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Full Text

Preamble

Holographic Metal-Insulator Transition in Higher Derivative Gravity

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Abstract

We introduce a Weyl term into the Einstein-Maxwell-Axion theory in four-dimensional spacetime. Up to first order in the Weyl coupling parameter , we construct charged black brane solutions without translational invariance in a perturbative manner. Among all holographic frameworks involving higher derivative gravity, we are the first to obtain metal-insulator transitions (MIT) when varying the system parameters at zero temperature. Furthermore, we study the holographic entanglement entropy (HEE) of strip geometry in this model and find that the second-order derivative of HEE with respect to the axion parameter exhibits maximization behavior near quantum critical points (QCPs) of MIT. This testifies to the conjecture in [1, 2] that HEE itself or its derivatives can be used to diagnose quantum phase transitions (QPT).

Introduction

Quantum phase transitions (QPT) [3] occur at absolute zero temperature when varying system parameters and are believed to account for some peculiar phenomena observed in novel condensed matter systems at finite temperature, such as the strange metal behavior. As a fundamental issue in condensed matter physics, QPT has attracted considerable interest from both theorists and experimentalists. However, QPT typically involves strong correlation physics where conventional perturbative techniques lose their power.

Recently, gauge/gravity duality [4–6] has provided a novel mechanism to implement QPT in a holographic approach, particularly for metal-insulator transitions (MIT); see the recent review [7] and references therein. One key ingredient for realizing MIT in holography is to introduce momentum dissipation by breaking translational symmetry while simultaneously deforming the near-horizon geometry to a new infrared fixed point that is dual to an insulating phase [8, 9]. A simple holographic model with momentum dissipation can be constructed by introducing a set of massless axion fields [10]. In its original version, only the metallic phase was found. Later, when a general potential of axions or an additional dilaton field was introduced into this Einstein-Maxwell-Axion (EMA) model, the insulating phase was also observed [11–16]. Recent investigations of the hydrodynamic and transport properties of the EMA model without translational invariance can be found in [17–19].

In this paper, we provide a new strategy to implement MIT by introducing higher-derivative terms into the EMA model. This represents the first realization of MIT in the framework of higher-derivative gravity. In four-dimensional spacetime, there are eight independent terms in a general four-derivative action [20], which may emerge as quantum corrections in the low-energy effective action of superstring theory [21, 22]. From the viewpoint of the dual conformal field theory (CFT), these terms correspond to corrections at finite ' t Hooft coupling and/or beyond the large-N limit. Here, we focus on a special term with coupling between the gauge field and the Weyl tensor, which has been dubbed the Weyl term [20, 23, 24]. In holographic literature, it has been shown that the presence of the Weyl term in four-dimensional Schwarzschild-AdS geometry induces non-trivial behavior of the conductivity in the dual theory [20], in contrast to the frequency-independent conductivity obtained without the Weyl term [25]. Specifically, the real part of the conductivity displays a peak or valley near zero frequency depending on the sign of the Weyl coupling parameter, implying that the frequency-dependent conductivity is particle-like or vortex-like, respectively [20] (also see [26–30]).

Once the Weyl term is taken into account, the equations of motion for this Einstein-Maxwell-Axion-Weyl (EMA-Weyl) theory become a set of third-order differential equations with high nonlinearity, which are very difficult to solve analytically. As a first step, we treat the Weyl coupling parameter as a small number and construct analytical solutions up to first order in this parameter. This strategy has previously been used to construct perturbative charged black hole solutions for higher-derivative gravity in [23, 31–35]. With the background solutions up to O( ), we study the thermodynamics of the dual field theory. Moreover, the direct current (DC) conductivity can be derived analytically, which allows us to study the phase structure at zero temperature directly. We demonstrate that MIT as a quantum critical phenomenon can be observed manifestly in this context.

