Realistic Flipped SU(5) from Orbifold SO(10) (Postprint)

Authors: Balazs,C, Kang,Z, Li,T, Wang,F, Yang,JM

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Full Text

Preamble

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Csaba Balazs[1], Zhaofeng Kang[2], Tianjun Li[2],[3], Fei Wang[1], Jin Min Yang[2]

[1] Key Laboratory of Frontiers in Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, P. R. China
[2] George P. and Cynthia W. Mitchell Institute for Fundamental Physics, Texas A&M University, College Station, TX 77843, USA

Abstract: We propose a realistic flipped SU(5) model derived from a five-dimensional orbifold SO(10) model. The Standard Model (SM) fermion masses and mixings are explained by combining the traditional Froggatt-Nielsen mechanism with the five-dimensional wave function profiles of the SM fermions. Employing tree-level spontaneous R-symmetry breaking in the hidden sector and extra(ordinary) gauge mediation, we obtain realistic supersymmetry breaking soft mass terms with non-vanishing gaugino masses. Including the messenger fields at the intermediate scale and Kaluza-Klein states at the compactification scale, we study gauge coupling unification. We show that the SO(10) unified gauge coupling is very strong and the unification scale can be much higher than the compactification scale. We briefly discuss proton decay as well.

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1. Introduction

As one of the most attractive extensions of the Standard Model (SM), supersymmetric Grand Unification Theories (GUTs) like SU(5) [?, ?] or SO(10) [?, ?] give us deep insights into problems such as charge quantization, neutrino masses and mixings, as well as the origin of the Yukawa sector. However, these theories still have some unsatisfactory features such as the doublet-triplet (D-T) splitting problem, rapid proton decay, and unrealistic SM fermion mass relations.

The SO(10) models are particularly interesting since they achieve both gauge interaction unification and SM fermion unification. One type of these models, where the gauge symmetry is broken down to the Georgi-Glashow SU(5), suffers from rapid proton decay and D-T splitting problems in the subsequent gauge symmetry breaking into the SM. In contrast, another type with symmetry breaking to flipped SU(5) might be more attractive because flipped SU(5) models can solve the D-T splitting problem via the missing partner mechanism as well as the dimension-five proton decay problem [?, ?, ?]. Although embedding flipped SU(5) into SO(10) can retrieve gauge unification, the missing partner mechanism does not work in four-dimensional models (for a possible solution, see Ref. [?]). A simple solution is to realize such embedding in a five-dimensional orbifold. Orbifold GUT models for SU(5) were proposed in [?, ?, ?] and widely studied thereafter in [?, ?, ?, ?, ?, ?, ?, ?, ?]. Orbifold SO(10) models with symmetry breaking to Pati-Salam models were studied in [?, ?], and earlier studies on orbifold SO(10) models with symmetry breaking to flipped SU(5) can be found in [?, ?].

In this paper we consider a realistic flipped SU(5) model derived from the five-dimensional orbifold SO(10) and study its phenomenological consequences. As we know, it is interesting to explain the SM fermion masses and mixings in the Minimal Supersymmetric Standard Model (MSSM) from a top-down approach. In particular, the Froggatt-Nielsen mechanism [**?**] can be very predictive in GUTs. Efforts to explain the flavor structure through the deformed Froggatt-Nielsen mechanism in orbifold SU(5) models were shown in [**?**, **?**, **?**], in which the SM fermion mass and mixing hierarchies are obtained via wave-function profiles of the SM fermions by adding bulk mass terms [**?**]. However, we find that in the flipped SU(5) model it is not as simple as in the ordinary SU(5) model to explain the SM fermion masses and mixings by such a Froggatt-Nielsen mechanism because of the flipping of the right-handed up- and down-type quarks. Besides, the neutrino masses and mixings obtained from the double see-saw mechanism set stringent constraints on the possible quark mass hierarchies in the flipped SU(5) model. Therefore, we will introduce an additional discrete Z_3 symmetry and combine the traditional Froggatt-Nielsen mechanism with the wave-function profiles of the SM fermions. In this way we can generate the observed SM fermion masses and mixings.

In addition, we will discuss the relevant problems of supersymmetry (SUSY) breaking. We use the tree-level spontaneously R-symmetry breaking model and (extra)ordinary gauge mediation to obtain realistic SUSY breaking soft mass terms with non-vanishing gaugino masses, which is in contrast to previous models with vanishing gaugino masses by direct gauge mediation. Moreover, by including the messenger fields at the intermediate scale and the Kaluza-Klein (KK) states at the compactification scale, we will study gauge coupling unification in detail.

Our study shows that the SO(10) unified gauge coupling is very strong, and the unification scale can be much higher than the compactification scale. We will also comment on proton decay.

This paper is organized as follows. In Section 2 we recapitulate the flipped SU(5) model. In Section 3 we present the orbifold SO(10) models where the gauge symmetry is broken down to flipped SU(5). In Section 4 we explain the SM fermion masses and mixings via the usual Froggatt-Nielsen mechanism and wave function profiles of the SM fermions. In Section 5 we discuss four-dimensional $\mathcal{N} = 1$ supersymmetry breaking via tree-level spontaneously R-symmetry breaking. In Section 6 we discuss the hidden sector and (extra)ordinary gauge mediation. In Section 7 we discuss gauge coupling unification with threshold corrections from messenger fields and KK states. In Section 8 we discuss the proton decay problem. Section 9 contains our conclusions.

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2. Flipped SU(5) Model

In this section we briefly review the four-dimensional flipped SU(5) model [**?**, **?**, **?**].

The gauge group for the flipped SU(5) model is $SU(5) \times U(1)_X$, which can be embedded in the SO(10) group. We define the generator $U(1)_{Y'}$ in SU(5) as

$$T_{U(1)_{Y'}} \equiv \text{diag}\left(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{1}{2}, \frac{1}{2}\right).$$

The hypercharge is given by $Q_Y = (Q_X - Q_{Y'})/5$.

The SM fermions transform under $SU(5) \times U(1)_X$ as follows

$$F_i = (10, 1), \quad \bar{f}_i = (\bar{5}, -3), \quad l_i^c = (1, 5),$$

where $i = 1, 2, 3$. The particle assignments are

$$F_i = (Q_i, D_i^c, N_i^c), \quad \bar{f}_i = (U_i^c, L_i), \quad l_i^c = E_i^c,$$

where Q_i and L_i are the quark and lepton doublet superfields, and U_i^c, D_i^c, E_i^c, and N_i^c are the charge conjugate superfields of the right-handed up-type quark, down-type quark, lepton, and neutrino, respectively.

