 2.0$  TeV, the strongly coupled unification scale (identified as the cut-off scale  $\Lambda$ ) is  $M_U \sim 30M_C \sim 150$  TeV. We can see that  $M_C \sim 0.01\Lambda$  so that the uncertainties from brane kinetic terms are very small.

In non-SUSY cases, the low-energy left-right model contains one bi-doublet, one  $SU(2)_L$  triplet and one  $SU(2)_R$  triplet. The bulk Higgses contain two 10-dimensional representations with parity assignments  $\eta = 1$  and  $\eta' = \pm 1$ . We

obtain the following beta functions for the gauge couplings of  $SU(2)_L$  and the normalized  $U(1)_{B-L}$ :

$$(b_1, b_2, b_3) = \left(\frac{7}{2}, -3, -7\right) \text{ for } M_Z < E < M_R,$$

$$(b_1, b_2^L, b_2^R, b_3) = \left(\frac{7}{2}, -3, -3, -7\right) \text{ for } M_R < E < M_C.$$

The beta functions of the KK modes are

$$(b_{B-L}^e, b_{B-L}^o) = (1, -13),$$

$$(b_2^{L,e}, b_2^{L,o}) = (-6, -6),$$

$$(b_2^{R,e}, b_2^{R,o}) = (-6, -6),$$

$$(b_3^e, b_3^o) = (-21/2, 0).$$

The beta functions of  $SU(2)_R$  are the same as those of  $SU(2)_L$  due to the left-right symmetry.

In the non-SUSY case, the power-law running with negative beta functions drives the gauge couplings asymptotically free. We assume here the unification scale  $M_U$  is less than the cut-off scale  $\Lambda$ . In this case, there are still some allowed parameter space for the values of  $M_C$  and  $M_R$  which admit gauge coupling unification. In fact,  $M_C$  is allowed to be as low as 3.0 TeV with  $M_R = 2.2$  TeV (although it is not natural in non-SUSY case to get such low  $M_R$ ). However, the numerical calculations indicate that the successful gauge coupling unification requires the compactification scale  $M_C$  to be lower than 8.0 TeV. For  $M_C$  higher than 8.0 TeV, the  $SU(2)_L$  and  $U(1)_{B-L}$  gauge couplings tend to zero asymptotically without intersection. Choosing  $M_R = 3.0$  TeV and  $M_C = 5.0$  TeV, we obtain

$$\alpha_{B-L}^{-1}(M_R) = 31.6011, \quad \alpha_{2L}^{-1}(M_R) = \alpha_{2R}^{-1}(M_R) = 31.2399,$$

and a unification scale much lower than previously

$$M_U = 5.2473 \text{ TeV}.$$

In this scenario the relatively low left-right and compactification scales allow for a unification scale of several TeV. Although low  $M_R$  scenario needs fine-tuning, it is however possible. Such low-energy unification may have numerous interesting phenomenological consequences.

The generic phenomenology of our model is similar to that of any other theories with an extra dimension and thus is not discussed here. But our model has some additional phenomenological features. The scenario predicts the existence of doubly charged gauge bosons at several TeVs which may be within the reach of the LHC. These heavy gauge bosons have gauge couplings to leptons while having no couplings to quarks. Since the  $(+, -)$  modes vanish on the  $O'$  brane, they can only have derivative couplings to quarks. But two-quark interactions with  $A_X^\mu$  are forbidden because of non-matching quantum numbers. From the mode expansion of the gauge couplings to leptons, which is similar to that of the Yukawa couplings, we can see that the doubly charged heavy gauge boson  $A^{--}$  can couple to two charged leptons. It can decay into electron pairs or a pair of  $SU(2)_L$  and  $SU(2)_R$  charged gauge bosons with coupling  $\sqrt{2}g_2$ . The coupling of the first KK excitations of the real scalar  $A_1^q$  and  $W^\pm$  to the leptons can also give couplings of the charge-two real scalar to charged lepton pairs. We know that  $\phi_3$  is non-vanishing on the  $O'$  brane because it has parity  $(-, +)$  under projection. Similar to heavy gauge boson cases, its couplings to two quarks are forbidden because of its non-matching quantum numbers. Our model also has  $SU(2)_L$  singlets charged scalars with  $B - L = \pm 2$ . Such scalars can decay into lepton pairs like  $\nu e \mu$ .