Motivated by our recent work [1, 2], we investigate the behavior of holographic entanglement entropy (HEE) near QPT in this holographic model. In condensed matter physics, numerous studies have revealed that entanglement itself or its derivatives display local extrema near QCPs [36–46]. Nevertheless, this phenomenon calls for deeper theoretical understanding. Recently, our series of works [1, 2] has disclosed that HEE or its first-order derivatives with respect to system parameters can be used to characterize QPT, establishing a holographic description of the relation between entanglement entropy and QPT. We have also proposed that it should be a universal feature that HEE or its derivatives can diagnose QPT in a generic holographic framework. The robustness of this proposal awaits further testing. Therefore, it is natural to ask whether the quantum critical phenomena observed in this model can also be captured by HEE. Interestingly, we demonstrate that the second-order derivative of HEE exhibits maximization behavior near QCPs. This observation not only justifies the conjecture in [1, 2] but also enriches our understanding of the scenario in which HEE characterizes QPT.

Our paper is organized as follows. We first introduce a Weyl term into the four-dimensional EMA theory and obtain perturbative black hole solutions in Section II, with a brief discussion of the thermodynamics of the background. In Section III, we calculate the DC conductivity of the dual system at absolute zero temperature and demonstrate that MIT occurs as quantum critical dynamics. We then move on to study HEE in Section IV and show that the second-order derivative of HEE with respect to the system parameter exhibits peaks near QPTs. In Section V, we summarize our results and discuss open questions for future investigation.

II. Einstein-Maxwell-Axion-Weyl Model

A. Setup and Equations of Motion

We consider a four-dimensional EMA theory with an additional Weyl term coupled to the Maxwell field (EMA-Weyl model), whose action reads as

$$S=\int d^4x\sqrt{-g}\left[R-2\Lambda-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{1}{2}\sum_{I=x,y}(\partial\phi_I)^2+\gamma C_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}\right]$$

where $F=dA$ and γ is the Weyl coupling parameter. ϕ_I is a set of free massless axion fields responsible for momentum dissipation [10]. $C_{\mu\nu\rho\sigma}$ is the Weyl tensor, defined as

$$C_{\mu\nu\rho\sigma}=R_{\mu\nu\rho\sigma}-\frac{1}{2}(g_{\mu[\rho}R_{\sigma]\nu}-g_{\nu[\rho}R_{\sigma]\mu})+\frac{R}{6}g_{\mu[\rho}g_{\sigma]\nu}.$$

It is straightforward to derive the equations of motion from the action in (1), which read

$$\nabla^2\phi_I=0,$$

$$\nabla_\mu\left[F^{\mu\nu}-4\gamma C^{\mu\nu\rho\sigma}F_{\rho\sigma}\right]=0,$$

$$R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}+\Lambda g_{\mu\nu}-\frac{1}{2}\left[F_{\mu\rho}F_\nu{}^\rho-\frac{1}{4}g_{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}+\partial_\mu\phi_x\partial_\nu\phi_x+\partial_\mu\phi_y\partial_\nu\phi_y-\frac{1}{2}g_{\mu\nu}((\partial\phi_x)^2+(\partial\phi_y)^2)\right]-\gamma(G_{1\mu\nu}+G$$

where

$$G_{1\mu\nu}=g_{\mu\nu}R_{\alpha\beta\rho\sigma}F^{\alpha\beta}F^{\rho\sigma}-3R_{(\mu|\alpha\beta\lambda|}F^{\alpha\beta}F^\lambda{}_{\nu)},$$

$$G_{2\mu\nu}=-g_{\mu\nu}R_{\alpha\beta}F^{\alpha\lambda}F^\beta{}_\lambda+g_{\mu\nu}\nabla_\alpha\nabla_\beta(F^{\alpha\lambda}F^\beta{}_\lambda)+2R_{\nu\alpha}F^{\alpha\beta}F_{\mu\beta}+2R_{\alpha\beta}F^\alpha{}_\mu F^\beta{}_\nu-2\nabla_\alpha\nabla_\beta(F^\alpha{}_\mu F^\beta{}_\nu)-g_{\mu\nu}\nabla^2(F_{\alpha\beta}$$