To break the GUT and electroweak gauge symmetries, two pairs of Higgs fields are introduced in the following representations

$$H = (10, 1), \quad \bar{H} = (\overline{10}, -1), \quad h = (5, -2), \quad \bar{h} = (\bar{5}, +2).$$

We label the states in the Higgs multiplets by the same symbols as in the SM fermion multiplets. Explicitly, the Higgs particles are

$$H = (Q_H, D_H^c, N_H^c), \quad \bar{H} = (Q_{\bar{H}}, D_{\bar{H}}^c, N_{\bar{H}}^c),$$

$$h = (D_h, D_h, D_h, H_d), \quad \bar{h} = (D_{\bar{h}}, D_{\bar{h}}, D_{\bar{h}}, H_u),$$

where H_d and H_u are the two Higgs doublets in the MSSM.

The flipped SU(5) model elegantly solves the D-T splitting problem via the missing partner mechanism. After N_H^c and $N_{\bar{H}}^c$ acquire vacuum expectation values (VEVs) which break the flipped SU(5) gauge symmetry down to the SM gauge symmetry, the superfields H and $\bar{H}$ will be eaten by the supersymmetric Higgs mechanism except for the D_H^c components. The superpotential term

$$W_{D-T} = \frac{1}{4\pi}\left(\lambda HHh + \bar{\lambda}\bar{H}\bar{H}\bar{h}\right)$$

couples D_H^c with D_h and $D_{\bar{H}}^c$ with $D_{\bar{h}}$ to form heavy eigenstates with masses $8\pi\lambda\langle N_H^c\rangle$ and $8\pi\bar{\lambda}\langle N_{\bar{H}}^c\rangle$. But the Higgs doublets remain massless since they do not have vector-like partners in H and $\bar{H}$. Thus, the doublets and triplets in h and $\bar{h}$ are split. Because the triplets in h and $\bar{h}$ only have small mixing through the effective μ-term, the Higgsino-exchange mediated proton decay is negligible, i.e., we do not have the dimension-five proton decay problem.

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3. Flipped SU(5) from Five-Dimensional Orbifold SO(10)

We consider the five-dimensional space-time $M_4 \times S^1/(Z_2 \times Z_2')$ comprising the Minkowski space M_4 with coordinates x^μ and the orbifold $S^1/(Z_2 \times Z_2')$ with coordinate $y \equiv x^5$. The orbifold $S^1/(Z_2 \times Z_2')$ is obtained from S^1 by modding out the equivalence classes

$$P : y \sim -y, \quad P' : y' \sim -y',$$

where $y' \equiv y + \pi R/2$. There are two inequivalent 3-branes located at $y = 0$ and $y = \pi R/2$ which are denoted as O and O', respectively.

The five-dimensional $\mathcal{N} = 1$ supersymmetric gauge theory has 8 real supercharges, corresponding to $\mathcal{N} = 2$ supersymmetry in four dimensions. The vector multiplet physically contains a vector boson A_M where $M = 0, 1, 2, 3, 5$, two Weyl gauginos $\lambda_{1,2}$, and a real scalar σ. In terms of four-dimensional $\mathcal{N} = 1$ language, it contains a vector multiplet $V(A_\mu, \lambda_1)$ and a chiral multiplet $\Sigma((\sigma + iA_5)/\sqrt{2}, \lambda_2)$ which transform in the adjoint representation of the gauge group. The five-dimensional hypermultiplet physically has two complex scalars ϕ and ϕ^c, a Dirac fermion Ψ, and can be decomposed into two four-dimensional chiral multiplets $\Phi(\phi, \psi \equiv \Psi_R)$ and $\Phi^c(\phi^c, \psi^c \equiv \Psi_L)$, which transform as conjugate representations of each other under the gauge group.

The general action for the gauge fields and their couplings to the bulk hypermultiplet Φ is [?, ?]

$$S = \int d^{5x} \left[\int d^2\theta \frac{1}{2kg^2} \mathrm{Tr}(W^\alpha W_\alpha + \mathrm{H.c.}) + \int d^4\theta \frac{1}{kg^2} \mathrm{Tr} \left(\frac{1}{2}(\partial_5 + \bar{\Sigma}) e^{-V} (-\partial_5 + \Sigma) e^V + \partial_5 e^{-V} \partial_5 e^V \right) \right.$$
$$\left. + \int d^4\theta (\Phi^c e^V \bar{\Phi}^c + \bar{\Phi} e^{-V} \Phi) + \left(\int d^2\theta \Phi^c (\partial_5 - \Sigma) \Phi + \mathrm{H.c.} \right) \right].$$

Possible kink mass terms can be added to hypermultiplets which will play a central role in reproducing the SM fermion masses and mixings in our paper.

We consider the flipped SU(5) gauge theory obtained from bulk SO(10) gauge theory via orbifolding in the five-dimensional $Z_2 \times Z_2'$ orbifold. We can choose proper boundary conditions to break SO(10) gauge symmetry down to flipped SU(5) on the O' brane at $y = \pi R/2$. The boundary conditions ((Z_2, Z_2') parities) for the bulk fields can be chosen so that the SO(10) representation decomposes in terms of flipped SU(5) as

$$V_g(45) = V^{++}_{24^0} + V^{+-}_{10^{-4}} + V^{-+}_{1^0} + V^{--}_{10^{+4}},$$

$$\Sigma_g(45) = \Sigma^{--}_{24^0} + \Sigma^{-+}_{10^{-4}} + \Sigma^{+-}_{1^0} + \Sigma^{++}_{10^{+4}},$$

$$\Phi(16)_1 = \Phi^{++}_{10^1} + \Phi^{+-}_{\bar{5}^{-3}} + \Phi^{-+}_{1^5},$$

$$\Phi(16)_2 = \Phi^{+-}_{10^1} + \Phi^{++}_{\bar{5}^{-3}} + \Phi^{+-}_{1^5},$$

$$\Phi(16)_3 = \Phi^{+-}_{10^1} + \Phi^{++}_{\bar{5}^{-3}} + \Phi^{+-}_{1^5},$$

$$H(10)_1 = H^{++}_{5^{-2}} + H^{+-}_{\bar{5}^{+2}},$$

$$H(10)_2 = H^{+-}_{5^{-2}} + H^{++}_{\bar{5}^{+2}}.$$

Also, the (Z_2, Z_2') parities for Φ^c and H^c are opposite to those of Φ and H. In order to explain the SM fermion masses and mixings, we choose the boundary conditions for **16** so that we have three types of wave function profiles for $\mathbf{10}_1$, $\bar{\mathbf{5}}_{-3}$, and $\mathbf{1}_5$, respectively. This is different from naive orbifold SO(10) models. Such boundary conditions are possible by introducing large brane mass terms for relevant fields to change Neumann boundary conditions into Dirichlet boundary conditions [**?**].

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4. The SM Fermion Masses and Mixings

It is well known that the SM fermion masses and mixings exhibit a hierarchical structure. The quark CKM mixings can be cast, in the Wolfenstein formalism, as [**?**]

$$V_{\mathrm{CKM}} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix},$$

where A is of order 1 while ρ and η are between λ and 1. The hierarchy is reflected in the dependence of various entries on different powers of $\lambda \approx 0.22$.

