Constraints on Cosmic Distance Duality Relation from Cosmological Observations Postprint

Authors: Meng-Zhen Lv, Jun-Qing Xia

Date: 2016-09-06T00:00:00+00:00

Abstract

In this paper, we use the model-dependent method to revisit the constraint on the well-known cosmic distance duality relation (CDDR). By using the latest SNIa samples, such as Union2.1, JLA and SNLS, we find that the SNIa data alone cannot constrain the cosmic opacity parameter ε, which denotes the deviation from the CDDR, $d_L = d_A(1+z)^{2+\varepsilon}$, very effectively. The constraining power on ε from the luminosity distance indicator provided by SNIa and GRB can hardly be improved at present. When we include other cosmological observations, such as the measurements of Hubble parameter, the baryon acoustic oscillations and the distance information from cosmic microwave background, we obtain the tightest constraint on the cosmic opacity parameter ε, namely the 68% C.L. limit: $\varepsilon = 0.023 \pm 0.018$. Furthermore, we also consider the evolution of ε as a function of z using two methods, the parametrization and the principal component analysis, and find no evidence for the deviation from zero. Finally, we simulate the future SNIa and Hubble measurements and find the mock data could provide very tight constraints on the cosmic opacity ε and verify the CDDR at high significance.

Full Text

Preamble

Constraints on Cosmic Distance Duality Relation from Cosmological Observations

Meng-Zhen Lva and Jun-Qing Xiab,∗

aKey Laboratory of Particle Astrophysics, Institute of High Energy Physics, Chinese Academy of Sciences, P.O. Box 918-3, Beijing 100049, China
bDepartment of Astronomy, Beijing Normal University, Beijing 100875, China

In this paper, we use a model-dependent method to revisit constraints on the well-known cosmic distance duality relation (CDDR). Using the latest SNIa samples—Union2.1, JLA, and SNLS—we find that SNIa data alone cannot constrain the cosmic opacity parameter , which denotes the deviation from the CDDR, $d_L = d_A(1+z)^{2+\varepsilon}$, very well. The constraining power on from the luminosity distance indicator provided by SNIa and GRB is difficult to improve at present. When we include other cosmological observations, such as measurements of the Hubble parameter, baryon acoustic oscillations, and distance information from the cosmic microwave background, we obtain the tightest constraint on the cosmic opacity parameter , namely the 68% C.L. limit: $\varepsilon = 0.023 \pm 0.018$. Furthermore, we also consider the evolution of as a function of z using two methods—parametrization and principal component analysis—and find no evidence for deviation from zero. Finally, we simulate future SNIa and Hubble measurements and find that mock data could provide very tight constraints on the cosmic opacity and verify the CDDR at high significance.

Introduction

In 1998, analyses of the redshift-distance relation of type Ia supernovae (SNIa) at low redshift $z < 2$ demonstrated that the Universe is undergoing an accelerated phase of expansion [**?**, **?**]. Currently, cosmological observations have provided tight constraints on distance measures: the luminosity distance d_L from SNIa measurements and the angular diameter distance d_A from baryon acoustic oscillation (BAO) measurements, which can be used to constrain different cosmological parameters in various theoretical models [**?**]. In general, the luminosity distance and angular diameter distance should satisfy the well-known cosmic distance duality relation (CDDR): $d_L = d_A(1+z)^2$.

This relation, also called the "Etherington relation" in the literature, holds only under three fundamental conditions: (i) spacetime is described by a metric theory of gravity; (ii) photons travel along unique null geodesics; (iii) the number of photons is conserved. Therefore, any departure from these conditions—such as deviation from a metric theory of gravity, photons not traveling along null geodesics, or variation in photon number—would reveal new physics beyond the standard model.

To test the CDDR, a model-independent method has been widely used in which researchers combine current datasets of d_L from SNIa or Gamma-Ray Burst (GRB) measurements with d_A from BAO or X-ray measurements at the same redshift to constrain the parameter $\eta = d_L/[d_A(1+z)^2]$ (e.g., see refs. \cite{4-16} and references therein). If η obtained from the d_L and d_A datasets differs from unity, the CDDR is violated. Recently, Ref. [**?**] used a new compilation of strong lensing systems to extract d_A information and obtained the constraint $\eta = -0.004^{+0.322}_{-0.210}$ (68% C.L.), together with the "Joint Luminosity Analyses" (JLA) compilation of SNIa [**?**].