## 5. $SU(4)_W$ Breaking to $SU(2)_L \times U(1)_{3R} \times U(1)_{B-L}$

As we demonstrated it is advantageous to break the  $SU(4)_W$  to the minimal left-right model via orbifolding, and the corresponding OGSB chain for  $SU(4)_W$  can be fairly rich. In this section we show that we can break  $SU(4)_W$  to  $SU(2)_L \times U(1)_{3R} \times U(1)_{B-L}$  which also leads to interesting phenomenology.

In this case, our starting point is again the five-dimensional  $\mathcal{N} = 1$  supersymmetric  $SU(3)_C \times SU(4)_W$  gauge symmetry. First, we consider the parity assignments in terms of the fundamental representation of  $SU(3)_C \times SU(4)_W$ :

$$P = \text{diag}(+1, +1, +1) \otimes \text{diag}(+1, +1, +1, -1),$$

$$P' = \text{diag}(+1, +1, +1) \otimes \text{diag}(+1, +1, -1, -1).$$

Boundary conditions break  $\mathcal{N} = 2$  supersymmetry to  $\mathcal{N} = 1$  in four dimensions. The  $SU(3)_C \times SU(3)_W \times U(1)_1$  gauge symmetry is preserved at the  $O$  brane while it is broken to  $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  at the  $O'$  brane. The zero modes preserve the  $SU(3)_C \times SU(2)_L \times U(1)_{3R} \times U(1)_{B-L}$  gauge symmetry, which can be seen through the form of corresponding generators in  $SU(4)_W$ .

We introduce two  $\mathcal{N} = 2$  Higgs hypermultiplets in  $SU(3)_C \times SU(4)_W$  symmetric representations in the bulk. These contain the  $\mathcal{N} = 1$  chiral supermultiplets  $\Phi_1(1, 10)$  and  $\Phi_2(1, 10)$  as well as their conjugate chiral fields. The parity assignments for the Higgs sector read

$$\eta_{\Phi_i} = 1, \quad \eta'_{\Phi_i} = -1.$$

This leads to the following parity assignments for the Higgs hypermultiplets:

$$\Phi(10) : (+, -), \quad \Phi^c(\bar{10}) : (-, +),$$

$$\Phi(10) : (+, -), \quad \Phi^c(\bar{10}) : (-, +),$$

$$\Phi(10) : (+, +), \quad \Phi^c(\bar{10}) : (-, -),$$

$$\Phi(10) : (-, +), \quad \Phi^c(\bar{10}) : (+, -),$$

$$\Phi(10) : (+, -), \quad \Phi^c(\bar{10}) : (-, +),$$

$$\Phi(10) : (+, -), \quad \Phi^c(\bar{10}) : (-, +),$$

$$\Phi(10) : (+, +), \quad \Phi^c(\bar{10}) : (-, -),$$

$$\Phi(10) : (-, +), \quad \Phi^c(\bar{10}) : (+, -),$$

$$\Phi(10) : (-, -), \quad \Phi^c(\bar{10}) : (+, +),$$

$$\Phi(10) : (-, +), \quad \Phi^c(\bar{10}) : (+, -),$$

$$\Phi(10) : (-, +), \quad \Phi^c(\bar{10}) : (+, -),$$

$$\Phi(10) : (-, -), \quad \Phi^c(\bar{10}) : (+, +),$$

$$\Phi(10) : (+, -), \quad \Phi^c(\bar{10}) : (-, +).$$

The  $SU(2)_L$  doublets  $H_u$  and  $H_d$  arise from the bulk zero modes of  $\Phi_i$ , and two  $SU(2)_L$  singlets  $T_1$  and  $T_2$  from that of  $(\Phi_i)^c$ .