Renormalization group evolution (RGE) of the charged fermion masses to a high scale ($\sim 10^{16}$ GeV) also reveals the following hierarchical structure:

$$(m_u : m_c : m_t) \sim 1 : \lambda^4 : \lambda^8,$$

$$(m_d : m_s : m_b) \sim 1 : \lambda^2 : \lambda^4,$$

$$(m_e : m_\mu : m_\tau) \sim 1 : \lambda^2 : \lambda^4,$$

with $m_b/m_t = \lambda^3$. In this section we discuss the explanation of the pattern of the SM fermion masses and mixings in the flipped SU(5) model.

In extra-dimensional models, a well-known approach to generate the SM fermion hierarchies is the so-called zero mode wave function profile [?]. A non-trivial wave function profile can be generated by bulk mass terms and the Yukawa couplings can be determined by the wavefunction overlap of the Higgs and matter fields. The bulk action for hypermultiplets $\{\Phi, \Phi^c\}$ with mass terms is

$$S = \int d^{5x} \left[\int d^4\theta (\Phi^\dagger \Phi + \Phi^c \Phi^{c\dagger}) + \left(\int d^2\theta \Phi^c (\partial_y + M_\Phi)\Phi + \text{H.c.} \right) \right].$$

In supersymmetric theories, matter multiplets with kink bulk mass terms still have zero modes. Depending on the sign of M_Φ, the zero mode is localized toward the O or the O' brane. The zero mode wave function of Φ has a suppression factor $\exp(-M_\Phi y)$ which means that the zero mode is localized near $y = 0$ for $M_\Phi > 0$ and near $y = \pi R/2$ for $M_\Phi < 0$. The M^{+-} (and M^{-+}) modes in the limit $M^{+-}\pi R/2 \gg 1$ (and $M^{-+}\pi R/2 \ll -1$) have the lightest KK mass $M_{\text{KK}} = 2|M_{zz'}| \exp(-|M_{zz'}|\pi R/2)$ which is less than $1/R$.

We assume that the Yukawa couplings are localized on the $y = \pi R/2$ brane with the general form

$$W = \int d^2\theta \frac{1}{M_*^{3/2}} y_{ijk} \Phi_i \Phi_j \Phi_k,$$

where the Yukawa couplings y_{ijk} are assumed to be around $\mathcal{O}(4\pi)$, and M_* is the cutoff scale of the theory. This results in the four-dimensional Yukawa couplings

$$W_{4D} = \lambda_{ijk} \phi_i \phi_j \phi_k,$$

where

$$\lambda_{ijk} \approx \sqrt{Z[M(\phi_i)] Z[M(\phi_j)] Z[M(\phi_k)]}\, y_{ijk},$$

with

$$Z[M(\phi_i)] = \frac{2M(\phi_i)}{e^{M(\phi_i)\pi R} - 1}.$$

Depending on the value of the bulk masses $M(\phi_i)$, we can have different suppression factors for the Yukawa couplings.

In this paper, we assume that the Higgs fields h and $\bar{h}$ are strongly localized on the symmetry breaking O' brane, which implies $M_h, M_{\bar{h}} \ll -1/R$.

Our goal is to explain the SM fermion masses and mixings based on the deformed Froggatt-Nielsen mechanism via wave function profiles, which is very difficult due to the flipping of the right-handed up- and down-type quarks. To solve this problem, we introduce an additional discrete symmetry and use the traditional Froggatt-Nielsen mechanism together with the wave function profiles to generate realistic SM fermion masses and mixings. After embedding the matter multiplets in flipped SU(5), we can have three types of profiles: $\mathbf{10}_1$ (Q_L, D^c_L, N^c_L) type, $\bar{\mathbf{5}}_{-3}$ (U^c_L, L_L) type, and the $\mathbf{1}_5$ (E^c_L) type. The relevant suppression profiles can be realized through different bulk mass terms.

Realistic neutrino masses can be generated using the double see-saw mechanism by introducing additional SM singlets N_i which mix with the ordinary neutrino sector. We can write the R-symmetry preserving interaction terms for the singlets as

$$W = y_s F_a H N_b + M_{ab} N_a N_b,$$

where we introduced an additional unit R-charge field ψ_2 which will also play a role in the SUSY breaking sector. After ψ_2 and the N^c_H components of H acquire VEVs, we can get the neutrino mass terms

$$\mathcal{L} = y_u^{ab} (\nu_L)_a (\nu^c_L)_b v_u + y_s^{ab} (\nu^c_L)_a N_b v_R + M_{ab} N_a N_b,$$

where $v_u = \langle \bar{h} \rangle$, $v_R = v_M / M_*$, and $M_{ab} \gg v_u$.

The neutrino mass matrix in the basis of (ν_L, ν^c_L, N) is

$$M_\nu = \begin{pmatrix} 0 & y_u v_u & 0 \\ (y_u v_u)^T & 0 & (y_s v_R)^T \\ 0 & (y_s v_R) & M \end{pmatrix}.$$

So we obtain the light Majorana neutrino masses as

$$m_\nu = (y_u v_u) \left[(y_s v_R) M^{-1} (y_s v_R)^T \right]^{-1} (y_u v_u)^T.$$

In the Froggatt-Nielsen mechanism, the Dirac neutrino mass matrix is proportional to the product of matrices F_i and $\bar{f}_i$ describing the fermion profiles:

$$M_{\text{Dirac}} \propto \begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix} \cdot (\bar{f}_1, \bar{f}_2, \bar{f}_3).$$

So the light neutrino mass matrix is

$$m_\nu \propto \begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix} \cdot (\bar{f}_1, \bar{f}_2, \bar{f}_3) \cdot (M_R)^{-1} \cdot \begin{pmatrix} \bar{f}_1 \\ \bar{f}_2 \\ \bar{f}_3 \end{pmatrix} \cdot (F_1, F_2, F_3).$$

From the tri-bimaximal (or bi-maximal) mixings in the neutrino sector, we can determine a possible ratio of the $\bar{f}_i$ profiles:

$$\bar{f}_1 : \bar{f}_2 : \bar{f}_3 \sim 1 : 1 : 1.$$

Thus, the neutrino mass matrix is proportional to

$$\begin{pmatrix} F_1^2 & F_1 F_2 & F_1 F_3 \\ F_2 F_1 & F_2^2 & F_2 F_3 \\ F_3 F_1 & F_3 F_2 & F_3^2 \end{pmatrix},$$

and the unitary transformation matrix is

$$U_L \sim \begin{pmatrix} 1 & -\sqrt{\frac{F_1}{F_2}} & 0 \\ \sqrt{\frac{F_1}{F_2}} & 1 & -\sqrt{\frac{F_2}{F_3}} \\ 0 & \sqrt{\frac{F_2}{F_3}} & 1 \end{pmatrix}.$$

Using the following four-dimensional effective Yukawa terms

$$W \sim \frac{1}{M_*} \tilde{S}_i F_j F_k h,$$

with SM singlet fields $\tilde{S}_i$ having profiles $\langle \tilde{S}_i \rangle \sim (1, 1, 1)$, we can obtain the ratios for the profiles of F_i:

$$F_1 : F_2 : F_3 \sim \lambda^8 : \lambda^4 : 1,$$

from the up-type quark mass ratio

$$(m_u : m_c : m_t) \sim 1 : \lambda^4 : \lambda^8,$$

and the $\bar{f}_i$ profiles.

