On the evaporation of solar dark matter: spin-independent effective operators (Postprint)

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Full Text

Preamble

ON THE EVAPORATION OF SOLAR DARK MATTER: SPIN-INDEPENDENT EFFECTIVE OPERATORS

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Abstract

As part of the effort to investigate the implications of dark matter (DM)-nucleon effective interactions on solar DM detection, this paper focuses on the evaporation of solar DM for a set of DM-nucleon spin-independent (SI) effective operators. To place the evaluation of the evaporation rate on more reliable ground, we calculate the non-thermal distribution of solar DM using Monte Carlo methods rather than adopting the Maxwellian approximation. We then specify relevant signal parameter spaces for solar DM detection for various SI effective operators. Based on our analysis, we determine the minimum DM masses for which DM-nucleon coupling strengths can be probed from solar neutrino observations. As an interesting application, our investigation also shows that the evaporation effect cannot be neglected in a recent proposal aiming to solve the solar abundance problem by invoking momentum-dependent asymmetric DM.

Introduction

As the nearest celestial body that is well understood and capable of stimulating and responding to phenomena associated with dark matter (DM), the Sun is presumed to be an ideal host for DM detection. On one hand, its deep gravitational well attracts and traps Galactic DM particles through scattering off solar elements if a DM-nucleon interaction exists at the weak scale. On the other hand, these captured DM particles may accumulate in the solar core and subsequently annihilate to primary and secondary high-energy neutrino fluxes that escape from the dense solar plasma, leaving a smoking-gun signature of their presence in the Sun. Presently, a number of terrestrial neutrino detection projects such as IceCube [1], Super-Kamiokande [2], Baikal Neutrino Project [3], and ANTARES [4] are dedicated to such observational missions.

In general, the neutrino flux at the detector location is related to solar DM annihilation through the following schematic relation:

$$d\Phi_\nu/dE_\nu = \frac{\Gamma_A}{4\pi d_\odot^2} \times dN_\nu/dE_\nu \tag{1.1}$$

where $d_\odot$ is the Sun-Earth distance, $d\Phi_\nu/dE_\nu$ and dN_ν/dE_ν represent the neutrino differential flux at Earth and the neutrino energy spectrum per DM annihilation event in the Sun, respectively. The total annihilation rate Γ_A can be expressed in terms of the number of trapped DM particles N_χ:

$$\Gamma_A = A_\odot N_\chi^2 \tag{1.2}$$

where $A_\odot$ denotes twice the annihilation rate of a pair of DM particles. The evolution of the solar DM number N_χ is described by the following equation:

$$\dot{N}_\chi = C_\odot - E_\odot N_\chi - A_\odot N_\chi^2 \tag{1.3}$$

which involves the DM capture rate $C_\odot$ (evaporation rate $E_\odot$) by scattering off atomic nuclei in the Sun, as well as the annihilation rate $A_\odot$. Equation (1.3) has an analytic solution:

$$N_\chi(t) = \frac{C_\odot \tanh(t/\tau_e)}{E_\odot/2 + (E_\odot/2)\tanh(t/\tau_e)} \tag{1.4}$$

with

$$\tau_e = (C_\odot A_\odot + E_\odot^2/4)^{-1/2} \tag{1.5}$$

representing the timescale for capture, evaporation, and annihilation processes to equilibrate. Once equilibrium is reached at the present day (i.e., $\tanh(t_\odot/\tau_e) \approx 1$ for $t_\odot = 4.5\times 10^9$ yr, the solar age), the annihilation output Γ_A also reaches its maximum value. As will be shown in Sec. 3.2, a GeV increment in the DM mass parameter results in 1-2 orders of magnitude reduction in the evaporation rate $E_\odot$ in the few-GeV region. Thus, depending on the ratio $E_\odot^2/(C_\odot A_\odot)$, such equilibrium can be categorized into two different scenarios:

1. $E_\odot^2/(C_\odot A_\odot) \ll 1$, when the evaporation effect can be neglected and the equilibrium is between annihilation and solar capture, i.e., $\Gamma_A \approx C_\odot/2$. In this case, we can either determine or constrain the strength of the DM-nucleon interaction from solar neutrino observations.

2. $E_\odot^2/(C_\odot A_\odot) \gg 1$, under which circumstance evaporation overwhelms annihilation for DM depletion, and the balance between evaporation and solar capture yields $\Gamma_A \approx (A_\odot C_\odot^2)/(4E_\odot^2)$. This not only implies a heavy suppression of the neutrino flux but also prevents us from extracting the coupling strength of the DM-nucleon interaction from any observed signals.