$$G_{3\mu\nu}=-g_{\mu\nu}R_{\alpha\beta}F^{\alpha\lambda}F^\beta{}_\lambda+2R_{\alpha\mu}F^{\alpha\beta}F_{\nu\beta}-2\nabla_\alpha\nabla_{(\mu}(F_{\nu)\beta}F^{\alpha\beta})+\nabla_\alpha\nabla^\alpha(F_{\lambda\mu}F^\lambda{}_\nu)+g_{\mu\nu}\nabla^2(F_{\alpha\beta}F^{\alpha\beta}).$$

As pointed out in the introduction, it is difficult to solve this system with full backreaction. Next, we construct a perturbative charged black brane solution to this system up to O( ). In this perturbative framework, we assume that is very small.

B. Charged Black Brane Solutions

We construct charged black brane solutions by solving the above equations (3), (4), and (5). To this end, we take the following ansatz:

$$ds^2 = -r^2 f(r)dt^2 + \frac{dr^2}{r^2 f(r)} + r^2 g(r)(dx^2 + dy^2),$$

$$A = A_t(r)dt, \quad \phi_x = kx, \quad \phi_y = ky,$$

where the UV boundary is located at $r \to \infty$. A non-zero $A_t(r)$ is introduced for a finite chemical potential. The special form of ϕ_I in (9) retains homogeneity and isotropy of spacetime while dissipating the momentum of the UV boundary CFT. Here, k is the system parameter and is also referred to as the axionic charge. This model does not have a manifest lattice wave vector but captures features of disorder, which is characterized by the axionic charge k [12, 16, 47].

Since the equations of motion for the axion fields ϕ_I in (3) are not influenced by corrections from the Weyl term, we only need to expand the functions $f(r)$, $g(r)$, and $A_t(r)$ in powers of γ up to first order:

$$f(r) = f_0(r) + \gamma Y(r), \quad g(r) = 1 + \gamma G(r), \quad A_t(r) = A_{t0}(r) + \gamma H(r),$$

where $f_0(r)$ and $A_{t0}(r)$ are leading-order solutions, while $G(r)$, $H(r)$, and $Y(r)$ are corrections of order O( ).

These functions can be determined by directly solving the equations of motion (3), (4), and (5) to zeroth and first order in . The solutions are:

$$f_0(r) = 1 - \frac{M}{r^3} + \frac{q^2}{r^4} + \frac{k^2}{r^2},$$

$$A_{t0}(r) = \mu - \frac{q}{r},$$

$$G(r) = -\frac{9r^4 - 296q^3}{45r^5} - \frac{104q^4}{45r^8} + \frac{20Mq^2}{9r^7} - \frac{16k^2 q}{9r^3} - \frac{9r^4 + 20k^2 q^2}{9r^6} + \frac{2r^4 + \cdots}{9r^4},$$

$$H(r) = \cdots,$$

$$Y(r) = \cdots.$$

Equations (11) and (12) are exactly the solutions of the EMA model proposed in [10].

In the above equations, there are five integration constants (μ, q, M, g_0, g_1), which are not independent from one another. We now derive the relations among these parameters.

First, one can show that the integration constants g_0 and g_1 can be eliminated by coordinate transformations $r \to r + \gamma g_0/2$, $x \to x(1-\gamma g_1/2)$, $y \to y(1-\gamma g_1/2)$, as well as a redefinition of the axion charge $k \to k(1+\gamma g_1/2)$. Then, up to O( ), Eqs. (11-15) can be reexpressed as:

$$f_0(r) = 1 - \frac{M}{r^3} + \frac{q^2}{r^4} + \frac{k^2}{r^2},$$

$$A_{t0}(r) = \mu - \frac{q}{r},$$

$$G(r) = \cdots,$$

$$H(r) = \cdots,$$

$$Y(r) = \cdots.$$

Furthermore, the location of the horizon r_h of these Weyl-corrected charged black brane solutions is determined by $f(r_h) = 0$. Meanwhile, to guarantee that A is well-defined on the horizon, we need to set $A_t(r_h) = 0$. We refer to these two conditions as the horizon conditions, which give the relations among (μ, q, M) as:

$$\mu = \cdots, \quad q = \cdots, \quad M = \cdots.$$

It is then straightforward to derive the Hawking temperature of this black brane as:

$$T = \frac{1}{4\pi r_h}\left[\cdots\right].$$

Note that we have expanded all the above quantities q, M, and T to O( ).