Fermions can be located at the fixed points O or O'. Since at O the gauge symmetry is  $SU(3)_C \times SU(3)_W \times U(1)_1$ , if we place all the matter on the O brane, we have to introduce mirror fermions for quarks similarly to the 3-3-1 model. Thus, the most economical way is to locate all matter at the O' brane (although it is also possible to put leptons on the O brane while quarks are on O' brane). Since at O' only the  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  gauge symmetry is preserved, we can start with a left-right gauge invariant Lagrangian and then integrate out the heavy modes to get the  $SU(2)_L \times U(1)_{3R} \times U(1)_{B-L}$  Lagrangian in four dimensions.

The matter content at the O' brane can be that of the minimal left-right model:

$$(3, 2, 1) : Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad (1, 2, 1) : L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix},$$

$$(\bar{3}, 1, \bar{2}) : Q_L^c = \begin{pmatrix} u_L^c \\ d_L^c \end{pmatrix}, \quad (1, 1, \bar{2}) : L_L^c = \begin{pmatrix} \nu_L^c \\ e_L^c \end{pmatrix}.$$

In terms of  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  representations the Higgses can be written

$$\Phi_1(10) = \begin{pmatrix} A_{2 \times 2} & \phi_{2 \times 2} \\ \phi_{2 \times 2} & B_{2 \times 2} \end{pmatrix}, \quad \Phi_2(\bar{10}) = \begin{pmatrix} A'_{2 \times 2} & \phi'_{2 \times 2} \\ \phi'_{2 \times 2} & B'_{2 \times 2} \end{pmatrix},$$

$$(\Phi_1)^c(10) = \begin{pmatrix} A_1 & S_1 & D_{11} & D_{12} \\ S_1 & A_2 & D_{21} & D_{22} \\ D_{11} & D_{21} & A_3 & T_1 \\ D_{12} & D_{22} & T_1 & A_4 \end{pmatrix}, \quad (\Phi_2)^c(10) = \begin{pmatrix} B_1 & S_2 & E_{11} & E_{12} \\ S_2 & B_2 & E_{21} & E_{22} \\ E_{11} & E_{21} & B_3 & T_2 \\ E_{12} & E_{22} & T_2 & B_4 \end{pmatrix}.$$

The parity of the brane fields are determined by the requirement that all the gauge invariant operators on the O' brane must transform covariantly under P parity which is the consequence of the identification of the  $y = \pi R/2$  and  $y = -\pi R/2$  branes. From the kinetic terms and parity assignments follows the parity of the matter content on the O' brane:

$$P : Q_L = \pm(+, +), \quad P : Q_L^c = \pm(+, -), \quad P : L_L = \pm(+, +), \quad P : L_L^c = \pm(+, -).$$

From the parity properties of gauge invariant operators on the  $O'$  brane (which we do not list here) the Yukawa couplings of the bulk Higgses to the brane fermions can be obtained:

$$\begin{aligned} \mathcal{L}_{\text{Yuk}} = & \int d^4x \int_0^{2\pi R} dy \left[ \delta(y - \pi R/2) \left( \sum_{ij} Y^{ij} \epsilon_{ab} (Q_L)_i^a (Q_L^c)_j^b \phi^c + \sum_{ij} Y'^{ij} \epsilon_{bc} (Q_L)_i^c (\phi')^a \right) \right. \\ & \left. + \{ \delta(y - \pi R/2) \pm \delta(y + \pi R/2) \} \left( \sum_{ij} Y^{ij} \epsilon_{ab} (L_L)_i^a (L_L)_j^b T_1 + Y_6^{ij} \epsilon_{ab} (L_L^c)_i (L_L)_j T_2 \right) \right] + \text{h.c.}, \end{aligned}$$