Apparently, this method for testing the CDDR is conservative and independent of the underlying cosmological model. However, the major problem is that current observations cannot provide luminosity distance d_L and angular diameter distance d_A for the same astronomical target simultaneously. Therefore, researchers must use d_A information from galaxy cluster observations and d_L from SNIa measurements at similar redshifts, which inevitably introduces large numerical errors in the determination of η.

On the other hand, there is another model-dependent method to study this relation. The CDDR is deeply connected with cosmic opacity [?, ?]. A variation in photon number during propagation toward us, which could be caused by simple astrophysical effects like interstellar dust, gas, and/or plasmas, or by exotic physics beyond the standard model, would affect SNIa luminosity distance measurements but not angular diameter distance determinations in a certain underlying cosmological framework, consequently modifying the CDDR. Assuming $\tau(z)$ denotes the cosmic opacity between an observer at $z = 0$ and a source at z, the flux received from the source would be attenuated by a factor $e^{-\tau(z)}$. Then the observed luminosity distance relates to the true luminosity distance as $d_{L,\mathrm{obs}}(z) = d_{L,\mathrm{true}}(z) \exp[\tau(z)/2]$, because intensity is inversely proportional to the square of the distance between source and observer. Ref. [?] introduced a parameter ε to study deviations from the Etherington relation of the form $d_L(z) = d_A(z)(1 + z)^{2+\varepsilon}$, where ε denotes the departure from transparency. Considering the small value of ε at low redshift, this is equivalent to assuming an optical depth parameterization $\tau(z) = 2\varepsilon z$.

The advantage of this method is that we can use high-precision d_L measurements to constrain cosmic opacity while avoiding large uncertainties from galaxy cluster measurements. Currently, the tightest constraint comes from combining the "Union2 Compilation" SNIa sample with Hubble parameter measurements $H(z)$: $\varepsilon = -0.01^{+0.08}_{-0.09}$ (95% C.L.) [?]. Until now, all measurements satisfy the CDDR at 68% confidence level.

In this paper, we focus on the model-dependent method to verify the CDDR and update constraints on cosmic opacity using the latest SNIa samples: "Union2.1 Compilation" (Union2.1) [?], "Joint Luminosity Analyses" (JLA) [?], and "Supernovae Legacy Survey" (SNLS) [?], along with Hubble parameter $H(z)$ measurements. Furthermore, we include GRB, BAO, and cosmic microwave background (CMB) distance information in our analyses to improve constraints on cosmic opacity. The paper is organized as follows: In Section II, we introduce the datasets used in our analyses. We present numerical results in Section III. Finally, Section IV provides conclusions and discussion.

II. Current Datasets

In our calculations, we rely on the following observational datasets: (i) SNIa and GRB distance moduli; (ii) Hubble parameter determinations; (iii) BAO in galaxy power spectra; (iv) CMB distance information.

A. Type-Ia Supernovae & Gamma-Ray Bursts

SNIa distance moduli provide the luminosity distance as a function of redshift z. In this paper, we use the latest Union2.1 compilation of 580 SNIa from the Hubble Space Telescope Supernova Cosmology Project [?]. The data are presented as tabulated distance moduli with errors. In this catalog, the redshift spans $0 < z < 1.414$, with approximately 95% of samples in the low-redshift region $z < 1$. The authors also provide covariance matrices with and without systematic errors. To be conservative, we include systematic errors in our calculations.

For comparison, we also consider two additional SNIa datasets: (1) 472 samples from the first three years of the SuperNova Legacy Survey (SNLS) program (123 low-z, 93 SDSS, 242 SNLS, and 14 Hubble Space Telescope) at $0.01 < z < 1.4$ [?]; (2) 740 samples from the SDSS-II/SNLS3 Joint Light-curve Analysis (JLA) at redshifts up to 1.30, including several low-redshift samples ($z < 0.1$), all three seasons from SDSS-II ($0.05 < z < 0.4$), and three years from SNLS ($0.2 < z < 1$) [?].

