

Neutrino Mass Ordering in Future Neutrinoless Double Beta Decay Experiments (Postprint)

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Abstract

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Full Text

Preamble

Chapter 1: Neutrino Mass Ordering in Future Neutrinoless Double Beta Decay Experiments

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Motivated by recent intensive experimental efforts on searching for neutrinoless double beta decays, we present a detailed quantitative analysis on the prospect of resolving neutrino mass ordering in the next generation ^{76}Ge -type experiments.

1. Introduction

The quest for neutrinoless double beta decay ($0\nu\beta\beta$) has experienced an interesting history. In 1939, combining the idea of double beta decay ($2\nu\beta\beta$) proposed by Goeppert-Mayer four years earlier and the notion of Majorana fermions formulated by Majorana two years prior, Furry for the first time discussed the possibility of observing two electrons but without any neutrinos during the transformation of the (A, Z) nucleus to the $(A, Z + 2)$ nucleus, i.e.,

$(A, Z) \rightarrow (A, Z + 2) + 2e^-$. At that time, based simply on phase space arguments, this $0\nu\beta\beta$ mode was believed to have a half-life around 10^{15} yr, much shorter than that of the corresponding $2\nu\beta\beta$ mode, $\sim 10^{21}$ yr. If such an estimation were true, one would have discovered $0\nu\beta\beta$ quite shortly, perhaps by the 1950s.

The first turning point in the $0\nu\beta\beta$ history was the discovery of parity violation in 1957 and the establishment of V-A theory, which concludes that if neutrinos were massless, $0\nu\beta\beta$ would be exactly forbidden. This discouraging result declared that the experimental search for $0\nu\beta\beta$ entered the “middle ages.” Although $0\nu\beta\beta$ experiments were still performed in the 1960s and 1970s, the main goal was to test the lepton number conservation law. Only in the early 1980s did the search for $0\nu\beta\beta$ embrace its “Renaissance” period, mainly because the grand unified theories developed around that time could naturally generate a Majorana mass term for neutrinos, and neutrinos with tens of eV masses were thought to be possible candidates for dark matter in cosmology.

In the current “contemporary ages,” the observation of neutrino oscillation, which indicates that neutrinos have masses, has truly boomed this field. Several experiments have been built or are under construction, and dozens of proposals have been made for future investigations. Such dedicated experimental efforts may enable us to extract rich physics results from $0\nu\beta\beta$ in the near future. For example:

- **Pinning down the nature of neutrinos.** According to the Schechter-Valle theorem, the observation of $0\nu\beta\beta$ would imply a Majorana mass term for neutrinos, proving the Majorana nature of neutrinos.
- **Constraining the absolute neutrino mass scale and lepton CP-violating phases.** This is because if $0\nu\beta\beta$ is mediated by the light neutrino Majorana mass term, the decay half-life $T_{1/2}^{0\nu}$ can then be written as

$$\left(T_{1/2}^{0\nu}\right)^{-1} = G^{0\nu} |M^{0\nu}|^2 m_{\beta\beta}^2$$

where $G^{0\nu}$ and $M^{0\nu}$ stand for the phase-space factor and the nuclear matrix element (NME), respectively, and $m_{\beta\beta}$ is the so-called effective neutrino mass. Although there still exist large uncertainties in the calculation of NME, in principle one can obtain the former two accurately within nuclear theory. Then, the measured limit on the half-life $T_{1/2}^{0\nu}$ would directly yield a constraint on $m_{\beta\beta}$, whose definition is given by

$$m_{\beta\beta} = \left| m_1 \cos^2 \theta_{12} \cos^2 \theta_{13} e^{2i\phi_1} + m_2 \sin^2 \theta_{12} \cos^2 \theta_{13} e^{2i\phi_2} + m_3 \sin^2 \theta_{13} \right|,$$

where m_i 's are neutrino masses, and θ_{ij} 's and ϕ_i 's are the mixing angles and CP-violating phases in the lepton mixing matrix, respectively.