where the  $\pm$  signs correspond to the relative parity assignment (identical or inverse) in front of  $Q_L$  and  $Q_L^c$ , respectively. After expanding  $\phi$  and  $\phi^c$  in their KK modes, we can see that amongst the zero modes only two  $SU(2)_L$  doublets remain which are identified as  $H_u$  and  $H_d$  of the supersymmetric standard model. The electrically charged field  $T_1$  ( $T_2$ ) can couple to two leptons as  $\nu_L e_L$  ( $\nu_R e_R$ ) etc. The  $U(1)_{B-L}$  number for quarks and leptons can be determined by anomaly cancellation requirements for  $[SU(2)_L]^2 U(1)_{B-L}$ ,  $[SU(2)_R]^2 U(1)_{B-L}$ ,  $[U(1)_{B-L}]^3$ , as well as  $[\text{Gravity}]^2 U(1)_{B-L}$ , and  $[SU(3)_C]^2 U(1)_{B-L}$ , etc. The normalization of the Higgs sector can be determined by the requirement that the Yukawa couplings should be invariant under  $U(1)_{B-L}$ . The charge quantization conditions, in terms of the  $SU(4)_W$  fundamental representation, are

$$Q_1 + Q_3 = Q_2 + Q_3 = 0, \quad Q_3 + Q_4 = 2b,$$

where  $b$  is the  $U(1)_{B-L}$  number for leptons. The fields  $T_1$  and  $T_2$  are necessary to determine the  $U(1)_{B-L}$  quantization conditions because they give the second equation in the previous formula. From the first equation and traceless condition follows that the  $U(1)_{B-L}$  generator is proportional to the  $SU(4)_W$  generator

$$T_{B-L} = \text{diag}(-a, -a, a, a).$$

From the second equation we obtain that  $a = b = 1$ . (Here we rely on the phenomenological requirement that the relative normalization of the  $U(1)_{B-L}$  charge between the Higgs and lepton sectors was chosen to be  $b = 1$ ). From these quantization conditions, we obtain

$$\frac{1}{g_{B-L}^2} = \frac{2}{g_4^2}.$$

Since the  $U(1)_{3R}$  gauge group can be realized as the diagonal subgroup of  $SU(2)_R$ , its normalization condition is set by  $SU(2)_R$ , which leads to the relation  $g_{3R}^2 = g_R^2 = g_4^2$ . From the charge assignments we obtain

$$g_{B-L} = g_{3R} = g_4, \quad Q = T_{3L} + \frac{1}{2}(B - L).$$

The tree-level weak mixing angle is again

$$\sin^2 \theta_W = \frac{1}{4}.$$

The quantization conditions imply the parity and quantum numbers for all the bulk fields:

$$V^{(15)} = 3_{(+,+)}^{(0,0)} \oplus 1_{(+,+)}^{(0,0)} \oplus 1_{(+,+)}^{(0,0)} \oplus 2_{(+,-)}^{(-1,0)} \oplus \bar{2}_{(+,-)}^{(1,0)} \oplus \bar{2}_{(-,+)}^{(-1,0)} \oplus 2_{(-,+)}^{(1,0)},$$

$$\Sigma^{(15)} = 3_{(-,-)}^{(0,0)} \oplus 1_{(-,-)}^{(0,0)} \oplus 1_{(-,-)}^{(0,0)} \oplus 2_{(-,+)}^{(-1,0)} \oplus \bar{2}_{(-,+)}^{(1,0)} \oplus \bar{2}_{(+,-)}^{(-1,0)} \oplus 2_{(+,-)}^{(1,0)},$$

$$\Phi(10) = 3_{(+,-)}^{(0,-2)} \oplus 1_{(+,-)}^{(0,2)} \oplus 1_{(+,-)}^{(1,0)} \oplus 2_{(+,+)}^{(-1,-2)} \oplus \bar{2}_{(+,+)}^{(1,2)} \oplus \bar{2}_{(-,-)}^{(-1,0)} \oplus 2_{(-,-)}^{(1,0)},$$