Therefore, from a theoretical standpoint it is interesting to pin down the parameter space where neutrino observations are relevant for DM detection. Conventionally, this purpose is fulfilled using a characteristic quantity, the evaporation mass m_{evap}, which is defined by the equation $E_\odot(m_{\text{evap}}) = t_\odot^{-1}$ for a given DM-nucleon coupling. Above the evaporation mass, one can safely assume that capture-annihilation equilibrium is reached.

The key challenge is to calculate the distribution of solar DM. While in Refs. [5,6] the authors adopt a Maxwellian distribution to describe the non-thermal equilibrium between solar DM particles and solar nuclei, studies in Refs. [7,8]

indicate a deviation from the Maxwellian form, such that the actual velocity distribution is suppressed at the tail and tends to be anisotropic at large radius. This deviation can be attributed to the fact that energetic collisions that send DM particles into high orbits occur predominantly near the hot solar core. Consequently, one expects a lower angular momentum distribution for high-energy orbits. To properly describe physical processes such as evaporation and energy transfer of solar DM, an accurate description of the tail of the velocity distribution is necessary.

In addition, since the evaporation mass has been studied thoroughly in the literature under the assumption of a constant DM-nucleon cross section [5,7-9], the quest naturally arises to extend the discussion to a broader set of DM-nucleon effective interaction operators. For instance, it is tempting to evaluate the evaporation rate for light asymmetric DM particles with a DM-nucleon scattering amplitude linearly proportional to the square of the transferred momentum q^2, because while the authors of Refs. [10,11] manage to resolve the disagreement between the solar model and helioseismological data with a preferred DM mass of 3 GeV and coupling strength of 10^{-37} cm^2, the evaporation effect is not included in their discussion. Given such small DM masses, evaporation may no longer be neglected in the buildup of solar DM, and a quantitative analysis is needed on this issue.

Thus, as a tentative study we investigate the implications of non-relativistic spin-independent (SI) effective operators on the solar DM distribution and evaporation mass. The set of 15 Galilean invariant operators introduced in Ref. [12] provides a comprehensive and convenient treatment for DM-nucleus interaction in DM direct detection. Following Ref. [8], we calculate the non-thermal distribution of solar DM by Monte Carlo methods and numerically compute evaporation rates for different SI DM-nucleus effective operators. Moreover, based on the calculated capture and evaporation rates, we also discuss the parameter space relevant for DM detection.

This paper is organized as follows. In Sec. 2 we briefly review the effective interaction between the DM particle and nucleus. In Sec. 3 we calculate the solar DM distribution and evaporation rate for various SI DM-nucleus interaction operators and discuss relevant implications for high-energy solar neutrino signals. Some interesting discussions are arranged in Sec. 4.

2. Effective Interaction Between DM and Nucleus

We discuss DM-nucleus scattering at low-energy scales in the context of the non-relativistic (NR) effective interaction theory [12,14-17], in which a set of linearly independent operators listed in Table 2.1 can be generated from the following five Hermitian operators: $\hat{1}, \hat{q}^2, i\hat{\mathbf{S}}_\chi \cdot \hat{\mathbf{q}}, i\hat{\mathbf{S}}_N \cdot \hat{\mathbf{q}}, \hat{\mathbf{S}}_\chi \cdot \hat{\mathbf{S}}_N, \hat{\mathbf{S}}_\chi \cdot \hat{\mathbf{v}}_\perp, \hat{\mathbf{S}}_N \cdot \hat{\mathbf{v}}_\perp$.

Here $\mathbf{q}$ is the transferred momentum from nucleon to the DM particle in a collision, and the transverse velocity is defined as $\hat{\mathbf{v}}_\perp = \mathbf{v} + \mathbf{q}/(2\mu_N)$, which satisfies $\mathbf{q} \cdot \hat{\mathbf{v}}_\perp = 0$ for the on-shell process, where $\mathbf{v} = \mathbf{v}_{\chi,i} - \mathbf{v}_{N,i}$ is the relative initial velocity between the DM particle and nucleon, and $\mu_N = m_\chi m_N/(m_\chi + m_N)$ is the reduced mass of the system. $\hat{\mathbf{S}}_\chi$ and $\hat{\mathbf{S}}_N$ are the spins of the DM particle and the nucleon, respectively.

While the operators presented in Table 2.1 exhaust all possible NR reductions of the Lorentz invariant spin-1/2 DM-nucleon interaction, up to corresponding coefficients dependent on the Galilean invariant scalar q^2, in this study we shall investigate the implication of all the SI operators $\hat{\mathcal{O}}_i^\dagger$ for the DM evaporation mass.