The reason to introduce $\tilde{S}_i$ is to explain the bottom quark masses and quark CKM mixings. We consider the discrete symmetry Z_3 for F_i in the following, and then the above Yukawa couplings for up-type quarks can be invariant under Z_3 by assigning suitable Z_3 quantum numbers to $\tilde{S}_i$.

So the up-type quark mass matrix is

$$M_u \sim M_{\text{Dirac}} \begin{pmatrix} \lambda^8 & \lambda^8 & \lambda^8 \\ \lambda^4 & \lambda^4 & \lambda^4 \\ 1 & 1 & 1 \end{pmatrix}.$$

This up-type quark mass matrix leads to the unitary transformation matrix

$$V_L^u \sim \begin{pmatrix} \lambda^8 & -\lambda^4 & 1 \\ \lambda^4 & 1 & \lambda^4 \\ 1 & -\lambda^4 & \lambda^8 \end{pmatrix},$$

defined by $M_u^{\text{diag}} = (V_L^u)^\dagger M_u (V_R^u)$.

From the $\bar{f}_i$ profiles and the charged lepton mass hierarchy

$$(m_\tau : m_\mu : m_e) \sim (1 : \lambda^2 : \lambda^4),$$

we can obtain the ratios of the l_i^c profiles:

$$(l_1^c, l_2^c, l_3^c) \sim (\lambda^4, \lambda^2, 1).$$

Thus, the charged lepton mass matrix is

$$M_e \sim \begin{pmatrix} \lambda^4 & \lambda^2 & 1 \\ \lambda^4 & \lambda^2 & 1 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix}.$$

The unitary transformation matrix for $M_e = (U_{e_L})^\dagger M_e^{\text{diag}} V_e^R$ can be obtained via the matrix $H = M_e M_e^\dagger$. Thus, the PMNS mixing matrix is given by

$$U_{\text{PMNS}} \sim (U_e^L)^\dagger U_L,$$

which can have tri-maximal (or bi-maximal)-like mixings.

The symmetric down-type quark mass matrix cannot be naively determined from the F_i profile ratios $(\lambda^8, \lambda^4, 1)$ to agree with the observed mass hierarchy:

$$(m_b : m_s : m_d) \sim 1 : \lambda^2 : \lambda^4.$$

In order to obtain realistic down-type quark mass ratios and quark CKM mixings, we introduce an additional discrete symmetry and use the traditional Froggatt-Nielsen mechanism. We consider an Abelian Z_3 flavor symmetry with three one-dimensional representations: a trivial representation $\mathbf{1}$, and two others, $\mathbf{1}'(\omega)$ and $\mathbf{1}''(\omega^2)$ where $\omega^3 = 1$. The representation of F_i in terms of Z_3 is presented in Table 1 .

The effective symmetric Yukawa terms for down-type quarks are

$$W = y_u h \left[S_1 F_1 F_1 + S_2 F_2 F_2 + S_3 F_3 F_3 + S_{12} F_1 F_2 + S_{13} F_1 F_3 + S_{23} F_2 F_3 \right].$$

With the suppression factors

$$\langle S_1 \rangle \sim \lambda^3, \quad \langle S_{12} \rangle \sim \lambda^6,$$

$$\langle S_2 \rangle \sim \lambda^7, \quad \langle S_{13} \rangle \sim \lambda^{12},$$

$$\langle S_3 \rangle \sim \lambda^{10}, \quad \langle S_{23} \rangle \sim \lambda^9,$$

we obtain the following mass matrix for down-type quarks

$$M_d \sim \begin{pmatrix} \lambda^{16} & \lambda^{15} & \lambda^{15} \\ \lambda^{15} & \lambda^{14} & \lambda^{14} \\ \lambda^{15} & \lambda^{14} & \lambda^{12} \end{pmatrix},$$

which leads to the unitary transformation matrix in the down-type quark sector

$$V_L^d \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix},$$

with $M_d^{\text{diag}} = (V_L^d)^\dagger M_d (V_L^d)$. The quark CKM mixing matrix is given by

$$V_{\text{CKM}} = (V_L^u)^\dagger (V_L^d) \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix},$$

which agrees with experimental data.

We know that $m_b : m_t = \lambda^3 : 1$, so if we set $m_t \sim \lambda^9$ and assume approximate $b - \tau$ unification $m_b \sim m_\tau$, we can obtain the profiles

$$(F_1, F_2, F_3) \sim (\lambda^8, \lambda^4, 1),$$

$$(\bar{f}_1, \bar{f}_2, \bar{f}_3) \sim (\lambda^9, \lambda^9, \lambda^9),$$

$$(l_1^c, l_2^c, l_3^c) \sim (\lambda^7, \lambda^5, \lambda^3).$$

Here we also assume that there are appropriate suppression factors for fields that contain h and $\bar{h}$, and then the total factor λ^9 may be absorbed in h and $\bar{h}$ at low energy. From the orbifolding procedure we know that the matter content in each generation arises from different boundary conditions. Using the profiles of F_i, $\bar{f}_i$, and l_i^c, we can easily obtain the bulk masses for various generations which we will not give explicitly here.

Finally, we briefly present another scenario in which the observed SM fermion masses and mixings can also be generated. We assume

$$(F_1, F_2, F_3) \sim (\lambda^7, \lambda^4, 1),$$

$$(\bar{f}_1, \bar{f}_2, \bar{f}_3) \sim (\lambda^{10}, \lambda^9, \lambda^9),$$

$$(l_1^c, l_2^c, l_3^c) \sim (\lambda^6, \lambda^5, \lambda^3),$$

with

$$\langle S_1 \rangle \sim \lambda^2, \quad \langle S_{12} \rangle \sim \lambda^4,$$

$$\langle S_2 \rangle \sim \lambda^6, \quad \langle S_{13} \rangle \sim \lambda^{12},$$

$$\langle S_3 \rangle \sim \lambda^8, \quad \langle S_{23} \rangle \sim \lambda^{10}.$$

From this we obtain that the down-type quark mass matrix is similar to that in Eq. (4.31). The up-type quark mass matrix, the charged lepton mass matrix, and the neutrino mass matrix are

$$M_u \propto \begin{pmatrix} \lambda^8 & \lambda^7 & \lambda^7 \\ \lambda^5 & \lambda^4 & \lambda^4 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \quad M_e \propto \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda \\ \lambda^3 & \lambda^2 & 1 \\ \lambda & 1 & 1 \end{pmatrix},$$

$$M_\nu \propto \begin{pmatrix} \lambda^2 & \lambda & \lambda \\ \lambda & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix}.$$

5. Gauge Mediated Supersymmetry Breaking with Spontaneously R-symmetry Breaking

We know from the previous orbifolding procedure that the five-dimensional $\mathcal{N} = 1$ SUSY, which is $\mathcal{N} = 2$ SUSY in four dimensions, reduces to $\mathcal{N} = 1$ SUSY in four dimensions. We need to further break the remaining $\mathcal{N} = 1$ SUSY and mediate the breaking effects to the SM sector.