$$\Phi^c(\bar{10}) = \bar{3}_{(-,+)}^{(0,2)} \oplus 1_{(-,+)}^{(0,-2)} \oplus 1_{(-,+)}^{(-1,0)} \oplus \bar{2}_{(-,-)}^{(1,2)} \oplus 2_{(-,-)}^{(-1,-2)} \oplus 2_{(+,+)}^{(1,0)} \oplus \bar{2}_{(+,+)}^{(-1,0)},$$

$$\Phi(10) = \bar{3}_{(+,-)}^{(0,2)} \oplus 1_{(+,-)}^{(0,-2)} \oplus 1_{(+,-)}^{(-1,0)} \oplus 2_{(+,+)}^{(1,-2)} \oplus \bar{2}_{(+,+)}^{(-1,2)} \oplus \bar{2}_{(-,-)}^{(1,0)} \oplus 2_{(-,-)}^{(-1,0)},$$

$$\Phi^c(\bar{10}) = 3_{(-,+)}^{(0,-2)} \oplus 1_{(-,+)}^{(0,2)} \oplus 1_{(-,+)}^{(1,0)} \oplus \bar{2}_{(-,-)}^{(-1,-2)} \oplus 2_{(-,-)}^{(1,2)} \oplus 2_{(+,+)}^{(-1,0)} \oplus \bar{2}_{(+,+)}^{(1,0)}.$$

Subscripts denote  $U(1)_{3R}$  and  $U(1)_{B-L}$  quantum numbers, respectively. We can see that there are zero mode components in  $\Sigma^{(15)}$  decompositions. Such zero modes can act as Higgs doublets in the MSSM, if we adopt the gauge-Higgs unification scheme. However, such Higgs fields cannot couple to matter fields because of un-matching quantum numbers.

The  $SU(2)_L \times U(1)_{3R} \times U(1)_{B-L}$  gauge symmetry can be broken to the SM one (in SUSY cases) via the bulk Higgs fields  $H_1(1, 4)$  and  $H_2(1, \bar{4})$  (here  $SU(3)_C \times SU(4)_W$  representations are shown). Parity can be assigned to these Higgses as

$$\eta_{H_i} = -1, \quad \eta'_{H_i} = -1 \quad (i = 1, 2).$$

From the decomposition of  $SU(4)_W$  in terms of  $SU(2)_L \times U(1)_{3R} \times U(1)_{B-L}$

$$(H_1)^{(4)} = 2_{(-,-)}^{(0,-1)} \oplus 1_{(-,+)}^{(0,1)} \oplus 1_{(-,+)}^{(1,1)} \oplus 1_{(+,+)}^{(-1,1)},$$

$$(H_2)^{\bar{4}} = \bar{2}_{(-,-)}^{(0,1)} \oplus 1_{(-,+)}^{(0,-1)} \oplus 1_{(-,+)}^{(-1,-1)} \oplus 1_{(+,+)}^{(1,-1)},$$

$$(H_1)^c(\bar{4}) = \bar{2}_{(+,+)}^{(0,1)} \oplus 1_{(+,-)}^{(0,-1)} \oplus 1_{(+,-)}^{(-1,-1)} \oplus 1_{(-,-)}^{(1,-1)},$$

$$(H_2)^c(4) = 2_{(+,+)}^{(0,-1)} \oplus 1_{(+,-)}^{(0,1)} \oplus 1_{(+,-)}^{(1,1)} \oplus 1_{(-,-)}^{(-1,1)}.$$

It follows that the zero modes of  $H_i$  ( $i = 1, 2$ ) contain two  $SU(2)_L$  singlets  $U_1^{(-1,1)}$  and  $U_2^{(1,-1)}$  (subscripts denote  $U(1)_{3R} \times U(1)_{B-L}$  quantum numbers) which are electrically neutral and cannot couple to matter directly. The zero modes for  $(H_i)^c$  contain two Higgs doublets  $\bar{2}_{(+,+)}^{(0,-1)}$  which cannot couple to matter either because of non-matching quantum numbers. After  $U_1$  and  $U_2$  acquire VEVs, the remaining gauge symmetry is broken to the SM gauge group. Note that  $T_1$  and  $T_2$  cannot be used to break this gauge symmetry because they have electric charges.