These two compilations differ from Union2.1 in three major aspects: (1) The two supernova nuisance parameters α and β from light-curve calibration are handled correctly rather than held fixed at their best-fit values; (2) They provide covariance between light-curve fits; (3) The luminosity distance accounts for differences between CMB frame and heliocentric frame redshifts, which is important for some nearby supernovae. Furthermore, the JLA compilation includes intrinsic dispersion and gravitational lensing effects in supernova magnitudes, while SNLS does not.

We also consider another luminosity distance indicator provided by GRBs, which can potentially measure luminosity distances to higher redshifts than SNIa. GRBs are not standard candles since their isotropic equivalent energetics and luminosities span 3–4 orders of magnitude. However, similar to SNIa, correlations between various properties of prompt emission and afterglow emission have been proposed to standardize GRB energetics (e.g., Ref. [?]). Recently, several empirical correlations between GRB observables have been reported, triggering intensive studies on using GRBs as cosmological "standard" candles. However, due to the lack of low-redshift long GRB data to calibrate these relations in a cosmology-independent way, the parameters of reported correlations are given assuming an input cosmology and obviously depend on the same cosmological parameters we aim to constrain. Thus, applying such relations to constrain cosmological parameters leads to biased results. Ref. [?] naturally eliminates this "circular problem" by marginalizing over free parameters involved in the correlations; additionally, some results show these correlations do not change significantly for a wide range of cosmological parameters [?, ?]. Therefore, we use the 69 GRBs over a redshift range $z \in [0.17, 6.60]$ presented in Ref. [?], while accounting in our statistical analysis for issues related to the circular problem discussed more extensively in Ref. [?] and for the fact that all correlations used to standardize GRBs have scatter and poorly understood physics.

In calculating the likelihood from SNIa and GRBs, we marginalize over the absolute magnitude M, a nuisance parameter, following Refs. [?, ?]:

$$\bar{\chi}^2 = A - \frac{B^2}{C} + \ln\left(\frac{C}{2\pi}\right)$$

where $A = \sum_i (\mu^i_{\rm data} - \mu^i_{\rm th})^2/\sigma_i^2$, $B = \sum_i (\mu^i_{\rm data} - \mu^i_{\rm th})/\sigma_i^2$, and $C = \sum_i 1/\sigma_i^2$.

B. Hubble Measurements

Hubble parameter measurements can serve as a complementary probe for constraining cosmological parameters. The Hubble parameter characterizes the expansion rate of our Universe at different redshifts and depends on the differential age of the Universe as a function of redshift:

$$H(z) = -\frac{1}{1+z}\frac{dz}{dt}$$

Therefore, measuring dz/dt can directly estimate $H(z)$, first proposed by Ref. [?]. They selected samples of passively evolving galaxies with high-quality spectroscopy and used stellar population models to constrain the age of the oldest stars in these galaxies. After computing differential ages at different redshifts, they obtained determinations of the Hubble parameter [?, ?].

The Hubble parameter can also be obtained from BAO measurements. By observing the typical acoustic scale in the line-of-sight direction, one can extract the expansion rate of the Universe at certain redshifts. Ref. [?] analyzed Hubble parameter information at redshifts $z = 0.24$ and $z = 0.43$ from Sloan Digital Sky Survey (SDSS) DR6 and DR7 data. Recently, these $H(z)$ data have been widely used to determine cosmological parameters, such as the effective number of neutrinos [?, ?], the equation of state of dark energy \cite{37-39}, cosmography scenarios [?, ?], and modified gravity models \cite{42-44}.