C. Thermodynamics

We briefly discuss the thermodynamics of the quantum field theory dual to the EMA-Weyl system using a standard approach (see e.g., [48]). To this end, we first construct the renormalized action S_{ren} by adding a boundary term to the original action (1):

$$S_{\text{ren}} = S + S_{\text{bdy}} = S + \int d^3x\sqrt{-h}(2K - 4),$$

where h is the determinant of the induced metric h_{ij} on the boundary and K is the trace of the extrinsic curvature on a slice with constant r. By straightforward calculation, the free energy density can be derived as:

$$\mathcal{F} = \cdots .$$

We then derive the charge density Q, entropy density s, pressure P, and energy density ϵ as:

$$Q = \cdots ,$$

$$s = \cdots ,$$

$$P = \cdots ,$$

$$\epsilon = \cdots ,$$

where $\epsilon = sT + \mu Q - P$ [48] has been used in the last equation. Again, all thermodynamic quantities are obtained up to O( ).

One can also check that s and ϵ correspond to the Wald entropy density [49] and ADM mass density [50, 51], respectively. In addition, for vanishing axions ($k = 0$), our results agree with those in [33, 34].

III. Metal-Insulator Transition

We study the MIT by analyzing the behavior of direct current (DC) conductivity. By definition, a MIT is reflected by an abrupt change in DC conductivity behavior, which is a macroscopic observable governed by quantum critical physics. Specifically, at zero temperature, the DC conductivity of a metallic phase behaves as $\partial_T \sigma_{\text{DC}} < 0$, while for an insulating phase it behaves as $\partial_T \sigma_{\text{DC}} > 0$. The critical point (line) of MIT is determined by $\partial_T \sigma_{\text{DC}} = 0$. Therefore, to demonstrate MIT from the EMA-Weyl model, we first calculate the DC conductivity.

In calculating DC conductivity, it is more convenient to work in coordinates $z \equiv r_h/r$. Note that for a given γ, the Weyl-corrected EMA black hole solutions are parametrized by two scaling-invariant parameters, $\hat{k} \equiv k/\mu$ and $\hat{T} \equiv T/\mu$. With this in mind, we rewrite the background solutions as follows:

$$ds^2 = \frac{1}{z^2}\left[-f(z)dt^2 + \frac{dz^2}{f(z)} + g(z)(dx^2+dy^2)\right],$$

$$A = A_t(z)dt,$$

where $f(z)$, $g(z)$, and $A_t(z)$ take the form:

$$f(z) = (1-z)p(z), \quad g(z) = 1+\gamma\left[\cdots\right],$$

$$p(z) = \gamma\left[240 + 100\hat{k}^2\mu^2(z^3-1) + 2\mu^2(13z^4-14) + (\mu^2-100)(z^3+z^2+z)\right] + 1 + z + (1-\hat{k}^2\mu^2)z^2 - \cdots,$$

$$A_t(z) = \mu(1-z) + \gamma\left[180 + 20\hat{k}^2\mu^2(-5-4z^2+9z^3) - 29\mu^2 + 74\mu^2 z^4 - 45z^3(4+\mu^2)\right].$$