In general, the breaking of SUSY requires the presence of R-symmetry [**?**]. However, an exact R-symmetry forbids gaugino masses, which is not acceptable. One possible solution is to explicitly break the R-symmetry by introducing small R-symmetry violation terms which leads to meta-stable vacua [**?**, **?**]. But there is, in general, some tension between acceptable gaugino masses and sufficiently long-lived vacua. The other possibility is to spontaneously break the R-symmetry in O' Raifeartaigh models.

We know that the generalized O' Raifeartaigh model can serve as the low-energy description of dynamical SUSY breaking in strongly coupled gauge theories. It is known that tree-level flat directions (pseudo-moduli) from local SUSY-breaking vacuum always exist in the O'Raifeartaigh framework [**?**, **?**]. In most O' Raifeartaigh models constructed before, the pseudo-moduli, which are charged under R-symmetry, break the R-symmetry by acquiring VEVs through a radiatively generated effective potential. It was shown in [**?**] that the necessary condition to break R-symmetry at one loop via Coleman-Weinberg potential is the existence of a field with R-charge $R \neq 0$ or 2, which is rather complicated to evaluate in detail.

It is, however, possible to spontaneously break R-symmetry by the tree-level VEVs of fields other than pseudo-moduli [**?**, **?**, **?**]. It is shown in [**?**] that a theory of this type with direct gauge mediation leads to vanishing gaugino masses at leading order in F. We want to use the generalized O' Raifeartaigh model in the hidden sector, with spontaneous R-symmetry breaking at tree level, to generate non-vanishing leading order gaugino masses through indirect gauge mediation.

We use a Carpenter-Dine-Festuccia-Mason (CDFM) like model [**?**, **?**, **?**] in the hidden sector to achieve tree-level spontaneous R-symmetry breaking:

$$W = -fX + m_{\psi_2}\tilde{\psi}_2 + m_{\psi_3}\tilde{\psi}_3 + \lambda_2 X\psi_2\tilde{\psi}_3 + \frac{m_2}{2}\psi_2^2.$$

The superpotential contains an R-symmetry with charges

$$R(X) = 2, \quad R(\psi_2) = R(\tilde{\psi}_3) = 1, \quad R(\tilde{\psi}_2) = 1, \quad R(\psi_3) = 3.$$

The tree-level scalar potential is

$$V = |-f + \lambda_2\psi_2\tilde{\psi}_3|^2 + |m_{\psi_2}\tilde{\psi}_2 + \lambda_2 X\tilde{\psi}_3 + 2m_2\psi_2|^2 + |m_{\psi_3} + \lambda_2 X\psi_2|^2.$$

We are interested in SUSY breaking without ψ_i and $\tilde{\psi}_i$ vanishing simultaneously. We can require $F_{\psi_2} = F_{\tilde{\psi}_3} = 0$ simultaneously by properly choosing $\tilde{\psi}_2$ and ψ_3 with arbitrary X. The reduced potential reads

$$V = |-f + \lambda\psi_2\tilde{\psi}_3|^2 + |m\psi_2|^2 + |m\tilde{\psi}_3|^2.$$

The minimum occurs at

$$\tilde{\psi}_3 = \frac{\lambda f - m^2}{\lambda m}, \quad |\psi_2| = |\tilde{\psi}_3|,$$

for $\lambda f > m^2$. The non-zero VEVs can be parameterized as follows:

$$\psi_2 = re^{i\theta}, \quad \tilde{\psi}_3 = re^{-i\theta}, \quad r = \sqrt{\frac{\lambda f - m^2}{\lambda^2}},$$

with the R-Goldstone boson labeled by θ. R-symmetry is broken everywhere in the pseudo-moduli space.

In this case with non-vanishing r, the SUSY breaking can be mediated to the visible sector via the messengers ϕ_i and $\tilde{\phi}_i$. We want to use the two gauge singlets ψ_2 and $\tilde{\psi}_2$ to couple to the messenger sector directly. In the SUSY breaking hidden sector, ψ_2 develops a non-zero VEV in its scalar component while $\tilde{\psi}_2$ gets a non-zero F-term. Their couplings to the messenger sector are

$$\langle\lambda'_{ij}(\psi_2 + \tilde{\psi}_2) + m_{ij}\rangle\phi_i\tilde{\phi}_j = M_{ij}\phi_i\tilde{\phi}_j,$$

where ϕ_i and $\tilde{\phi}_j$ are messenger fields transforming in the $(5, -2)$ and $(\bar{5}, 2)$ representations of flipped SU(5), respectively. We can also introduce additional messengers in $(10, 1)$ and $(\overline{10}, -1)$ representations of flipped SU(5). We use the following form for M_{ij} with $\det\lambda'_{ij} \neq 0$ and $\det m_{ij} = 0$:

$$W = \lambda'(\psi_2 + \tilde{\psi}_2)\sum_i \phi_i \tilde{\phi}_i + m' \sum_j \phi_j \tilde{\phi}_j,$$

with $R(\phi_i) + R(\tilde{\phi}_j) = 2$ in the second term.

The new terms do not spoil the original SUSY breaking vacuum. In terms of the total superpotential, we have

$$F_{\psi_2} = \lambda_2 X \tilde{\psi}_3 + m\tilde{\psi}_2 + 2m_2\psi_2 + \lambda' \sum_i \tilde{\phi}_i,$$

$$F_{\tilde{\psi}2} = m\psi_2 + \lambda' \sum_i \phi_i.$$

With $\phi_i = \tilde{\phi}_i = 0$, the messenger sector will not spoil the SUSY breaking vacua which have $F^*_{\psi_2} \neq 0$ and $F^*_{\tilde{\psi}2} \neq 0$.