Since the atomic nucleus is a composite of bound nucleons, its structural effect must be taken into consideration in the analysis of DM-nucleus interaction. Interestingly, in addition to the conventional nuclear form factor that describes the mass distribution within a nucleus, other types of DM and nuclear response functions arise from various underlying DM-nucleon interactions. For example, the operator $\hat{\mathbf{v}}_\perp$ can be divided into center-of-mass and relative motion components. While the former represents the effective operator at the nucleus scale, the latter corresponds to the probability density current of the bound nucleon and gives rise to a nuclear response function (Δ response in Ref. [12]) associated with the nuclear orbital angular momentum in the long-wavelength limit. Nevertheless, compared with the conventional form factor that corresponds to W_M in Refs. [14-17], response functions coming from nuclear intrinsic motion (W_Δ in Refs. [14-17]) can be safely neglected if isospin symmetry is respected. This is directly observable from the nuclear response functions provided in Ref. [18]: isoscalar response functions W_Δ^{00} are much smaller than W_M^{00} for the unpaired solar elements (e.g., ^{14}N, ^{23}Na, and ^{27}Al). Additionally, these W_Δ responses associated with unpaired elements suffer significant abundance suppression in the Sun.

Therefore, assuming the DM particle couples to protons and neutrons with equal strengths, the effects of response Δ can be neglected for operators $\hat{\mathcal{O}}_1$, $\hat{\mathcal{O}}_5$, $\hat{\mathcal{O}}_8$, and $\hat{\mathcal{O}}_{11}$, and hence we simply utilize the conventional Helm form factor to account for nuclear internal structure when investigating the implications of various SI interactions on DM evaporation on a case-by-case basis. As a result, the DM-nucleus differential cross section for operators $i = 1, 5, 8, 11$ can be expressed in terms of the transferred momentum q as follows:

$$\frac{d\sigma_i}{dq^2} = \frac{c_i^2}{4\pi v_{\rm rel}^2} A^2 F_N^2(q^2) P_i(v_{\rm rel}^2, q^2) \tag{2.2}$$

where c_i (carrying dimension of mass^{-2}) is the nucleon coupling constant for operator $\hat{\mathcal{O}}_i$, $P_i(v_{\rm rel}^2, q^2)$ is the corresponding DM response function listed explicitly in Table 2.2, and A is the atomic number of the target nucleus.

In Table 2.2, j_χ represents the spin of the DM particle, $\hat{\mathbf{v}}_{\perp,A} = \mathbf{v}_{\rm rel} + \mathbf{q}/(2\mu_A)$ is the nucleus transverse velocity with $\mathbf{v}_{\rm rel} = \mathbf{v}_{\chi,i} - \mathbf{v}_{A,i}$ being the relative incoming velocity of the DM-nucleus system, and μ_A the corresponding reduced mass. $F_N^2(q^2) = [3j_1(qR_1)/(qR_1)]^2 e^{-q^2 s^2}$ is the Helm form factor, with $j_1(x) = \sin(x)/x^2 - \cos(x)/x$ being the spherical Bessel function of the first kind, $R_1 = \sqrt{R_0^2 + s^2}$ with $R_0 \simeq 1.23 A^{1/3}$ fm, and $s \simeq 1$ fm [19].

3. Distribution and Evaporation of Solar DM

In this section we discuss the distribution and evaporation of solar DM. Since evaporation occurs predominantly at the high end of the velocity distribution, its evaluation relies on an accurate description thereof. We determine the solar DM distribution by solving the Boltzmann equation numerically and then separately calculate the evaporation rate for various effective SI DM-nucleon interaction operators.

3.1. High End of the Velocity Distribution in the Sun

To date, there are two effective strategies in the literature for determining the solar DM distribution. In the "Brownian motion" method pioneered by the author of Ref. [7], the distribution sample is obtained by simulating the motion of a single DM particle wandering in the Sun. While this method is efficient in describing the bulk of the velocity distribution, it becomes impractical for computing the tail of the distribution, for which a huge and uneconomical base of event samples is required to generate sufficient statistics.

Therefore, to determine the distribution of solar DM, we resort to essentially the same method as outlined in Ref. [8]. Our discussion begins with the assumption that the presence of solar DM does not significantly impact the solar structure; i.e., the feedback from accumulating DM particles is assumed to be negligible. The Boltzmann equation is linear due to the absence of DM self-interaction and can be further simplified as the following master equation when expressed with the convenient parameters E (total energy per unit mass) and L (angular momentum per unit mass) [8]:

$$\frac{df(E,L)}{dt} = \sum_{E',L'} f(E',L')S(E',L';E,L) - \sum_{E',L'} f(E,L)S(E,L;E',L') \quad (3.1)$$

where $f(E,L)$ is the distribution function of solar DM and $S(E,L;E',L')$ represents the scattering matrix element for the transition process $(E,L) \to (E',L')$.