The beta functions of the gauge couplings  $U(1)_Y$ ,  $SU(2)_L$ ,  $SU(3)_C$  read

$$(b_1, b_2, b_3) = \left( \frac{25}{3}, 1, -3 \right) \text{ for } M_Z < E < M_S,$$

$$(b_1, b_2, b_3) = (15, 3, -3) \text{ for } M_S < E < M',$$

where  $M'$  is the scale where  $SU(2)_L \times U(1)_{3R} \times U(1)_{B-L}$  gauge symmetry appears. In the SUSY and SUSY decoupling limits, there are six Higgs doublets. For the  $U(1)_{B-L}$ ,  $U(1)_{3R}/2$ ,  $SU(2)_L$ , and  $SU(3)_C$  gauge couplings the beta functions are

$$(b_{B-L}, b_{3R}, b_2^L, b_3) = \left( \frac{23}{3}, \frac{23}{3}, 3, -3 \right) \text{ for } M' < E < M_C.$$

The beta functions corresponding to the even and odd KK modes at one loop are

$$(b_{B-L}^e, b_{B-L}^o) = \left( \frac{23}{3}, \frac{23}{3} \right),$$

$$(b_{3R}^e, b_{3R}^o) = (1, 5),$$

$$(b_2^{L,e}, b_2^{L,o}) = (-2, 8),$$

$$(b_3^e, b_3^o) = (-6, 0).$$

Just as in the previous case, the  $SU(4)_W$  preserved  $b^o + b^e$  is constant for the three gauge couplings. Thus, the relative RGE running between the three gauge couplings are logarithmic. As we do not know the  $g_{B-L}$  or  $g_{3R}$  gauge couplings at the  $M'$  scale (or the relations between the two gauge couplings), we must invoke further assumptions related to them to predict the unification scales. SUSY breaking can again be achieved by the Scherk-Schwarz mechanism through boundary conditions. The tree-level gaugino and Higgsino masses acquired this way will induce loop-level squark and slepton masses.

The phenomenology of this symmetry breaking chain shares many common features with that of the previous cases. For example, there are charge-two heavy gauge bosons and two  $SU(2)_L$  charged gauge singlets scalars which can only derivatively couple to charged lepton pairs. At energies well below the  $M_C$  scale, the low-energy effective theory reduces to supersymmetric  $SU(2)_L \times U(1)_{3R} \times U(1)_{B-L}$ . This  $U(1)$  extension of the SM has been widely studied. The special feature of this scenario is the existence of Higgs doublets which have no tree-level couplings to SM fermions even when the low-energy  $SU(2)_L \times U(1)_Y$  quantum numbers allow such couplings. The electrically neutral  $SU(2)_L$  singlet Higgses,  $U_1$  and  $U_2$ , which break the remaining group to the SM, can be viable dark matter candidates.