In the case of tree-level spontaneous R-symmetry breaking, we parameterize

$$\langle \psi_2 + \tilde{\psi}_2 \rangle = M + \theta^2 F,$$

with

$$M = \sqrt{\frac{\lambda f - m^2}{\lambda^2}}, \quad F = mM.$$

We can use the wave function renormalization technique proposed in [**?**] to calculate the gaugino masses and squark masses if we require $m \ll M$. Then the supersymmetry breaking soft mass terms are

$$m_{\tilde{f}} = 2C_{\tilde{f}} \left(\frac{\alpha_r}{4\pi}\right)^2 \Lambda_S^2, \quad \Lambda_G = \frac{F}{M} \log\det M,$$

$$\Lambda_S^2 = \frac{|F|^2}{\partial_{\psi_2}\partial_{\psi_2^*} \sum_i (\log |M_i^2|)^2}.$$

In our case, the messengers couple to the SUSY breaking fields which in general leads to the non-constant determinant

$$\langle \lambda'_{ij}(\psi_2 + \tilde{\psi}_2) + m_{ij} \rangle = (\psi_2 + \tilde{\psi}_2)^{n_G} g(m', \lambda'),$$

with

$$n_G = \sum_i \left(2 - R(\phi_i) - R(\tilde{\phi}_i)\right),$$

similarly to the case of (extra)ordinary gauge mediation [?]. In our messenger sector with $\det \lambda'_{ij} \neq 0$, we have

$$\langle \lambda'(\psi_2 + \tilde{\psi}_2) + m' \rangle = (\psi_2 + \tilde{\psi}_2)^N \det \lambda'.$$

Thus, as we can see, the gaugino masses at leading order in F are non-vanishing.

On the other hand, it is problematic to have a massless R-Goldstone boson. Fortunately, such a massless mode can become massive through gravitational effects. For example, we can add a constant term W_0 to the original superpotential W_1 to tune the cosmological constant to zero (or to a tiny value). Such a constant term will explicitly break the R-symmetry and then contribute to the R-axion mass. The value of the constant W_0 in the total superpotential $W = W_0 + W_1$ can be determined from the scalar potential in supergravity [?]:

$$V(\phi^\dagger, \phi) = e^{K/M_{\rm Pl}^2}\left[(K^{-1})_i^j \left(W_i + \frac{WK_i}{M_{\rm Pl}^2}\right)\left(W^{*j} + \frac{W^* K^j}{M_{\rm Pl}^2}\right) - 3\frac{|W|^2}{M_{\rm Pl}^2}\right],$$

with the derivatives of the Kähler potential K defined as

$$K_i(\phi^\dagger, \phi) = \frac{\partial K}{\partial \phi^i}, \quad K_i^j = \frac{\partial^2 K}{\partial \phi_j^\dagger \partial \phi^i}.$$

A vanishing cosmological constant term in the scalar potential requires W_0 to be

$$|W_0| \sim \langle W_{1,i} K^{-1}_{ij^*} K^{*j} - 3W_1 \rangle.$$

Then the axion acquires the following mass [?]:

$$m_a^2 \sim \frac{f_a^2}{M_{\rm Pl}^2}\frac{F^2}{M^2},$$

where f_a is the axion coupling

$$f_a^2 = \sum_{i,j} (v_i Q_i)(v_j^* Q_j)\langle K_{ij^*}\rangle \sim M^2.$$

Requiring the axion coupling f_a to lie in the astrophysically and cosmologically allowed window [?]

$$0.5 \times 10^9 \text{ GeV} < f_a \sim M < 2.5 \times 10^{12} \text{ GeV},$$

we can estimate the SUSY breaking scale

$$0.5 \times 10^{14} \text{ GeV}^2 \lesssim F \lesssim 2.5 \times 10^{17} \text{ GeV}^2,$$

with the requirement that the gaugino masses $\alpha_g F/(4\pi M)$ are at the order of TeV.

The axion mass is estimated to lie within 1 GeV to 1 TeV, which may be constrained by cosmological effects similar to moduli fields [?]. In our scenario, the gravitino acquires a mass

$$m_{3/2} \sim \frac{F}{\sqrt{3} M_{\text{Pl}}},$$

with order 10^{-5} GeV $\lesssim m_{3/2} \lesssim 10^{-2}$ GeV and is the LSP.

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6. Gauge Coupling Unification

The bulk gauge symmetry SO(10) is broken down to flipped SU(5) on the O' brane by boundary conditions. We need to break the remaining gauge symmetry further down to the SM gauge group. This step is realized via the antisymmetric Higgs fields H and $\bar{H}$. The Higgs fields can acquire VEVs through the superpotential

$$W = Y(H\bar{H} - v^2),$$

where Y is a SM singlet field. To preserve SUSY, the F-term flatness conditions for the chiral fields Y, H, and $\bar{H}$ give

$$F_Y = H\bar{H} - v^2 = 0,$$

$$F_H = Y\bar{H} - 8\pi\lambda H h = 0,$$

$$F_{\bar{H}} = YH - 8\pi\bar{\lambda}\bar{H}\bar{h} = 0,$$

and then we have $\tan\beta \sim \mathcal{O}(1)$. So we can anticipate that $\langle H\rangle \sim \langle\bar{H}\rangle \sim v \equiv M_{23}/g_{23}$, where g_{23} is the $SU(3)_C \times SU(2)_L$ unified gauge coupling.

There are two possibilities for the mass scale v, which characterizes the breaking of flipped SU(5): large GUT-breaking ($(g_5 v)^2 \gg M_C \equiv 1/R$) and small GUT-breaking ($(g_5 v)^2 \ll M_C$). The large GUT-breaking scenario [**?**, **?**] greatly changes the mass spectra of the gauge bosons that correspond to the broken generators of flipped SU(5). In this case there is no approximate flipped SU(5) unification era for the orbifold zero modes. Thus, we are only interested in the small GUT-breaking scenario in which the flipped SU(5) breaking effects on the brane are negligible. In this case we have an approximate α_2 and α_3 unification era upon M_{23}.

From the missing-partner mechanism, we know that the triplet components of h and $\bar{h}$ are much heavier than the doublet components which will be considered as H_d and H_u, respectively. We assume that the mass scale for the N_F pairs of messengers $(5, -2)$ and $(\bar{5}, 2)$ (and for the N_G pairs of $(10, 1)$ and $(\overline{10}, -1)$) is $M_E \sim M \gg \sqrt{F}$ and is determined by the R-axion constraints to lie between 0.5×10^9 GeV and 2.5×10^{12} GeV. For simplicity, we also assume that the Yukawa couplings among the messenger fields, the SM fermions, and Higgs fields are negligibly small.

In the small GUT-breaking scenario, the gauge couplings α_2 and α_3 unify into SU(5) first. After that, SU(5) unifies with $U(1)_X$ into SO(10). The RGE running of the gauge couplings are

$$\frac{d\alpha_i^{-1}}{d\ln E} = -\frac{b_i}{2\pi},$$

where E is the energy scale and b_i are the beta functions. The running of the gauge couplings for $U(1)_Y$, $SU(2)_L$, and $SU(3)_C$ are given by

$$(b_1, b_2, b_3) = \left(\frac{33}{5}, 1, -3\right) \quad \text{for } M_Z < E < M_S,$$

$$(b_1, b_2, b_3) = \left(\frac{33}{5}, 1, -3\right) \quad \text{for } M_S < E < M_E,$$

$$(b_1, b_2, b_3) = \left(\frac{33}{5}, N_F + 3N_G + 1, N_F + 3N_G - 3\right) \quad \text{for } M_E < E < M_{23}.$$

The gauge coupling of $U(1)_Y$ is normalized to the SU(5) generator: $g_Y^2 = \frac{3}{5} g_5^2$.