In Table I we adopt 25 Hubble parameter measurements used in Ref. [?]. Additionally, we include the direct measurement of the current Hubble constant H_0 obtained from the re-analysis of Ref. [?] using a revised geometric maser distance to NGC 4258 from Ref. [?]:

$$H_0 = (70.6 \pm 3.3)\ \text{km s}^{-1}\text{Mpc}^{-1} \quad [?]$$

C. BAO

BAO provides an efficient method for measuring the expansion history by using features in the clustering of galaxies within large-scale surveys as a ruler to measure the distance-redshift relation. Since current BAO data are not accurate enough, one can only determine an effective distance:

$$D_V(z) = \left[(1+z)^2 D_A^2(z) \frac{cz}{H(z)}\right]^{1/3}$$

In this paper, we use BAO measurements of $r_{\rm drag}/D_V(z)$ from the 6dF Galaxy Redshift Survey (6dFGRS) at low redshift ($z = 0.106$) [?], the $D_V/r_{\rm drag}(z)$ measurement of the BAO scale based on a re-analysis of the Luminous Red Galaxy (LRG) sample from SDSS Data Release 7 at median redshift ($z = 0.35$) [?], the BAO signal of $D_A(z)$ and $H(z)$ from BOSS CMASS DR9 data at redshift ($z = 0.57$) [?], the $D_V/r_{\rm drag}(z)$ BAO measurement from the WiggleZ survey at $z = 0.44$, $z = 0.60$, and $z = 0.73$ [?], and the latest BAO measurement of $D_A(z)$ and $H(z)$ at high redshift $z = 2.34$ from the analysis of Ly- forest of BOSS quasars [?].

D. CMB Distance Information

CMB measurements are sensitive to the distance to the decoupling epoch via the locations of peaks and troughs of acoustic oscillations. Here we use the "distance information," following the Planck measurement [?], which includes the "shift parameter" $\mathcal{R}$, the "acoustic scale" l_A, and the photon decoupling epoch z_*. $\mathcal{R}$ and l_A correspond to the ratio of angular diameter distance to the decoupling era over the Hubble horizon and the sound horizon at decoupling, respectively:

$$\mathcal{R} = \sqrt{\Omega_m H_0^2} \frac{\chi(z_*)}{c}, \quad l_A = \pi \frac{\chi(z_*)}{\chi_s(z_*)}$$

where $\chi(z_*)$ and $\chi_s(z_*)$ denote the comoving distance to z_* and the comoving sound horizon at z_*, respectively. The decoupling epoch z_* is given by Ref. [?]:

$$z_* = 1048 \left[1 + 0.00124(\Omega_b h^2)^{-0.738}\right] \left[1 + g_1(\Omega_m h^2)^{g_2}\right]$$

with

$$g_1 = \frac{0.0783(\Omega_b h^2)^{-0.238}}{1 + 39.5(\Omega_b h^2)^{0.763}}, \quad g_2 = \frac{1 + 21.1(\Omega_b h^2)^{1.81}}{1 + 21.1(\Omega_b h^2)^{1.81}}$$

We calculate the likelihood of CMB distance information as:

$$\chi^2 = (x^i_{\rm th} - x^i_{\rm data})(C^{-1})_{ij}(x^j_{\rm th} - x^j_{\rm data})$$

where $x = (\mathcal{R}, l_A, z_*)$ is the parameter vector and $(C^{-1})_{ij}$ is the inverse covariance matrix for CMB distance information.

III. Numerical Results

In our analysis, we perform a global fit using the COSMOMC package [?], a Monte Carlo Markov Chain (MCMC) code. Besides the cosmic opacity parameter ε, we vary the following cosmological parameters with top-hat priors: dark matter energy density $\Omega_c h^2$, baryon energy density $\Omega_b h^2$, Hubble parameter h, and constant dark energy equation of state w. For JLA and SNLS datasets, we include two additional nuisance parameters α and β from light-curve calibration.

A. Constant   from Various Datasets

The latest constraint on the cosmic opacity parameter ε comes from combining the "Union2 Compilation" SNIa sample with Hubble parameter measurements $H(z)$ in Ref. [?]. It is worth revisiting constraints on ε from recent SNIa and Hubble parameter measurements, along with other useful probes like GRBs, BAO, and CMB distance information.

First, we use the latest SNIa datasets—Union2.1, JLA, and SNLS—to obtain limits on ε. In Figure 1 [Figure 1: see original paper], we show one-dimensional constraints on ε and Ω_m from Union2.1 data alone, as well as the two-dimensional contour between them. The constraint on ε is very weak: $\varepsilon = 0.11 \pm 0.17$ (68% C.L.) and does not improve significantly even with the latest SNIa sample, compared to Ref. [?]. However, this constraint differs slightly from that result when using SNIa data alone. We also use the old Union2008 SNIa data alone to constrain opacity and obtain a similar result. Therefore, this difference might be due to different fitting methods, parameterizations, or treatments of Union data systematics in the calculations.