- **Determining the neutrino mass ordering.** This is due to the presence of a lower bound for the above effective neutrino mass $m_{\beta\beta}$ in the inverted neutrino mass ordering, according to the current global fit results.

- **Restricting new physics contributions.** As $0\nu\beta\beta$ is a rare process, any new physics that contributes to it should be constrained by the obtained data.

In this work we focus on the question of determining neutrino mass ordering in the next generation $0\nu\beta\beta$ experiments. Our goal is to provide a quantitative description of experimental power to distinguish between normal neutrino mass ordering (NO) and inverted neutrino mass ordering (IO). For simplicity, we only consider ^{76}Ge -type experiments, as their background is found to be nearly flat in the signal region. Moreover, we adopt Bayesian analysis as our statistical method.

2. Description of $0\nu\beta\beta$ Experiments

Since we are discussing the future generation $0\nu\beta\beta$ experiments, it would be beneficial to review what the indispensable ingredients are if one wants to build an ultimate $0\nu\beta\beta$ experiment:

- **Large amount of $\beta\beta$ -decaying isotopes.** The number of signal events $N^{0\nu}$ is directly related to the fiducial mass of $\beta\beta$ -decaying isotopes. Taking ^{76}Ge as an example, we have

$$N^{0\nu} = \frac{\ln 2 \cdot N_A \cdot \mathcal{E} \cdot \epsilon}{M_{Ge}} \cdot \frac{1}{T_{1/2}^{0\nu}}$$

where N_A is Avogadro's constant, $M_{Ge} = 75.6$ g/mol is the molar mass of ^{76}Ge , and \mathcal{E} and ϵ are the exposure and detection efficiency, respectively. Since the exposure \mathcal{E} is defined as the product of the fiducial mass and the experiment operating time, increasing the mass of isotopes is helpful to reduce the detection time. In Table 1 we list the typical sizes of the fiducial mass in the previous, current and next generation $0\nu\beta\beta$ experiments.

- **Good energy resolution.** Ideally, all the above signal events should appear at the Q-value of the corresponding $0\nu\beta\beta$ process. For ^{76}Ge , this Q-value is 2039 keV. However, because of finite energy resolution, these signal events spread out and can be easily contaminated by background events, especially those from the intrinsic $2\nu\beta\beta$ process. Because ^{76}Ge -type experiments employ the “detector = studied substance” scheme, a very high energy resolution can be reached. In this work we take FWHM (full width at half maximum) as 3 keV.
- **Low background.** It has been found that reducing the level of background is very helpful for improving the half-life sensitivity. Moreover, if possible, one should pursue the zero background limit, so that a single $0\nu\beta\beta$ event would be enough to pin down the Majorana nature of neutrinos. For the next generation ^{76}Ge -type experiments we will choose $BI = 10^{-4}$ counts/keV/kg/yr, with BI standing for background index. BI values for other generations of $0\nu\beta\beta$ experiments are also given in Table 1.

- **Good detection efficiency.** $0\nu\beta\beta$ occurs so rarely that any signal event would be precious. Here we take the typical value of $\epsilon = 0.65$ for ^{76}Ge -type experiments.
- **Good scalability.** By scalability we mean the ability to scale the experimental set-up to a larger size. This is very important for building a large $0\nu\beta\beta$ experiment stage by stage.

Currently, we have two commissioning ^{76}Ge -type $0\nu\beta\beta$ experiments, namely GERDA and MAJORANA DEMONSTRATOR. In the first phase of GERDA, a lower limit of $T_{1/2}^{0\nu} > 2.1 \times 10^{25}$ yr at 90% confidence level was reported for an exposure of $21.6 \text{ kg} \cdot \text{yr}$ and a BI of about 10^{-2} . The GERDA Phase-II and MAJORANA DEMONSTRATOR are expected to increase the exposure to about $200 \text{ kg} \cdot \text{yr}$ each, and to reduce BI to the 10^{-3} level simultaneously. More excitingly, these two collaborations are also discussing the possibility of building a future large scale ^{76}Ge (LSGe) experiment together, which may eventually reach an exposure of around $10^4 \text{ kg} \cdot \text{yr}$ and $BI = 10^{-4}$. It is this possibility that motivates us to perform a detailed statistical analysis, so as to find out at which level the currently undetermined neutrino mass ordering can be resolved.