In the messenger sector we introduce N_F pairs of $(5, -2)$ and $(\bar{5}, 2)$ as well as N_G pairs of $(10, 1)$ and $(\overline{10}, -1)$ multiplets.

The unification of α_2 and α_3 determines the unification scale M_{23} which is independent of M_E:

$$2\pi[\alpha_2^{-1}(M_Z) - \alpha_3^{-1}(M_Z)] = \ln\left(\frac{M_S}{M_Z}\right)^{23/6}\left(\frac{M_{23}}{M_E}\right)^4\left(\frac{M_{23}}{M_S}\right)^4.$$

After the unification of the α_2 and α_3 couplings, the flipped $SU(5)\times U(1)_X$ gauge group will further unify into SO(10). The $U(1)_Y$ generator is the combination of $U(1)_X$ and the diagonal generator of SU(5). After normalizing $U(1)_Y$ to SU(5), that is $\alpha_Y = 5\alpha_{\rm em}/(3\cos^2\theta_W)$, the relation between the flipped SU(5) gauge couplings and the $U(1)_Y$ gauge coupling at M_{23} can be obtained:

$$\alpha_5^{-1}(M_{23}) = \alpha_Y^{-1}(M_{23}) - \frac{2}{5}\alpha_X^{-1}(M_{23}).$$

Here we normalize the $U(1)_X$ gauge coupling $g_X Q_X$ so that the Q_X charge has a factor $1/\sqrt{40}$ consistently with unification into SO(10). As mentioned before, in orbifold models with kink masses, the lightest KK modes can be as light as $2M\exp(-M\pi R/2)$. We assume that the lightest KK mode is heavier than M_{23}.

The bulk matter multiplets of flipped SU(5) at M_{23} will give (from the **16** and **16′** representations of SO(10)) $N_G + 1$ pairs of chiral fields in the $(10, 1)$ and $(\overline{10}, -1)$ representation (including N_G pairs of messengers); N_F pairs of $(5, -2)$ and $(\bar{5}, 2)$ messenger multiplets (from **10** representation of SO(10)); and $N_f = 3$ families of $((10, 1), (\bar{5}, -3), (1, 5))$ multiplets (from **16** representation of SO(10)) to account for the MSSM matter content.

After integrating out contributions from all the KK modes, the one-loop gauge couplings have the form [**?**]

$$\alpha_a^{-1}(\mu) = \alpha_a^{-1}(M_*) + \frac{b_a}{2\pi}\ln\frac{M_*}{\mu} + \frac{1}{2\pi}\Delta_a,$$

where the cutoff scale $M_* \sim M_U$ is assumed to be large enough compared to other mass parameters of the theory. Here μ is the scale below the lightest massive KK modes but higher than M_{23}, Δ_a are threshold corrections due to massive KK modes while b_a are the 1-loop beta functions due to zero modes. The bare couplings consist of several pieces [**?**]:

$$\alpha_a^{-1}(M_*) = \frac{M_*\pi R}{g_{5a}^2} + \frac{\gamma_a}{2\pi},$$

where γ_a are the coefficients of UV-sensitive linearly divergent corrections. In orbifold GUT which is strongly coupled at M_*, g_{5a}^2 and γ_a are universal. So we have

$$\alpha_5^{-1}(M_*) = \alpha_X^{-1}(M_*).$$

The KK threshold correction Δ_a can be calculated for SU(5) to be

$$\Delta_{\mathrm{SU}(5)} = 5\ln\left(\frac{Z_1^{10}Z_2^{10}Z_3^{10}}{Z_1^{\bar{5}}Z_2^{\bar{5}}Z_3^{\bar{5}}}\right)+\ln\left(\frac{Z_1^m\cdots Z_{2N_F}^m}{Z_1^n\cdots Z_{2N_G}^n}\right)+\frac{\pi R}{2}\sum_{i=1}^{N_f}(M_{10i}+M_{5i}+M_{1i})+\frac{\pi R}{2}\sum_{i=1}^{2N_F}M_{mi}+\frac{\pi R}{2}\sum_{i=1}^{2N_G}M_{ni},$$

while for $U(1)_X$ they are

$$\Delta_{U(1)_X} = \ln(Z_1^{10}Z_2^{10}Z_3^{10})+\ln(Z_1^{\bar{5}}Z_2^{\bar{5}}Z_3^{\bar{5}})+\ln(Z_1^1Z_2^1Z_3^1)+\ln\left(\frac{Z_1^m\cdots Z_{2N_F}^m}{Z_1^n\cdots Z_{2N_G}^n}\right)+\frac{\pi R}{2}\sum_{i=1}^{N_f}(M_{10i}+M_{5i}+M_{1i})+\frac{\pi R}{2}\sum_{i=1}^{2N_F} M$$

Here $Z(M)$ is the profile suppression factor which appears in Eq. (4.7). The various profiles can be deduced from the hierarchy in Section 3.

The zero mode contributions to the SU(5) and $U(1)_X$ beta functions above M_{23} are calculated as

$$(b_5, b_X) = (N_F + 3N_G - 5, N_F + 3N_G + 1).$$

Combining the previous expressions and the RGE running to M_{23}, we can obtain in our model the relation of the gauge couplings at M_{23}:

$$2\pi(\alpha_5^{-1}-\alpha_X^{-1})(M_{23}) = 5\ln\frac{M_*}{M_{23}}-\frac{1}{2}\ln(Z_1^{10}Z_2^{10}Z_3^{10})-\frac{1}{2}\ln(Z_1^{\bar{5}}Z_2^{\bar{5}}Z_3^{\bar{5}})-\frac{1}{2}\ln(Z_1^1Z_2^1Z_3^1)+\frac{1}{2}\ln(Z_1^m\cdots Z_{2N_F}^m)-\frac{1}{2}\ln(Z_1^n$$

It is interesting to note that in our case when $N_G = 0$ with N_F messenger fields $(5, -2)$ and $(\bar{5}, 2)$, the cutoff (strongly coupled unification) scale of the theory is independent of the messenger profiles. Substituting the various profiles into the above expression, we obtain

$$2\pi(\alpha_5^{-1} - \alpha_X^{-1})(M_{23}) = -\frac{1}{2}\ln(\lambda^{12}) - \frac{1}{2}\ln(\lambda^{27}) - \frac{1}{2}\ln(\lambda^{15}) + 5\ln\frac{M_*}{M_{23}} + 17.034.$$

Our weak scale inputs [?]

$$M_Z = 91.1876 \pm 0.0021 \text{ GeV}, \quad \sin^2\theta_W(M_Z) = 0.2312 \pm 0.0002,$$

$$\alpha_{\mathrm{em}}^{-1}(M_Z) = 127.906 \pm 0.019, \quad \alpha_3(M_Z) = 0.1187 \pm 0.0020,$$

fix the numerical values of the standard $U(1)_Y$ and $SU(2)_L$ couplings at the weak scale:

$$\alpha_1^{-1}(M_Z) = \frac{5\alpha_{\rm em}^{-1}(M_Z)}{3\cos^2\theta_W} = (59.00048)^{-1},$$

$$\alpha_2^{-1}(M_Z) = \frac{\alpha_{\rm em}^{-1}(M_Z)}{\sin^2\theta_W} = (29.5718)^{-1}.$$

The unification scale M_{23} can be determined after we set the soft SUSY breaking mass scale M_S. For example, we can choose $M_S = 600$ GeV and obtain

$$M_{23} = 2.633 \times 10^{16} \text{ GeV}.$$

We present the RGE running of the various gauge couplings below M_{23} in Fig. 1 [Figure 1: see original paper] for $N_G = 0$ and $N_G = 2$, respectively. In addition, we present the strongly coupled unification scales from our numerical calculations for $N_G = 0$ in Table 2 . These results are independent of the messenger scale M_E and the messenger numbers N_F.