In the two-dimensional contour of Figure 1, there is a strong positive correlation between ε and Ω_m. The reason is that larger ε values correspond to more flux received from the source. Therefore, supernovae are not as faint as expected from a matter-dominated Universe, meaning an accelerating Universe might not be necessary. When including ε, the constraint on Ω_m is significantly enlarged, as shown in the one-dimensional distribution of Ω_m in Figure 1. Assuming a transparent Universe, the constraint on Ω_m shrinks dramatically to $\Omega_m = 0.30 \pm 0.04$ (68% C.L.) from Union2.1 data alone.

Besides the Union2.1 sample, the JLA and SNLS SNIa samples have been widely used in recent literature. Therefore, we also use JLA and SNLS data to constrain ε individually. We obtain similar constraints at 68% confidence level: $\varepsilon = 0.17 \pm 0.18$ and $\varepsilon = 0.19 \pm 0.19$ from JLA and SNLS data, respectively. This result implies that the constraining power on ε from the luminosity distance indicator provided by SNIa is difficult to improve at present, due to the strong correlation with Ω_m. In Figure 2 [Figure 2: see original paper], we show two-dimensional contours between ε and Ω_m (red contours) from Union2.1, JLA, and SNLS, respectively.

To improve constraints on ε, we also include GRB data. GRBs are another luminosity distance indicator that can potentially measure luminosity distances to higher redshifts than SNIa. In Figure 2, we show the two-dimensional contour in the (ε, Ω_m) plane from the GRB sample alone (magenta contours). Since GRBs are also luminosity distance indicators, their constraining power on ε is limited as well, due to the strong correlation between ε and Ω_m. However, because of the high redshift of GRB samples, the allowed region of large ε values is clearly reduced: $\varepsilon = 0.03 \pm 0.13$ (68% C.L.), and the significance of models with large ε is suppressed. Furthermore, the direction of correlation between ε and Ω_m differs from that obtained from SNIa data alone. When using Union2.1 and GRB data together, the constraint on ε improves slightly: $\varepsilon = 0.003 \pm 0.12$ (68% C.L.). Note that the GRB sample suffers from the "circular problem" when using GRBs as cosmological standard rulers. We account in our statistical analysis for issues related to the circular problem discussed more extensively in Ref. [?].

The Hubble parameter as a function of redshift, $H(z)$, is a useful measurement employed in many works. Here, we use this dataset to study the cosmic opacity parameter ε. Since $H(z)$ data provide good constraints on Ω_m and there is a strong positive correlation between ε and Ω_m, $H(z)$ data can also significantly improve constraints on ε indirectly. In Figure 2, we show constraints from $H(z)$ data (green contours), which apparently have no direct constraint on ε. However, when combining $H(z)$ data with SNIa or GRB datasets, the constraint on ε is dramatically reduced: the 68% C.L. constraint is $\varepsilon = -0.008 \pm 0.048$ from Union2.1+GRB+$H(z)$ data. This limit is similar to that obtained from Union2+$H(z)$ data in Ref. [?]. We also show constraints from JLA+GRB+$H(z)$ and SNLS+GRB+$H(z)$ data combinations in the other two panels of Figure 2, respectively. The constraints on ε are quite similar, with JLA+GRB+$H(z)$ giving a slightly better constraint, while SNLS+GRB+$H(z)$ gives a slightly worse one.

Finally, we include BAO and CMB distance information to narrow constraints on Ω_m and consequently improve limits on the cosmic opacity parameter ε. First, using BAO measurements together with Union2.1, GRB, and $H(z)$ data to constrain ε, we find the constraint improves significantly: $\varepsilon = 0.009 \pm 0.024$ (1). This constraint is tighter than that obtained in Ref. [?], due to several latest precise BAO measurements we employ. Then we replace BAO measurements with CMB distance information in the calculation. The CMB distance information contains information from the early Universe at $z \sim 1100$, clearly different from other probes at $z \lesssim 2.5$, but it provides tighter constraints on the matter energy density, indirectly affecting the study of cosmic opacity. Using CMB distance information from Planck measurements, we obtain an even tighter constraint than from BAO: $\varepsilon = 0.025 \pm 0.020$ (1). When combining all these data together, due to the strong constraining power on Ω_m from BAO and CMB data, the constraint on ε improves significantly: $\varepsilon = 0.023 \pm 0.018$ (68% C.L.).