In fact, to fully describe the physical state of a bound DM particle we still need an extra parameter, say a temporal parameter τ, to label the position in the periodic orbit defined by energy and angular momentum. However, we approximate both the distribution function and scattering matrix elements as independent of τ in Eq. (3.1). The reason is that a small DM-nucleus cross section, or equivalently a large mean free path, leads to a slowly increasing probability for a renewed collision, which implies an insensitive reliance of the distribution and scattering matrix on τ.

The scattering matrix $S(E, L; E', L')$ is determined via simulation, and the weighting method is adopted to facilitate computation. Specifically, we first calculate the probability for a trapped DM particle to collide with solar elements on its trajectory at a fixed time interval Δt, and then multiply this probability as a weight with the tally of simulated transition events to evaluate the scattering matrix efficiently. The numerical integration of bound DM orbits is based on the Standard Solar Model (SSM) GS98 [20], and five solar elements (H, ^{4}He, ^{14}N, ^{16}O, and ^{56}Fe) are included in the simulation of DM-nucleus scattering. With random numbers that help select both the colliding solar element and its velocity, as well as the scattering angle in the center-of-mass (CM) frame, we determine the outgoing state of the scattered DM particle after a coordinate transformation back to the solar reference frame. Further details on thermal collisions are arranged in Appendix A.

It is not difficult to observe that Eq. (3.1) represents a Markov process. We evolve it with discrete time steps Δt until $f(E, L)$ converges to the limiting distribution $f_\chi(E, L)$. For illustration, we present the equilibrium distribution $f_\chi(E, L)$ for the DM-nucleon interaction operator $\hat{\mathcal{O}}_1$ in Fig. 3.1. The parameters E and L are nondimensionalized in units of an energy reference value $GM_\odot/R_\odot$ and an angular momentum value $(GM_\odot R_\odot)^{1/2}$, where G is Newton's constant and $M_\odot$ is the solar mass. These values are constructed from a length unit (the solar radius $R_\odot = 6.955 \times 10^5$ km), a time unit $(GM_\odot/R_\odot^3)^{-1/2} \simeq 436$ s, from which the DM velocity v_χ can also be expressed in terms of a reference value $(GM_\odot/R_\odot)^{1/2} \simeq 436$ km/s.

Finally, by convolving $f_\chi(E, L)$ with $\phi_{EL}(r, v_\chi)$, the distribution function of radius r and velocity v_χ for orbit (E, L), we obtain the DM distribution function:

$$f_\chi(r, v_\chi) = \sum_{E,L} f_\chi(E, L)\phi_{EL}(r, v_\chi) \tag{3.2}$$

For illustration, we present the distribution function of radius r after integrating out velocity v_χ and vice versa for the orbit $(E = 1.225, L = 0.124)$ in Fig. 3.2.

Although the Maxwellian form of the DM velocity distribution fails to describe the tail of the actual velocity distribution, as mentioned in Sec. 1, it suffices to approximate the bulk of the non-thermal distribution, on which physical processes such as DM annihilation can be evaluated easily and accurately. The approximate thermal distribution is expressed as $f_{\text{th}} \propto \exp(-m_\chi E/T_\chi)$, with the effective temperature parameter T_χ. T_χ is determined by requiring no net energy transfer from solar nuclei to shuttling DM particles once steady state is achieved, corresponding to the following energy-moment equation [5]:

$$\int_0^{R_\odot} n_A(r) \left[\frac{m_A T_\chi + m_\chi T_\odot(r)}{m_A m_\chi}\right]^{1/2} [T_\odot(r) - T_\chi] e^{-m_\chi V(r)/T_\chi} r^2 dr = 0 \tag{3.3}$$

where m_A and $n_A(r)$ are the mass and local number density of element A, $T_\odot(r)$ is the temperature within the Sun, and $V(r)$ is the gravitational potential as a function of radius r.

Table 3.3 shows the effective temperature T_χ for benchmark DM masses from 1 GeV to 100 GeV. For a DM particle weighing tens of GeV, the effective temperature T_χ can be approximated as the solar central temperature $T_\odot(0)$.

For contrast, we compare the simulated velocity distribution f_χ to the approximate thermal distribution f_{th} in Figs. 3.3 and 3.4 for effective operators $\hat{\mathcal{O}}_1$, $\hat{\mathcal{O}}_5$, $\hat{\mathcal{O}}_8$, and $\hat{\mathcal{O}}_{11}$ in terms of the ratio f_χ/f_{th}. The DM velocity v_χ stretches no further than the escape velocity at the solar core $v_{\mathrm{esc}}(0)$.

3.2. Evaporation, Capture, and the Minimum Testable Mass of Solar DM

In Ref. [8], the author provided a thorough discussion on DM evaporation under the assumption of a constant DM-nucleon cross section, which corresponds to operator $\hat{\mathcal{O}}_1$ in the context of effective operators. Now we extend the discussion to include other SI effective operators $\hat{\mathcal{O}}_5$, $\hat{\mathcal{O}}_8$, and $\hat{\mathcal{O}}_{11}$. Our interest focuses on the scenario in which the Sun is optically thin to DM particles.