The dimensionless Hawking temperature $\hat{T} \equiv T/\mu$ is given by:

$$\hat{T} = \frac{12-\mu^2-4\hat{k}^2\mu^2}{\mu(\mu^2-60)} + \cdots.$$

We calculate the DC conductivity in the dual field theory using the scheme proposed in [52] (also see [53]). We turn on a constant electric field from the beginning, instead of an alternating current (AC) electric field. Specifically, we take the following consistent ansatz:

$$\delta A_x = -E_x t + a_x(z), \quad \delta g_{tx} = z^2\left[h_{tx}(z) + \gamma G(z) h_{tx}(z)\right], \quad \delta\phi_x = \chi_x(z).$$

The key point of this method is to find the conserved current in the bulk [52, 53], which in this model is:

$$J^x = -\sqrt{-g}\left(F^{zx} - 4\gamma C^{zx\alpha\beta}F_{\alpha\beta}\right).$$

Up to O( ), J^x can be expressed as:

$$J^x = -Qh_{tx} + fa'_x + \gamma\left[\cdots\right],$$

where we have denoted $J^t = Q$, which is the conserved electric charge density. The DC conductivity can then be obtained from:

$$\sigma_{\mathrm{DC}} = \frac{J^x}{E_x}.$$

As revealed in [52, 53], given a conserved current J^x along the radial direction, it is sufficient to determine the DC conductivity from the requirement of regularity of the perturbation variables at the horizon $z = 1$. We illuminate this procedure below.

First, to have a well-defined gauge field at the horizon, we require $a'_x(z) \sim f(z)$. Second, when momentum conservation is violated, h_{tx} should be finite at the horizon, and we can extract this value from the $t-x$ component of the Einstein equation, which after evaluation at $z = 1$ reads:

$$\left[6(-6 + 2k^2 + f'(-4 + \gamma G') + f'') + A_t'^2(-3 + 8\gamma f' - 4\gamma f'') - 2fa'_x A'_t(3 + 4\gamma f'')\right]_{z=1} = 0.$$

Finally, combining the relevant equations, the DC conductivity can be expressed as a function of $(\hat{k}, \mu, \gamma)$:

$$\sigma_{\mathrm{DC}} = 1 + \frac{\hat{k}^2\mu^2 + 15\mu^2 - 2}{16}\gamma + \cdots.$$

[Figure 1: see original paper] shows the DC conductivity σ_{DC} as a function of temperature $\hat{T}$ for specific values of $\hat{k}$ (left plot for $\hat{k} = 0.4$ and right plot for $\hat{k} = 0.8$) and γ.

[Figure 2: see original paper] (left plot) shows $\partial_{\hat{T}}\sigma_{\mathrm{DC}}$ as a function of $\hat{k}$ at zero temperature for different Weyl parameters γ. The right plot shows the phase diagram in the $(\gamma, \hat{k})$ plane for MIT at zero temperature in the Weyl-corrected EMA geometry. The transverse blue line corresponds to the critical line $\hat{k} \approx 0.58116$.

Note that $\gamma \neq 0$ here. Armed with the conductivity expression, we study MIT by examining the behavior of σ in the zero-temperature limit. First, we directly see that when $\gamma = 0$, the DC conductivity is independent of temperature, as observed in [10]. However, when $\gamma \neq 0$, the DC conductivity becomes temperature-dependent (Fig. 1). In particular, we find that given a nonzero γ, a MIT occurs when varying the system parameter $\hat{k}$ (see the left plot in Fig.

2). To demonstrate the MIT more explicitly, we plot the phase diagram in the $(\gamma, \hat{k})$ plane at zero temperature in Fig. 2 (right plot). The quantum critical line (blue line in the right plot of Fig. 2) is determined by $\partial_{\hat{T}}\sigma_{\rm DC} = 0$ at zero temperature, which corresponds to $\hat{k} = \frac{1}{15}(5+\sqrt{5785}) \approx 0.58116$. This quantum critical line is independent of γ. Specifically, we observe that for $\gamma > 0$, the transition from metallic phase to insulating phase occurs when increasing $\hat{k}$. For $\gamma < 0$, however, the opposite scenario is obtained.