Compared with constraints in Ref. [?], the statistical error bar on ε has shrunk by a factor of 2, due to new BAO and CMB distance information in our calculations. Meanwhile, since the new BAO and CMB data favor larger values of Ω_m [?], the combined dataset slightly favors positive ε, due to the positive correlation between ε and Ω_m. However, the significance level is only about 1.2 .

More importantly, besides correlation with Ω_m, ε is also strongly correlated with the dark energy equation of state w. In Figure 3 [Figure 3: see original paper], we show one-dimensional distributions of w and ε and two-dimensional contours between w and ε from current data combinations. There is a positive correlation between w and ε from the Union2.1+GRB+$H(z)$ data combination. The reason is that larger ε values correspond to more flux received from the source, making supernovae brighter than expected from the standard ΛCDM Universe with $w = -1$. Consequently, large values of w are favored by the data. The constraint on ε is slightly weaker than discussed above, $\varepsilon = -0.016 \pm 0.053$ (68% C.L.), due to this degeneracy. We also obtain constraints on w, as shown in Figure 3: $w = -1.12 \pm 0.20$ and $w = -1.10 \pm 0.19$ for models with free ε and with $\varepsilon = 0$, respectively.

Furthermore, including more precise BAO and CMB data, we find this positive correlation between ε and w becomes much stronger, as shown by the blue contours in Figure 3. Compared with Eq. (12), the limit on ε becomes significantly weaker: $\varepsilon = 0.009 \pm 0.031$ (68% C.L.). Meanwhile, constraints on w also differ: at 68% confidence level, $w = -1.04 \pm 0.07$ and $w = -1.05 \pm 0.04$ for models with free ε and with $\varepsilon = 0$, respectively.

B. Parametrization & PCA

In the above subsection, we considered only a constant cosmic opacity parameter ε. However, ε should generally vary as a function of redshift during cosmic evolution. Therefore, following the parametrization approach in dark energy studies, we parameterize ε in three forms [?, ?]:

- $\varepsilon = \varepsilon_{z0} + \varepsilon_z z$ (z-expansion)
- $\varepsilon = \varepsilon_{a0} + \varepsilon_a (1 - a)$ (a-expansion)
- $\varepsilon = \varepsilon_{\ln 0} + \varepsilon_{\ln} \ln(1 + z)$ (ln-expansion)

Using Union2.1, GRB, $H(z)$, BAO, and CMB data together (all data), we obtain constraints on the free parameters in these three parametrization forms, listed in Table II.

First, we see that the obtained median values of ε_{i0} in the three parametrization forms are similar and all consistent with zero, with 68% C.L. limits $\varepsilon_{z0} = 0.030 \pm 0.024$, $\varepsilon_{a0} = 0.052 \pm 0.051$, and $\varepsilon_{\ln 0} = 0.041 \pm 0.036$, respectively. Regardless of parametrization form, ε_{i0} always denotes the current value of cosmic opacity at $z = 0$.

On the other hand, constraints on ε_i for the three parametrization forms differ considerably in both median values and statistical error bars. Combining constraints on ε_{i0} and ε_i, we can plot the evolution of cosmic opacity ε as a function of z and its statistical error bars $\Delta\varepsilon(z)$:

$$\Delta\varepsilon(z) = \sqrt{(\Delta\varepsilon_{i0})^2 + 2\Delta\varepsilon_{i0}\Delta\varepsilon_i f_i \mathrm{cov}(\varepsilon_{i0}, \varepsilon_i) + (\Delta\varepsilon_i f_i)^2}$$

where $\Delta\varepsilon_{i0}$ and $\Delta\varepsilon_i$ denote the obtained 1 statistical error bars on ε_{i0} and ε_i, respectively, $\mathrm{cov}(\varepsilon_{i0}, \varepsilon_i)$ is the correlation between ε_{i0} and ε_i, and f_i denotes z, $(1-a)$, and $\ln(1+z)$ for the three parametrization forms, respectively. We obtain similar evolutions of $\varepsilon(z)$, which are consistent with zero in the redshift region $z \in [0, 5]$ for all three parametrization forms, as shown in Figure 4 [Figure 4: see original paper].