Following Ref. [8], we start with the quantity $R_A(w \to v)$, which represents the probability per unit volume for a DM particle with initial velocity w to be scattered to final velocity v by nucleus A:

$$R_A(w \to v) = n_A \left\langle \frac{d\sigma}{dv} \right\rangle f_A(\mathbf{u}_A) \tag{3.4}$$

where $d\sigma/dv$ is the differential cross section for the DM-nucleus system, which depends on their relative velocity $w - \mathbf{u}_A$, and $\langle\cdots\rangle$ denotes the average over the thermal velocity distribution of element A. The Maxwellian distribution $f_A(\mathbf{u}_A)$ is written as:

$$f_A(\mathbf{u}_A) = (\sqrt{\pi}u_0)^{-3} \exp\left(-\frac{u_A^2}{u_0^2}\right) \tag{3.5}$$

where $u_0 = \sqrt{2T_\odot/m_A}$. For conciseness, we postpone the explicit expression of Eq. (3.4) to Appendix B.

Next, given DM velocity w and the escape velocity v_{esc}, the evaporation rate per unit volume can be written as:

$$\Omega_+(w, v_{\mathrm{esc}}) = \sum_A \int_{v>v_{\mathrm{esc}}} R_A(w \to v) dv \tag{3.6}$$

where the summation is taken over all solar elements. Finally, by convolving $\Omega_+(w, v_{\rm esc})$ with the DM distribution $f_\chi(r, w)$ determined from simulation, we express the DM evaporation rate as:

$$E_\odot = \int \Omega_+(w, v_{\rm esc}) f_\chi(r, w) dr dw \tag{3.7}$$

where $\Omega_+(w, v_{\rm esc})$ depends on the radial coordinate r through the distributions of solar nuclei and the escape velocity $v_{\rm esc}(r)$, both described with the SSM GS98 [20].

Given $j_\chi = 1/2$, the evaporation rates for various SI effective operators are expressed with the following fitting functions:

$$E_\odot^{(1)} \approx 10^{-2.81} \left(\frac{m_\chi}{1\ \text{GeV}}\right)^{1.08} \left(\frac{\sigma_p}{10^{-40}\ \text{cm}^2}\right)\ \text{s}^{-1} \tag{3.8a}$$

$$E_\odot^{(5)} \approx \left[10^{-2.01} \left(\frac{m_\chi}{1\ \text{GeV}}\right)^{1.17} + \left(\frac{1\ \text{GeV}}{m_\chi}\right)^{0.09}\right] \left(\frac{c_5}{10^{-1}\ \text{GeV}^{-2}}\right)^2\ \text{s}^{-1} \tag{3.8b}$$

$$E_\odot^{(8)} \approx \left[10^{-2.42} \left(\frac{m_\chi}{1\ \text{GeV}}\right)^{1.26} + \left(\frac{1\ \text{GeV}}{m_\chi}\right)^{0.24}\right] \left(\frac{c_8}{10^{-3}\ \text{GeV}^{-2}}\right)^2\ \text{s}^{-1} \tag{3.8c}$$

$$E_\odot^{(11)} \approx 10^{-1.78} \left(\frac{m_\chi}{1\ \text{GeV}}\right)^{1.26} \left(\frac{c_{11}}{10^{-4}\ \text{GeV}^{-2}}\right)^2\ \text{s}^{-1} \tag{3.8d}$$

These approximate the numerical results with better than 10% accuracy in the DM mass range $2-5$ GeV. For convenience, we invoke the DM-nucleon cross section $\sigma_p = c_1^2 m_N^2/\pi$ instead of coupling parameter c_1 in Eq. (3.8a).

Here we briefly review the solar capture rate $C_\odot$ and the annihilation coefficient $A_\odot$. The standard procedure for evaluating $C_\odot$ is developed in the literature [21-23]. Given the Galactic DM distribution unperturbed by solar influence, we first derive the collision event rate using the Liouville theorem and angular momentum conservation in the solar central force field, and by demanding the momentum transfer be large enough for capture, we then extract the capture rate from the total collision event rate. While discussions on capture rates for various DM-nucleon effective operators can be found in Refs. [18,24], here we present numerical results for $j_\chi = 1/2$ in the DM mass range $2-5$ GeV as the following fitting functions dependent on DM mass $x = (m_\chi/1\ \text{GeV})$:

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$$C_\odot^{(1)} \approx \frac{0.008241x^6}{1.17023 + 17.9214x + 15.0294x^2 + 6.30696x^3 + 1.43792x^4 + 0.170425x^5} \left(\frac{\sigma_p}{10^{-40}\ \text{cm}^2}\right) 10^{25}\ \text{s}^{-1} \tag{3.9a}$$

$$C_\odot^{(5)} \approx \frac{6.73314 + 0.00379191x^6}{12.5207x + 9.48633x^2 + 3.63890x^3 + 0.771875x^4 + 0.0849675x^5} \left(\frac{c_5}{10^{-1}\ \text{GeV}^{-2}}\right)^2 10^{26}\ \text{s}^{-1} \tag{3.9b}$$

$$C_\odot^{(8)} \approx \frac{6.33402 + 0.00408605x^6}{11.3047x + 8.86278x^2 + 3.54797x^3 + 0.775692x^4 + 0.0882098x^5} \left(\frac{c_8}{10^{-3}\ \text{GeV}^{-2}}\right)^2 10^{26}\ \text{s}^{-1} \tag{3.9c}$$

$$C_\odot^{(11)} \approx \frac{4.69007 + 0.00322178x^6}{8.90451x + 6.98704x^2 + 2.69955x^3 + 0.592700x^4 + 0.0683059x^5} \left(\frac{c_{11}}{10^{-4}\ \text{GeV}^{-2}}\right)^2 10^{25}\ \text{s}^{-1} \tag{3.9d}$$

In evaluating these capture rates, we adopt the isothermal DM halo model with local density $\rho_\chi = 0.3$ GeV/cm^3 and a Maxwellian velocity distribution with dispersion $v_0 = 220$ km/s, truncated at the Galactic escape velocity of 544 km/s.

The annihilation coefficient $A_\odot$ can be expressed in terms of the thermal cross section $\langle\sigma v\rangle_\odot$ and the effective occupied volume of solar DM V_{eff} as:

$$A_\odot \equiv \frac{\langle\sigma v\rangle_\odot}{V_{\text{eff}}} \tag{3.10}$$

While the signal regions ($\tanh(t_\odot/\tau_e) \approx 1$) are presented as darker-colored areas in Fig. 3.5, the lighter counterparts correspond to the region where $\tanh(t_\odot/\tau_e) < 0.9$ for reference. In the red (blue) area, evaporation (annihilation) plays a sub-dominant role in the evolution of the solar DM number. The purple belt represents the transition zone where both evaporation and annihilation effects are of equal importance. The 90% C.L. upper bounds (yellow dashed lines) are inferred from the binned data of CDMSlite [25].

The effective volume can be described with the fitting function:

$$V_{\text{eff}} = 6.9 \times 10^{30} \left(\frac{100\ \text{GeV}}{m_\chi}\right)^{3/2}\ \text{cm}^3 \tag{3.11}$$

Now we explore the parameter space where the solar neutrino observational approach is effective for DM detection, putting our intuitive discussion from

Sec. 1 onto concrete computation. On one hand, as mentioned in Sec. 1, to ensure the full strength of the neutrino flux it is required that $\tanh(t_\odot/\tau_e) \approx 1$, for which we adopt the criterion $t_\odot/\tau_e \geq 3.0$. On the other hand, to specify the parameter region for annihilation- and evaporation-dominated scenarios, we set the criteria as $E_\odot^2/(4C_\odot A_\odot) \leq 0.1$ and $E_\odot^2/(4C_\odot A_\odot) \geq 10$, respectively.

For concreteness, in Fig. 3.5 we show the relevant parameter regions for annihilation- and evaporation-dominated regimes for SI effective operators $\hat{\mathcal{O}}_1$, $\hat{\mathcal{O}}_5$, $\hat{\mathcal{O}}_8$, and $\hat{\mathcal{O}}_{11}$, assuming the canonical s-wave thermal annihilation cross section $\langle\sigma v\rangle_\odot = 3 \times 10^{-26}$ cm^3/s, although p-wave annihilation is also possible. Also shown in Fig. 3.5 (in yellow dashed lines) are the 90% C.L. upper limits on DM-nucleon coupling strengths imposed by the second run of CDMSlite [25], derived using Poisson statistics based on the event spectrum, signal efficiency, and detector resolution presented in Ref. [25], along with astrophysical parameters consistent with the capture rate calculation.

Given this quantitative analysis, we can draw clear boundaries among different signal topologies. For instance, for effective interaction $\hat{\mathcal{O}}_1$ with DM-nucleon cross section $\sigma_p = 10^{-40}$ cm^2, the assumption of equilibrium between capture and annihilation is only valid for DM particles heavier than 2.99 GeV, while for DM masses smaller than 2.68 GeV, one can no longer extract the coupling strength of the DM-nucleon interaction from observed neutrino flux because the number of DM particles $N_\chi = C_\odot/E_\odot$ becomes independent of cross section σ_p [9]. Additionally, if the DM-nucleon σ_p is smaller than roughly 10^{-44} cm^2, equilibrium among capture, evaporation, and annihilation has not yet been achieved at present, and hence the signal flux is suppressed. The neutrino observation approach is no longer effective for probing solar DM.