In [20, 26–30], the transport behavior of the boundary field theory dual to Schwarzschild-AdS geometry has been studied. The optical conductivity near zero frequency displays a Drude-like peak for $\gamma > 0$, while for $\gamma < 0$ the conductivity exhibits a valley near zero frequency. As argued in [20] (also see [26–30]), the Drude-like behavior for $\gamma > 0$ could be described by the collision and motion of charged particles, while the valley behavior for $\gamma < 0$ should be depicted by the collision of vortices. Further, it is revealed in [20, 26–30] that for small γ, an EM duality transformation relates the equations of motion of the Maxwell field at γ and at $-\gamma$, leading to $\sigma(\omega,\gamma) \approx 1/\sigma(\omega,-\gamma)$ in the dual boundary theory. Later, a broader class of particle-vortex duality was revealed in [54, 55], which differs from that in [20, 26–30].

From Fig. 1 we find that in our model $\sigma_{\rm DC}(\gamma, T)$ exhibits an interesting mirror symmetry $\sigma_{\rm DC}(\gamma, T) \approx \text{const.}-\sigma_{\rm DC}(-\gamma, T)$ when $\hat{k}$ is fixed, which can be viewed as a special particle-vortex duality as investigated in [54, 55]. It can be deduced from the conductivity expression that $\partial_T\sigma(\gamma, T)$ is an odd function of γ, which is also numerically depicted in the left plot of Fig. 2. Thus, we have a "metal-insulator" duality when changing the sign of γ, as illustrated in the right plot of Fig. 2. A concrete and analytical derivation of this duality in our present model, however, would be more complicated and difficult than that in [20, 26–30] due to the involvement of finite charge density and momentum dissipation. We leave this issue for future investigation.

Our present holographic EMA-Weyl model is dual to a boundary field theory with finite density and momentum dissipation. For weak momentum dissipation (small k), the transport behaves as metallic for $\gamma > 0$, which may be described by the motion and collision of particles [20]. With increasing $\hat{k}$, the motion of particles is suppressed and the system undergoes a phase transition from a metallic phase to an insulating phase. It is deduced from the conductivity expression that an opposite scenario happens for $\gamma < 0$, resulting in the phase structure illustrated in the right plot of Fig. 2.

IV. Holographic Entanglement Entropy Close to QCPs

We study the HEE for the dual field theory living on the boundary. It has been revealed in [1, 2] that HEE or its first-order derivative with respect to system parameters exhibit local extrema near QCPs and thus can be used to diagnose QPT in holographic frameworks. It is also conjectured in [2] that higher-order derivatives of HEE probably play a similar role in characterizing

QPT in holographic models. Inspired by this observation, we compute the HEE in our present model.

In comparison with previous holographic models with MIT, one nice feature of our current model is that analytical solutions for black branes with zero temperature are derived, allowing us to directly compute HEE in a bulk geometry at zero temperature.

Before calculating HEE explicitly, it is worthwhile to point out a key difference in the holographic description of entanglement entropy in higher-derivative gravity. The original Ryu-Takayanagi formula [56, 57] only holds for Einstein gravity. In [58], an alternative prescription for HEE is proposed for Lovelock gravity. This prescription reproduces the universal contribution to the EE for the dual CFT in four- and six-dimensional spacetimes. Further, a general formula for HEE in higher-derivative gravity is proposed in [59]. This formula includes the Wald entropy as the leading term and a correction from extrinsic curvature, usually dubbed the anomaly term of HEE. For our model, it is easy to check that the anomaly term of HEE vanishes for the Weyl-corrected action (1). Therefore, on a slice with fixed time, it is valid to calculate the HEE of our present model with the leading term of the formula proposed in [59], which is

$$S_{\mathrm{EE}} = -2\pi \int_{\Sigma} d^2x\sqrt{h}\frac{\partial \mathcal{L}}{\partial R_{\mu\nu\rho\sigma}}\epsilon_{\mu\nu}\epsilon_{\rho\sigma},$$

where $\mathcal{L}$ is the Lagrangian density of action (1), $\epsilon_{\mu\nu}$ is the Levi-Civita symbol, and h is the determinant of the induced metric on the surface Σ that minimizes the functional S_{EE}. In this EMA-Weyl model, the formula (39) can be evaluated as

$$S_{\mathrm{EE}} = 4\pi \int_{\Sigma} d^2x\sqrt{h}\left[1+\frac{\gamma}{3}F^2\right].$$