Since current data are not accurate enough, it is worthwhile discussing constraints on cosmic opacity ε from future measurements. Here, we use future SNIa observations from WFIRST and $H(z)$ measurements from the BOSS experiment (similar analyses using Euclid measurements can be found in Refs. [**?**, **?**]). The fiducial models are taken from the best-fit values of the all-data combination in the ΛCDM framework with $\varepsilon = 0$.

According to the updated report by the Science Definition Team [**?**], we obtain 2725 SNIa over the region $0.1 < z < 1.7$ with redshift bin $\Delta z = 0.1$. The photometric measurement error per supernova is $\sigma_{\text{meas}} = 0.08$ magnitudes. The intrinsic dispersion in luminosity is assumed to be $\sigma_{\text{int}} = 0.09$ magnitudes (after correction/matching for light-curve shape and spectral properties). Another contribution to statistical errors is gravitational lensing magnification, $\sigma_{\text{lens}} = 0.07 \times z$ mags. The overall statistical error in each redshift bin is then:

$$\sigma_{\text{stat}} = \sqrt{\frac{(\sigma_{\text{meas}})^2 + (\sigma_{\text{int}})^2 + (\sigma_{\text{lens}})^2}{N_i}}$$

where N_i is the number of supernovae in the i-th redshift bin. The systematic error per bin is estimated as $\sigma_{\text{sys}} = 0.01(1+z)/1.8$. Therefore, the total error per redshift bin is:

$$\sigma_{\text{tot}} = \sqrt{(\sigma_{\text{stat}})^2 + (\sigma_{\text{sys}})^2}$$

For BAO simulations, we focus on BOSS [**?**], part of the SDSS-III survey operating from 2009-2014. Using the 2.5m SDSS telescope, it will measure redshifts of 1.5 million luminous galaxies in the range $0.1 < z < 0.7$ (as well as Ly absorption toward 160,000 high-redshift quasars at $z \sim 2.5$), covering $\sim 10,000$ deg^2 of high-latitude sky. The forecast precision for $H(z)$ is 1.8%, 1.7%, and 1.2% in redshift bins centered at $z = 0.35$, 0.6, and 2.5, respectively.

The standard deviation on the cosmic opacity parameter from simulated mock data is $\Delta\varepsilon = 0.005$ for constant ε, corresponding to a transparency bound

$\Delta\tau < 0.003$ (95% C.L.) for redshifts between 0.2 and 0.35. This limit is six times tighter than current results. Besides constant ε, we also constrain $\varepsilon(z)$ in three parametrization forms and list the standard deviations of ε_{i0} and ε_i in Table II. Due to improved measurement precision, future data could narrow current constraints by a factor of ~ 2 and verify the evolution of the cosmic opacity parameter $\varepsilon(z)$.

Besides the parametrization method, we also perform a model-independent analysis by imposing several parameters ε_n representing cosmic opacity in different redshift bins. Due to the low precision of current observational data, constraints on these coefficients are relatively weak. To reduce the dimensionality of parameter space, we adopt the principal component analysis (PCA) method [**?**, **?**], widely used in cosmological data analyses.

In practice, we divide the redshift region of the Union2.1 sample ($0 < z < 1.42$) into 7 bins, ensuring a similar number of SNIa samples in each bin. We thus have 7 additional free parameters ε_n ($n = 1, ..., 7$) denoting cosmic opacity in each bin. We set another parameter ε_8 to denote cosmic opacity at $z > 1.42$, which can only be constrained by high-redshift GRB data. Using the all-data combination, we obtain constraints on ε in each bin, as shown in the last plot of Figure 5 [Figure 5: see original paper]. We find that the constraint on ε_1 is much weaker than in the other seven bins, which should be considered as noise to be reduced by the PCA method.