3.1. Generating Pseudo-data Sample

We begin by generating pseudo-data samples to be used in the subsequent statistical analysis. Following GERDA's analysis of its first phase data, we consider a spectral region that spans from 2022 keV to 2061 keV with a bin size of 1 keV. See Fig. 1 for an example of the simulated event spectrum. The red curve represents the total un-binned signal and background events, while the binned results are given by the gray histogram, where Poisson statistics is assumed.

In Ref. [6], we indeed adopt the above generated spectrum, which incorporates statistical fluctuation, to perform analysis on the exclusion limit and discovery potential in upcoming ^{76}Ge -type experiments. However, for the current discussion on the determination of neutrino mass orderings, using the above statistical-fluctuation-included spectrum would be very time-consuming. This is because for each set of input parameters, one needs to generate and then analyze hundreds of pseudo-data samples to obtain sufficient statistics.

For simplicity, here we instead take the so-called Asimov data set, which is obtained by simply assuming the expected number of events for each bin in the spectrum. In terms of the example given in Fig. 1, the Asimov data set corresponds to the events under the red curve. Apparently, no statistical fluctuations are included in this Asimov data set. However, it has been found that the analysis of the Asimov data yields a good approximation to the median projection of experiments. For our current discussion such a median projection is sufficient.

3.2. Bayesian Analysis

We adopt Bayesian analysis as our statistical method. Bayesian analysis resides on the well-known Bayes' theorem, which describes the degree of belief in a certain hypothesis H_i , given the data set D . Specifically, Bayes' theorem states that

$$\Pr(H_i|D) = \frac{\Pr(D|H_i) \Pr(H_i)}{\Pr(D)}$$

where $\Pr(H_i)$ and $\Pr(H_i|D)$ are the prior and posterior probabilities of the hypothesis H_i , respectively. The probability of obtaining the data D , given the hypothesis H_i to be true, is denoted as $\Pr(D|H_i)$ and is also called the evidence Z_i . Lastly, $\Pr(D)$ stands for the overall probability of observing the data D , and it is equal to $\sum_i \Pr(D|H_i) \Pr(H_i)$ because of the normalization condition $\sum_i \Pr(H_i|D) = 1$.

Following the above formalism, we take our two competing hypotheses as H_{NO} and H_{IO} , and the above discussed pseudo-data samples serve as our data set D . To find out the more favorable hypothesis, we compute the posterior odds

$$\frac{\Pr(H_{\text{NO}}|D)}{\Pr(H_{\text{IO}}|D)} = \frac{\Pr(D|H_{\text{NO}})}{\Pr(D|H_{\text{IO}})} \cdot \frac{\Pr(H_{\text{NO}})}{\Pr(H_{\text{IO}})}$$

where the ratio of evidences $B \equiv Z_{\text{NO}}/Z_{\text{IO}}$ is termed the Bayes factor. If a priori we have no preference for a particular mass ordering, the above posterior odds is then directly reflected by the Bayes factor B . Furthermore, to interpret the value of this posterior odds or the Bayes factor, we adopt the Jeffreys scale given in Table 2.

The computation of Z_{NO} is performed via

$$Z_{\text{NO}} = \int \Pr(D|\Theta, \text{NO}) \pi(\Theta) d^N \Theta$$

(similar computation for Z_{IO}). Θ stands for the parameters in the hypothesis H_{NO} , and here they are three lepton mixing angles θ_{ij} , two CP-violating phases $\{\phi_1, \phi_2\}$, two neutrino mass-squared differences and the lightest neutrino mass m_0 . Their prior probability distributions are collectively denoted as $\pi(\Theta)$. For the mixing angles and mass-squared differences, we assume Gaussian priors with central values and 1σ errors taken from Ref. [5]. For CP-violating phases, uniform priors are chosen, and for the lightest neutrino mass m_0 we adopt two different priors: a uniform prior within $[0, 0.2]$ eV and a logarithmic prior on $[10^{-5}, 0.2]$ eV. Finally, we obtain $\Pr(D|\Theta, \text{NO})$ by fitting the pseudo-data set with the above sampled parameters in the realm of H_{NO} .