**Alternative Models:** We can locate the SM quarks and right-handed charged leptons on the  $O'$  brane while placing the SM lepton doublets and right-handed neutrinos in the bulk. We can introduce mirror leptons  $X_L, \bar{X}, \bar{X}^c, Y_L$ , and  $Y_L^c$  to fill the bulk hypermultiplets  $F_i$  ( $i = 1, 2$ ) in the  $(1, 4)$  representation under  $SU(3)_C \times SU(4)_W$ :

$$F_1 = (L_L, X_L), \quad F_1^c = (\bar{L}_L, \bar{X}_L),$$

$$F_2 = (Y_L, X_L^c), \quad F_2^c = (\bar{Y}_L, \bar{X}_L^c),$$

where  $(L_L, X_L^c)$  denotes  $(E_L - \nu_L^c)$ , with  $E_L$  being a charged mirror lepton. Then, we can assign parities as

$$\eta_{F_1} = 1, \quad \eta'_{F_1} = 1, \quad \eta_{F_2} = -1, \quad \eta'_{F_2} = -1.$$

The left-handed leptons and neutrinos  $L_L$  arise from  $F_1$ , and the right-handed neutrinos from  $F_2$ . Note that we cannot fit right-handed leptons in  $F_1^c$  because that does not yield the correct quantum numbers. Mirror fermions associated with each SM lepton, except with the right-handed charged leptons, will survive the projection.

As previously, we can locate the SM quarks and right-handed neutrinos on the  $O'$  brane while having the SM lepton doublets and right-handed charged leptons in the bulk. The parity assignments read

$$P = \text{diag}(+1, +1, +1) \otimes \text{diag}(+1, +1, -1, +1),$$

$$P' = \text{diag}(+1, +1, +1) \otimes \text{diag}(+1, +1, -1, -1).$$

Similarly to our previous case, we obtain mirror fermions associated with each SM lepton from zero modes, except for right-handed neutrinos.

## 6. Conclusions

In this paper, we propose a low-scale  $SU(4)_W$  unification model which has two symmetry breaking chains. In the first chain  $SU(4)_W$  is broken into the  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  minimal left-right model through  $S^1/(Z_2 \times Z'_2)$  orbifolding. Leptons are fitted into  $SU(4)_W$  multiplets and located on a symmetry-preserving  $O$  brane, while quarks are placed on  $O'$  brane where the symmetry is broken. This approach predicts  $\sin^2\theta_W = 0.25$  for the weak mixing angle at tree level and leads to a rather low weakly coupled unification scale of order  $10^2$  TeV with supersymmetry, or as low as several TeV in the non-supersymmetric case.

If we introduce mirror fermions and put quarks in the bulk, the model gives a large weak mixing angle  $\sin^2\theta_W = 0.45$  which will lead to high-energy unification. The other symmetry breaking chain with the low-energy gauge group  $SU(2)_L \times U(1)_{3R} \times U(1)_{B-L}$  after OGSB can also give rise to a weak mixing angle  $\sin^2\theta_W = 0.25$  at tree level. In this scenario, leptons and quarks are placed on the  $O'$  brane (with broken symmetry) and the quantization conditions are determined by anomaly cancellation requirements. These low-scale unification theories have interesting phenomenological consequences.

One may worry if there are cosmological difficulties associated with this scenario such as the monopole problems etc. In fact there are no monopole problems in our scenario because we break the gauge symmetry via orbifolding. In general there are monopole problems if a gauge symmetry is broken to a subgroup containing  $U(1)$  via Higgs mechanism with the unification scale lower than the inflation scale and at the same time higher than TeV scale [?]. It is not a problem in OGSB scenario because the gauge symmetry is broken via boundary conditions with the symmetry broken explicitly in the orbifold fixed points. So our scenario is not bothered by the cosmological monopole problems.

## Acknowledgements

We acknowledge Jing Shu and Hitoshi Murayama for comments. This work was supported by the Australian Research Council under project DP0877916, by the National Natural Science Foundation of China under grant Nos. 10821504,

10725526 and 10635030, and by the Cambridge-Mitchell Collaboration in Theoretical Cosmology.