We compute the HEE in a bulk geometry at zero temperature. We consider a strip geometry on the dual boundary system that has length $L_y \to \infty$ in the y-direction and finite width $\hat{l}$ in the x-direction. Given that both h and F are functions of the radial coordinate z only, we can label the surface Σ by the location of its bottom z_* in the z-direction. Since the boundary is asymptotically AdS_4, the HEE for each background solution receives a vacuum contribution. Here we define the HEE as $\hat{S} \equiv (S_{\mathrm{EE}}-S_{\mathrm{vac}})/2\pi L_y$ with the vacuum contribution S_{vac} subtracted out. The scaling-invariant width l and HEE S can be expressed as:

$$l = 2\mu\int_0^{z_*} dz\frac{(\gamma F^2(z)+3)^2 g_{yy}(z)}{\sqrt{g_{xx}(z)g_{zz}(z)}\sqrt{(\gamma F^2(z)+3)^2 g_{xx}(z)g_{yy}(z)-(\gamma F^2(z_*)+3)^2 g_{xx}(z_*)g_{yy}(z_*)}},$$

$$S = \frac{\mu}{2}\int_{z_*}^{0} dz \frac{(\gamma F^2(z)+3)\sqrt{g_{xx}(z)g_{yy}(z)g_{zz}(z)}}{\sqrt{(\gamma F^2(z)+3)^2 g_{xx}(z)g_{yy}(z)-(\gamma F^2(z_*)+3)^2 g_{xx}(z_*)g_{yy}(z_*)}}.$$

[Figure 3: see original paper] shows each curve representing $\partial^2 S/\partial\hat{k}^2$ vs. $\hat{k}$ with γ specified by the plot legends. The red dashed line indicates $\hat{k}_c = 0.58116$.

[Figure 4: see original paper] shows the contour plot of $\partial^2 S/\partial\hat{k}^2$ over $(\gamma, \hat{k})$ at $l = 2.22 \times 10^6$ and $\hat{T} = 0$. The values of $\partial^2 S/\partial\hat{k}^2$ can be read from the plot legends. The red dashed curve is the quantum critical line shown in Fig. 2.

We study the relation between HEE and QPT in this Weyl-corrected EMA model. In scenarios where HEE characterizes QPT, the first-order derivative of HEE [2] or HEE itself [1] characterizes QPT with local extrema near QCPs. However, in the present model, HEE itself as well as its first derivative are featureless. Instead, we find that the second-order derivative of HEE with respect to the system parameter $\hat{k}$ exhibits peaks near the critical points of MIT. As seen from Fig. 3, $\partial^2 S/\partial\hat{k}^2$ reaches its local extremum near the critical value $k_c = 0.58116$, regardless of γ. To demonstrate this phenomenon more transparently, we show the contour plot of $\partial^2 S/\partial\hat{k}^2$ over $(\gamma, \hat{k})$ in Fig. 4. From this plot, it is easily seen that the local extrema (ridge for $\gamma < 0$ and valley for $\gamma > 0$) of $\partial^2 S/\partial\hat{k}^2$ lie close to the quantum critical line. We note that the connection between $\partial^2 S/\partial\hat{k}^2$ and the critical line is prominent only for $l > 10^5$. The larger l is, the better $\partial^2 S/\partial\hat{k}^2$ diagnoses the QPT. Specifically, in the limit of large l, we find that S/l converges to the Wald entropy density s, indicating that s is also a good indicator of MIT in the present holographic model. A similar phenomenon was observed in [1]. This is in accordance with CMT results that the entanglement measure characterizing QPT becomes more prominent with increasing block size l.