4. Discussions

As mentioned in Sec. 1, the authors of Refs. [10,11] introduce weakly interacting asymmetric dark matter (ADM) with generalized form factors in an attempt to solve the solar abundance problem. Without annihilation, ADM may accumulate to such an amount that its presence can slightly affect solar structure. Assuming the evaporation rate is zero, it is found that the following SI interaction between a 3 GeV ADM and nucleon gives the best result:

$$\sigma = \sigma_0 \left(\frac{q}{q_0}\right)^2 \tag{4.1}$$

where the coupling $\sigma_0 = 10^{-37}$ cm^2 and the reference momentum $q_0 = 40$ MeV. The translation between the context of the generalized form factor and effective operator $\hat{\mathcal{O}}_{11}$ is realized through the relation:

$$\sigma_0 \left(\frac{q}{q_0}\right)^2 = \frac{c_{11}^2}{4\pi} A^2 \frac{j_\chi(j_\chi+1)}{3} \left(\frac{q}{m_N}\right)^2 \tag{4.2}$$

which gives $c_{11} = 1.87 \times 10^{-3}$ GeV^{-2} for $j_\chi = 1/2$.

For these best-fit parameters, we calculate the evolution of solar DM with and without evaporation in Fig. 4.1. It is evident that evaporation significantly constrains the growth of the DM number N_χ and freezes it at a number $\sim 10^4$ times smaller than the value without evaporation, indicating an inconsistency for the model in Eq. (4.1) to alleviate discrepancies between the SSM and helioseismological observables. Note that although we evaluate the evaporation rate by neglecting the interplay between the accumulated DM population and solar nuclei background, our calculation still holds in the ADM scenario because the relevant effects only result in minor changes to solar structure. It should also be noted that such inconsistency has been confirmed from the experimental aspect by DM direct detection: CRESST-II ruled out this particular model at 90% C.L. [27]. To evade constraints from direct detection, the same authors recently proposed a spin-dependent (SD) v^2 interaction as an alternative solution in Ref. [28]. We leave discussion of the relevant evaporation effect in the SD scenario for future work.

Finally, we discuss a subtlety underlying the methodology applied to calculate the steady distribution $f_\chi(E, L)$ in Sec. 3: to what extent does the Markov chain approach describe the realistic evolution of the solar DM distribution, considering that both replenishment and leakage of DM particles are not reflected in the master equation Eq. (3.1)? To address this, we explicitly write down the differential increment of the solar DM number in a time step δt:

$$' = \quad + C_\odot \delta t \; - N_\chi(t) \mathbf{E} \delta t \tag{4.3}$$

where vectors $^T = (\xi_1, \xi_2, \cdots, \xi_n)$ and $'^T = (\xi'_1, \xi'_2, \cdots, \xi'_n)$ denote the normalized probability for the n states at times t and $t + \delta t$, respectively, and $^T = (\eta_1, \eta_2, \cdots, \eta_n)$ represents the distribution for newly captured DM particles in time interval δt. The Markov transition matrix $\mathbf{S}$ is expressed as:

$$\mathbf{S} = \begin{pmatrix} 1 - \sum_{i \neq 1} S_{i1} & S_{12} & \cdots & S_{1n} \\ S_{21} & 1 - \sum_{i \neq 2} S_{i2} & \cdots & S_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ S_{n1} & S_{n2} & \cdots & 1 - \sum_{i \neq n} S_{in} \end{pmatrix} \tag{4.4}$$

with element S_{ji} being the probability for transition $i \to j$. Matrix $\mathbf{E} = \mathrm{diag}(e_1, e_2, \cdots, e_n)$ describes leakage due to evaporation, with e_i being the evaporation rate for the i-th state.

It is evident from Eq. (4.3) that the equilibrium distribution of the Markov chain $_{\mathrm{eq}}$, which satisfies $_{\mathrm{eq}} = {}_{\mathrm{eq}}\mathbf{S}$, well approximates the realistic distribution so long as the fractional change of DM number is negligible in the relaxation time $\delta t = t_{\mathrm{relax}}$, i.e.:

$$\left|\frac{N_\chi(t+t_{\rm relax})-N_\chi(t)}{N_\chi(t)}\right| \ll 1 \tag{4.6}$$

Therefore, for a time step $\delta t \gg t_{\rm relax}$, it is reasonable to assume that solar DM equilibrates to its limit distribution instantaneously, and the descriptions of the distribution and total number of solar DM decouple and can be treated separately. Under such circumstances, one determines the evaporation rate using the steady distribution function and in turn integrates Eq. (1.3) to obtain the number of solar DM in a self-consistent way. Note that for simplicity annihilation is not included in our discussion, which however does not cause any loss of generality in our conclusion.