In this scenario with N_F pairs of $(5, -2)$ and $(\bar{5}, 2)$ messengers, the strongly coupled unification is possible due to the threshold contributions of the bulk matter profiles. The unification of flipped SU(5) into SO(10) is not possible with such a choice of messengers in four dimensions or in orbifold models without kink mass terms.

If we adopt a nonzero N_G and set the profile for $(10, 1)$ and $(\overline{10}, -1)$ to be $\mathcal{O}(1)$, we can get

$$2\pi(\alpha_5^{-1} - \alpha_X^{-1})(M_{23}) = (N_G - 5)\ln\frac{M_*}{M_{23}} + 5\ln\frac{M_*}{M_{23}} - \frac{1}{2}\ln(\lambda^{12}) - \frac{1}{2}\ln(\lambda^{27}) - \frac{1}{2}\ln(\lambda^{15}) + \frac{1}{2}\ln(Z_1^n \cdots Z_{2N_G}^n) + 17.034,$$

with the last step obtained by taking $Z_i^n = 1$. The numerical results for the strongly coupled unification scale and non-zero N_G are given in Table 3 . In fact, it is more advantageous to choose the case with $N_G \neq 0$ not only because it can realize successful unification in four dimensions and ordinary orbifold models without kink mass terms, but also because it can satisfy the consistency requirements that the strongly coupled unification scale M_U is much higher than $M_C \equiv \pi R/2$.

7. Proton Decay

One of the unique GUT predictions is proton decay. There are several sources in SUSY GUT models: (i) the conventional lepto-quark vector gauge boson exchange which leads to dimension-six baryon number violating operators; (ii) new contributions from supersymmetry.

The dominant new contribution in SUSY GUTs comes from the F-type dimension-five baryon number violating operators

$$\mathcal{O}_{\Delta B\neq 0} = \frac{1}{M_T}\epsilon_{ijk}\epsilon_{\alpha\beta}Q_i^\alpha Q_j^\beta \tilde{L}_k,$$

which can arise from triplet Higgsino exchange in the presence of a triplet Higgsino mass insertion term $M_T\tilde{H}_T\tilde{\bar{H}}_T$. Although this operator cannot induce proton decay at the lowest order because it is composed of squarks and sleptons, it can cause proton decay once gaugino loops are included. Thus, we anticipate a proton lifetime $\tau_P \sim (M_T/M_{\rm SUSY})^2$ which may not be consistent with the unification scale and then cause a problem. In the previous discussions we pointed out that the D-T splitting problem in SUSY GUTs is intimately related to the dimension-five proton decay problem.

In flipped SU(5), the problem of D-T splitting can be naturally solved via the elegant missing partner mechanism. In particular, the mixing term between the triplet Higgsinos is absent due to R-symmetry, thus it will not cause proton decay.

The direct μ-term $\mu\bar{h}h$ is forbidden by the R-symmetry because of the following reason. From the superpotential we have

$$R(HHh) + R(\bar{H}\bar{H}\bar{h}) = R(\bar{h}h) + 2R(H\bar{H}) = 4.$$

The superpotential terms where H and $\bar{H}$ acquire VEVs indicate that $R(\bar{H}H) = 0$, which means $R(\bar{h}h) = 4$. It is obvious that such a μ-term is prohibited by R-symmetry. An effective μ-term can be generated through the Giudice-Masiero mechanism [**?**] by introducing some gauge singlets Z with R-charge 4. The effective Kähler potential is

$$\mathcal{K} \sim \frac{1}{\Lambda^2}Z^\dagger h\bar{h} + \text{h.c.} + \frac{1}{\Lambda^2}Z^\dagger Z h^\dagger h + \frac{1}{\Lambda^2}Z^\dagger Z\bar{h}^\dagger\bar{h} + \cdots,$$

while the $B\mu$-term $Z^\dagger Z h\bar{h}/\Lambda^2$ is forbidden in the potential. After the singlet Z gets a VEV

$$\langle Z\rangle = Z_0 + \theta^2 F_Z,$$

which breaks SUSY and R-symmetry, an effective μ-term can be generated: $\mu \sim F_Z/\Lambda$. Although the $B\mu$-term is forbidden by R-symmetry, such a term can arise from gaugino loops and can be naturally small compared to the μ-term. The possible UV completion, which gives the interaction between the singlet Z and the hidden SUSY breaking sector, is rather complicated. Thus, for simplicity we will not present a realistic model here. The small effective μ-term will not reintroduce the proton decay problem since the decay process will have an additional suppression factor $(\mu/M_H)^2$.

We can impose R-parity to forbid dimension-four proton decay interactions. Additional interactions leading to dangerous dimension-five operators, besides those by heavy Higgsino exchange, can be introduced on the gauge symmetry breaking O' brane as follows:

$$W \sim [\delta(y - \pi R/2) + \delta(y + \pi R/2)] \frac{(\psi_2)^2}{M_*^3} \lambda_{abcd} F_a \bar{f}_b l_c^c,$$

after ψ_2 acquires a VEV. Here a, b, c, d are family indices and the R-charge of the gauge singlets is $R(\psi_2) = 1$. It corresponds to an effective dimension-five operator suppressed by $M_{\rm Pl}/M^2 \sim 10^{30}$ GeV. Such operators will certainly not violate the current proton decay lower bound.

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8. Conclusions

We proposed a realistic flipped SU(5) model from an orbifolded SO(10) model. The SM fermion masses and mixings were obtained via the traditional Froggatt-Nielsen mechanism and the five-dimensional wave function profiles of the SM fermions. The breaking of $\mathcal{N} = 1$ supersymmetry after orbifolding was realized via tree-level spontaneous R-symmetry breaking in the hidden sector and extra(ordinary) gauge mediation. We generated realistic SUSY breaking soft mass terms with non-vanishing gaugino masses. In addition, we studied gauge coupling unification in detail by including the messenger fields at the intermediate scale and the KK states at the compactification scale. We found that the SO(10) unified gauge coupling is very strong and the unification scale can be much higher than the compactification scale. Finally, we briefly commented on proton decay.

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