Finally, we notice a mild discrepancy between the ridge/valley and the quantum critical line. Such a discrepancy might be ascribed to the first-order approximation of our calculation. Furthermore, one peculiar feature of our model is that the Wald entropy density is non-zero for both metallic and insulating phases even at zero temperature, so the thermal contribution to HEE cannot be ignored and its effect on this discrepancy is unclear. At this stage, we propose that mutual information might play a better role in diagnosing QPT. We intend to study this in the future.

V. Discussions and Open Questions

We have constructed perturbative black brane solutions to the EMA-Weyl gravity model and studied the electrical transport properties. A MIT is observed in our model, representing the first realization of MIT in holographic models with higher-derivative gravity. We have also investigated the relation between HEE and MIT, finding that the second-order derivative of HEE with respect to the system parameter $\hat{k}$ exhibits peaks or valleys near the critical points of MIT. Our results further testify to the conjecture in [1, 2] and enrich the scenario of HEE characterizing QPTs. Certainly, it can be expected that HEE characterizing QPT with even higher orders of derivatives could be observed in holographic models.

After a series of works on the relation between HEE and QPT, it becomes urgent to understand the underlying reasons that lead to different derivative orders of HEE diagnosing QPT in the holographic approach, which is also an open problem in condensed matter theory. Previously, it was argued in CMT literature (for instance in [60]) that the derivative order of entanglement that becomes extremal or divergent might be related to the order of QPT, which is determined by the behavior of the free energy of the system. Based on our results, however, this correspondence is not observed in the holographic framework. Nevertheless, it is instructive to summarize and compare what we have observed in this series of work:

1. In [1], HEE itself exhibits local extrema near the QCPs of MIT. The ground state entropy density is vanishing for insulating phases while non-vanishing for the metallic phase, reflecting an AdS_2 near-horizon geometry.
2. In [2], it is the first-order derivative of HEE with respect to the system parameter that diagnoses the QCPs of MIT. In this circumstance, both metallic and insulating phases have vanishing ground state entropy density.
3. In the present paper, the second-order derivative of HEE with respect to the relevant parameter $\hat{k}$ characterizes the QPT. Correspondingly, both metallic and insulating phases have non-vanishing ground state entropy density.

Therefore, we propose that the derivative order of HEE which signals QPT might be related to the behavior of ground state entropy density in the holographic approach. We also expect that a well-designed quantity that removes the thermal contribution from HEE, for instance mutual information, might play a crucial role in unveiling the relation between the derivative order of HEE and QPT. We leave all these important issues for further study.

We now point out other interesting topics worthy of further investigation:

First, it would be interesting to incorporate holographic superconductors into our current framework. In [61], a Weyl-corrected holographic superconductor without backreaction was constructed. An important feature is that the ratio ω_g/T_c of gap frequency ω_g over critical temperature T_c of the superconducting phase transition runs with the Weyl parameter γ. In particular, when $\gamma < 0$, the value of ω_g/T_c is lower than $\omega_g/T_c \approx 8$ in the usual holographic superconductor [62–64]. These results have been confirmed in subsequent works; see for example

[65–72]. It would be interesting to see how the ratio ω_g/T_c is affected by γ and $\hat{k}$ in our model.

Another worthwhile improvement to our current work is to obtain black brane solutions with full backreaction, which involves solving differential equations beyond second order. When full backreaction is considered, several important issues could be addressed. First, our present perturbative EMA-Weyl background has AdS_2 IR geometry at zero temperature, which is associated with a finite ground state entropy density. It would be valuable to examine the behavior of the ground state entropy density with full backreaction. Second, the computation of optical conductivity with full backreaction will reflect a more accurate phase structure. Furthermore, we could further study the mild discrepancy between the ridge of HEE and the critical line when full backreaction is considered. In addition, it would also be interesting to include superconductors in our present model with full backreaction.

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