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Appendix A: Collision Probability

As mentioned in Sec. 3.1, we need to calculate collision probabilities in the time interval Δt prior to sampling scattering events, and then fold these probabilities as weights with scattering samples to determine the transition matrix $S(E,L;E',L')$ efficiently. Here we provide a brief discussion on collision probability.

Considering that DM collisions are described by a Poisson process, the collision probability in time interval Δt can be expressed as:

$$P_c = 1-\exp\left(-\int_0^{\Delta t}\lambda(\tau)d\tau\right) \tag{A.1}$$

where $\lambda = n_A\langle f_A(\mathbf{u}_A)\sigma(|\mathbf{w}-\mathbf{u}_A|)\rangle$ is the collision rate. The Galilean invariant σ can be obtained by integrating the differential cross section in Eq. (2.2). However, it should be noted that for DM masses around a few GeV, the typical momentum transfer in thermal collisions is of order MeV, so we can neglect the Helm form factor for bound DM scattering processes.

Here we take operator $\hat{\mathcal{O}}_{11}$ as a specific example to illustrate how to calculate λ. First, it is straightforward to obtain the cross section:

$$\sigma_{11}(v_{\rm rel}) = \int 2\pi q dq \frac{d\sigma_{11}}{dq^2} = \text{constant}\times v_{\rm rel}^2 \tag{A.3}$$

where

$$\text{constant} = \frac{4j_\chi(j_\chi+1)}{3\pi}\left(\frac{A^2\mu_A^4}{m_N^2}\right)c_{11}^2 \tag{A.4}$$

We then input the $v_{\rm rel}^2$ dependence into the integration in Eq. (A.2) as follows:

$$\lambda = \text{constant} \times \int d_A^{3u} f_A(\mathbf{u}_A)|\mathbf{w}-\mathbf{u}_A|^2 = \text{constant} \times \frac{(5u_0^2+2w^2)}{4} \tag{A.5}$$

The analytic integration is performed using Mathematica. Thus, once the DM particle motion is specified, the collision probability can be evaluated explicitly with Eq. (A.1). As an illustration, a segment of the solar DM trajectory is shown in Fig. A.1.

Appendix B: Calculation of the Scattering Event Rate

In this appendix we provide a detailed discussion on the scattering event rate $R_A(w \to v)$ at which a DM particle scatters from initial velocity w to final velocity v off a thermal bath composed of element A per unit volume. Except for a few notations, our discussion follows closely the original calculation in Refs. [8,30].

After a coordinate transformation from the solar system to the CM system, Eq. (3.4) is expressed as an integration over transformed coordinates $(\mathbf{s}, \mathbf{t})$:

$$R_A(w \to v) = n_A(\eta_A^+)^2 m_A^2 \int d^{3s} d^{3t} f_A(\mathbf{u}_A)\frac{d\sigma}{dq^2}\delta^3(\mathbf{w}-\mathbf{v})\Theta(s+t-w)\Theta(w-|s-t|)\Theta(s+t-v)\Theta(v-|s-t|) \tag{B.1}$$

where $\eta_A^+ \equiv 1 + m_\chi/m_A$, $\mathbf{s} = (m_\chi \mathbf{w} + m_A \mathbf{u}_A)/(m_A + m_\chi)$ and $\mathbf{t} = m_A(\mathbf{w} - \mathbf{u}_A)/(m_A + m_\chi)$ are the CM velocity and DM incoming velocity in the CM frame, respectively. The scattering amplitude $\mathcal{M}$ depends on $(\mathbf{s}, \mathbf{t})$ through the transferred momentum $\mathbf{q} = m_\chi(\mathbf{t}' - \mathbf{t})$, with $\mathbf{t}'$ being the DM outgoing velocity in the CM frame.

By illustrating the relevant kinetic relation in Fig. B.1, we express the term $\delta^3(\mathbf{w} - \mathbf{v})$ as:

$$\delta^3(\mathbf{w}-\mathbf{v}) = \frac{2}{m_\chi^2 t^2}\delta(\cos\theta_{t't} - \cos\theta_{t't})\delta(\phi_{t't} - \phi_{t't}) \tag{B.2}$$

In practice, we numerically calculate Eq. (B.1) for various DM-nucleon effective interactions rather than finding an analytic expression as has been done for the simplest case $\hat{\mathcal{O}}_1$ in Ref. [8].

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Note: Figure translations are in progress. See original paper for figures.