3.3. Numerical Results

We now present the final numerical results on the discrimination of two neutrino mass orderings in the next generation ^{76}Ge $0\nu\beta\beta$ experiments, see Fig. 2.

In Fig. 2, the left and right panels correspond to the uniform and logarithmic priors on the lightest neutrino mass m_0 , respectively. Given a true value of $m_{\beta\beta}$ and an exposure \mathcal{E} , we can calculate the Bayes factor B and show its contours as orange, black and blue curves for $\ln(B) = 2.5, 1$ and -1 , respectively. Thick solid and dashed curves distinguish two different choices of NME: $M^{0\nu} = 4.6$ and 5.8 for the former and latter, respectively. For reference, we also plot the lower bound of $m_{\beta\beta}$ in IO: the red horizontal line represents the result calculated using the best-fit values of neutrino mixing parameters, while the dark (light) band is for the result using 1σ (3σ) ranges.

From Fig. 2, we make the following observations:

- An exposure of at least 500 (or 2500) $\text{kg} \cdot \text{yr}$ is needed in order to report a weak (or moderate) evidence for NO. According to the Jeffreys scale in Table 2, the weak and strong evidence should be understood as a degree of belief of 75.0% and 92.3%, respectively.
- In the next generation of ^{76}Ge -based $0\nu\beta\beta$ experiments with an exposure of 10^4 $\text{kg} \cdot \text{yr}$, we are only able to reach a value of $\ln(B) \gtrsim 2.5$, indicating moderate evidence for NO. To obtain strong evidence ($\ln(B) = 5$), one needs to further increase the exposure or reduce the level of background.

4. Summary and Conclusion

The question of whether the neutrino is its own antiparticle, namely the nature of the neutrino, has led to a long history of searching for $0\nu\beta\beta$ experimentally. In the course of such a dedicated search, one also realizes that studying $0\nu\beta\beta$ may help us distinguish the currently unresolved neutrino mass ordering. Motivated by this latter fact, we have performed a detailed quantitative analysis, aiming to find out at which statistical level one can exclude the inverted neutrino mass ordering in the next generation ^{76}Ge -type $0\nu\beta\beta$ experiments. Our findings indicate that the next generation ^{76}Ge -type $0\nu\beta\beta$ experiments indeed have some sensitivity to the discrimination of neutrino mass orderings, though not in a very decisive manner.

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References

5. M. C. Gonzalez-Garcia, M. Maltoni and T. Schwetz, JHEP 1411, 052 (2014) [arXiv:1409.5439 [hep-ph]].

6. J. Zhang and S. Zhou, Phys. Rev. D 93, no. 1, 016008 (2016) [arXiv:1508.05472 [hep-ph]].
7. J. J. Gomez-Cadenas, J. Martin-Albo, M. Mezzetto, F. Monrabal and M. Sorel, Riv. Nuovo Cim. 35, 29 (2012) [arXiv:1109.5515 [hep-ex]].
8. N. Abgrall et al. [Majorana Collaboration], Adv. High Energy Phys. 2014, 365432 (2014) [arXiv:1308.1633].
9. M. Agostini et al. [GERDA Collaboration], Phys. Rev. Lett. 111, no. 12, 122503 (2013) [arXiv:1307.4720].
10. Neutrinoless Double Beta Decay Report to the Nuclear Science Advisory Committee, April 24, 2014 (<http://science.energy.gov/np/nsac/reports>).
11. G. Cowan, K. Cranmer, E. Gross and O. Vitells, Eur. Phys. J. C 71, 1554 (2011) [Eur. Phys. J. C 73, 2501 (2013)] [arXiv:1007.1727].
12. M. Hobson et al., eds., Bayesian methods in cosmology (Cambridge University Press, 2010).

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