First, we perform an orthogonal transformation on the original parameter space ε_n to obtain a set of linearly uncorrelated variables q_n:

$$q = W\varepsilon$$

where F is the Fisher matrix describing the curvature of the likelihood function in parameter space, Λ is a diagonal matrix consisting of eigenvalues of F, and $W \in SO(8)$ represents the transformation matrix. In practice, we obtain the Fisher matrix by inverting a covariance matrix generated from the MCMC procedure, $F = C^{-1}$. Second, we reconstruct $\varepsilon(z)$ after truncating some noisy modes with small eigenvalues, since large eigenvalue modes dominate variation of the likelihood function. We find that most eigenmodes have comparable eigenvalues, except the worst one which should be considered as noise.

Reconstructed $\varepsilon(z)$ with different numbers of modes is plotted in Figure 5. We see that large nonzero expected values in the first two bins disappear when the worst mode is dropped. For the reconstructed result, the fewer modes we retain, the less influence from noise we obtain. However, the fewer modes we retain, the larger the distortion between the PCA result and the real result. We must optimize the number of modes to balance noise reduction and information loss. Following conventions in Ref. [**?**], the risk function is:

$$\text{risk} = \text{bias}^2 + \text{variance} = \|\varepsilon_{\text{reconst}} - \varepsilon\|^2 + \sum_{i=1}^{n} \sigma_i^2$$

where bias^2 stands for quadratic reconstruction error, σ_i is the diagonal element of matrix Λ, and n is the number of used eigenmodes. In practice, we find the risk function reaches its minimum when $n = 5$. Based on this result shown in Figure 5, there is no obvious deviation from zero in these reconstructions.

IV. Summary

Verification of the well-known Etherington relation provides a useful way to search for new physics beyond the standard model. Different from the model-independent method, which uses d_L and d_A information at the same redshift to constrain parameter η, we adopt the model-dependent method, where photon number is not conserved during propagation, and use the latest observational measurements—SNIa, GRB, $H(z)$, BAO, and CMB distance information—to study the cosmic opacity parameter ε. We summarize our main conclusions:

- Using the latest SNIa samples (Union2.1, JLA, and SNLS), we find that SNIa data alone cannot constrain the cosmic opacity parameter ε well. The constraining power on ε from the luminosity distance indicator provided by SNIa and GRB is difficult to improve at present, due to strong degeneracy between ε and Ω_m.
- The Hubble parameter as a function of redshift is a useful measurement employed in many works. Unlike SNIa and GRB data, $H(z)$ measurements can only indirectly improve constraints on ε through the strong correlation between ε and Ω_m. When including $H(z)$ data in the analysis, statistical error bars shrink by a factor of ~ 3.
- We also use BAO and CMB distance information to constrain ε and obtain the tightest constraint: $\varepsilon = 0.023 \pm 0.018$ (68% C.L.). This corresponds to a transparency bound $\Delta\tau < 0.018$ (95% C.L.) for redshifts between 0.2 and 0.35.
- There is strong degeneracy between ε and the dark energy equation of state w. Larger ε values correspond to more flux received from the source, making supernovae brighter than expected from the standard ΛCDM Universe with $w = -1$.
- Similar to parametrizations of the dark energy equation of state, we use three parametrization forms to describe the evolution of ε as a function of z. Besides parametrization, we also use the PCA method and find no obvious deviation from zero in these reconstructions.
- Finally, we simulate future SNIa observations from WFIRST and $H(z)$ measurements from the BOSS experiment. We find that future mock

data could provide very tight constraints on cosmic opacity ε and verify the Etherington relation at high significance.

Acknowledgments

We thank Yi-Fu Cai, Si-Yu Li, Taotao Qiu, and Ming-Jian Zhang for useful discussions. J.-Q. Xia is supported by the National Youth Thousand Talents Program and the National Science Foundation of China under grant No. 11422323. This research is also supported by the Strategic Priority Research Program "The Emergence of Cosmological Structures" of the Chinese Academy of Sciences, grant No. XDB09000000.

∗Electronic address: xiajq@bnu.edu.cn