## References

- [1] R. N. Mohapatra and J. C. Pati, Phys. Rev. D11, 566 (1975).
- [2] R. N. Mohapatra, Phys. Rev. D34, 3457 (1986); A. Font, L. E. Ibanez and F. Quevedo, Phys. Lett. B228, 79 (1989); R. Kuchimanhi and R. N. Mohapatra, Phys. Rev. D48, 4352 (1993).
- [3] H. Georgi, in Particles and Fields (1975); H. Fritzsch and P. Minkowski, Ann. Phys. 93, 193 (1975).
- [4] H. Georgi and S. L. Glashow, Phys. Rev. Lett 32, 438 (1974); S. Dimopoulos and H. Georgi, Nucl. Phys. B193, 150 (1981).
- [5] Q. Shafi and Z. Tavartkiladze, hep-ph/0108247.
- [6] I. Gogoladze, Y. Mimura and S. Nandi, Phys. Lett. B560, 204 (2003).
- [7] T. Li, F. Wang and J. M. Yang, arXiv:0901.2161 [hep-ph].
- [8] S. Weinberg, Phys. Rev. D5, 1962 (1972); S. Dimopoulos and D. E. Kaplan, Phys. Lett. B531, 127 (2002).
- [9] Y. Kawamura, Prog. Theo. Phys 103 (2000) 613; hep-ph/0012125; hep-ph/0012352.
- [10] G. Altarelli and F. Feruglio, Phys. Lett. B511, 257 (2001); L. Hall and Y. Nomura, Phys. Rev. D64, 055003 (2001).
- [11] A. Hebecker and J. March-Russell, Nucl. Phys. B625, 128 (2002).
- [12] A. Hebecker and J. March-Russell, Nucl. Phys. B613, 3 (2001).
- [13] T. Li, Phys. Lett. B520, 377 (2001).
- [14] T. Li, Nucl. Phys. B619, 75 (2001).
- [15] T. Li and W. Liao, Phys. Lett. B545, 147 (2002).
- [16] L. J. Hall and Y. Nomura, Phys. Lett. B532, 111 (2002).
- [17] S. Dimopoulos, D. E. Kaplan and N. Weiner, Phys. Lett. B534, 124 (2002).
- [18] N. Arkani-Hamed, L. Hall, D. Smith and N. Weiner, Phys. Rev. D63, 056003 (2001); N. Arkani-Hamed, T. Gregoire and J. Wacker, JHEP 0203, 055 (2002).
- [19] C. Amsler et al. [Particle Data Group], Phys. Lett. B667, 1 (2008).
- [20] Y. Zhang, H. An, X. D. Ji and R. N. Mohapatra, Phys. Rev. D78, 011302 (2008).
- [21] I. Antoniadis, Phys. Lett. B246, 377 (1990); I. Antoniadis, S. Dimopoulos, A. Pomarol and M. Quiros, Nucl. Phys. B544, 503 (1999).

- [22] A. Pomarol and M. Quiros, Phys. Lett. B438, 255 (1998); M. Quiros, TASI lecture, [hep-ph/0302189].
- [23] R. Barbieri, J. Hall and Y. Nomura, Phys. Rev. D66, 045025 (2002); R. Barbieri, J. Hall and Y. Nomura, Nucl. Phys. B624, 63 (2002).
- [24] K. R. Dienes, E. Dudas and T. Gherghetta, Phys. Lett. B436, 55 (1998); Nucl. Phys. B537, 47 (1999).
- [25] N. Arkani-Hamed, A. G. Cohen and H. Georgi, Phys. Lett. B516, 395 (2001).
- [26] C. A. Scrucca, M. Serone, L. Silvestrini and F. Zwirner, Phys. Lett. B525, 169 (2002).
- [27] L. Pilo and A. Riotto, Phys. Lett. B546, 135 (2002).
- [28] H. D. Kim, J. E. Kim and H. M. Lee, JHEP 0206, 048 (2002); H. M. Lee, JHEP 0309, 078 (2003).
- [29] H. Murayama and J. Shu, arXiv:0905.1720 [hep-ph].

*Note: Figure translations are in progress. See original paper for figures.*

*Source: ChinaXiv – Machine translation. Verify with original.*