Spectra of heavy-light mesons in a relativistic model (Postprint)

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Abstract

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Full Text

Preamble

Spectra of Heavy-Light Mesons in a Relativistic Model

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The spectra and wave functions of heavy-light mesons are calculated within a relativistic quark model derived from the instantaneous Bethe-Salpeter equation by applying the Foldy-Wouthuysen transformation on the heavy quark. The kernel we choose is based on scalar confining and vector Coulomb potentials. The Hamiltonian for the heavy-light quark-antiquark system is calculated up to order $1/m^2 Q$. *The results are in good agreement with available experimental data except for the masses of the anomalous D{s0}(2317) and D*{s1}(2460) states.

The newly observed charmed meson states can be accommodated successfully in the relativistic model, and their assignments are presented: the D{sJ}(2860) can be interpreted as the $|1^3/_2D_1$ and $|1^5/_2D_3$ states being the J^P = 1^- and 3^- members of the 1D family in our model.

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I Introduction

Great experimental progress has been achieved in studying the spectroscopy of heavy-light mesons in the last decades, especially in the charm sector [1–8]. Several new excited charmed meson states were discovered in addition to the low-lying states. For D_J mesons, the resonances $D(2740)^0$, D*(2760)$^{0,+}$ [1], D*(2650) and D*(3000) [2] were observed, while for D_{sJ} mesons, the resonances D^+{sJ}(2700) [5] and D^+{sJ}(3040) [6] were established. As for the b-flavored meson sector, several higher excitation states were studied in experiment as well as the ground states of B and B_s mesons [9]. The heavy-light $Q\bar{q}$ system plays an important role in understanding the strong interactions between quark and antiquark, while the spectroscopy provides a powerful test of the theoretical predictions based on the quark model in the standard model.

Heavy-light mesons have been investigated extensively in relativistic potential models in Refs. [10–16], where many relativistic potential models are constructed by modifying or relativizing nonrelativistic quark potential models. For heavy-light systems, one needs a model that can include the relativistic effects of the light quark. In this work we resort to the Bethe-Salpeter equation, which is originally relativistic. Many studies of mesons were carried out in the Bethe-Salpeter approach [17–22]. It is very difficult to solve the Bethe-Salpeter equation, especially when we consider states with large angular momentum number. In order to study mesons systematically, we need to reduce the Bethe-Salpeter equation.

In previous work [23], we applied the instantaneous approximation and derived an equation equivalent to the Bethe-Salpeter equation. The Hamiltonian for the heavy-light quark-antiquark system was expanded to order 1/m_Q by Foldy-Wouthuysen transformation on the heavy quark. We found that the leading Hamiltonian is actually not Dirac-like. The interactions we obtain by reducing the equivalent equation are essentially different from the Breit interaction, which is widely used in studying quark-antiquark systems as a nonrelativistic or semirelativistic model [24–26].

In this paper we extend and improve our study of the heavy-light meson spectra. The running coupling constant is considered. Moreover, $1/m^2$_Q correction is calculated. We find that the $1/m^2$_Q correction to the mass of the mesons is about 50 to 100 MeV. The correction is too large to be ignored if we are to obtain reliable spectra. The parameters in the equations are determined by fitting the spectra of the heavy-light meson states presented in the Particle Data Group [9]. The large discrepancy from experimental data in the previous work is decreased, and our results are more consistent with experiments. The assignments of the newly observed heavy-light charmed meson states are presented based on the theoretical results.

This paper is organized as follows. In Section II we derive the effective Hamiltonian for the heavy-light quark-antiquark system. Section III presents the solution of the relativistic wave equation, and Section IV the perturbative corrections. Section V contains the numerical results and discussion. In Section VI we provide a brief summary.

II The Model

The Bethe-Salpeter Equation for the quark-antiquark system can be written as [27]:

$$(\not{p}_1 - m_1)\chi(p)(\not{p}_2 + m_2) = (2\pi)^4 \int K(p, p', P)\chi(p'), \tag{1}$$

where the total momentum P (in the center-of-mass reference, $P = (E, 0)$) and the relative momentum p are defined as:

$$P = p_1 + p_2, \quad p = \alpha_1 p_2 - \alpha_2 p_1, \quad \alpha_i = \frac{m_i}{m_1 + m_2}, \quad i = 1, 2.$$

Here we choose the kernel in a form that can be reduced to the effective Hamiltonian used in Ref. [16]. The kernel is based on scalar confining potential and vector Coulomb potential which includes the transverse interaction of the gluon exchange. The kernel can be written as:

$$K(p, p', P) = \gamma_\mu^{(1)} \gamma^{(2)\mu} V_v(k^2) + V_s(k^2),$$

where k is the transferred four-momentum, $k = p - p'$. By taking the instantaneous approximation, we can perform the integration over p_0 and the wave function of the instantaneous Bethe-Salpeter equation decouples from the time coordinate. Now we can transform the instantaneous Bethe-Salpeter equation into coordinate space by Fourier transformation. Details of this derivation can be found in Refs. [28–31].

In previous work [23], we derived that the instantaneous Bethe-Salpeter equation is equivalent to the following equation:

$$\omega_1 + \omega_2 + (h_1 + h_2)U(h_1 + h_2)\phi = 0, \tag{7}$$

where $\omega_i = \sqrt{p^2 + m_i^2}$, $h_i = \frac{\mathbf{p}\cdot\alpha^{(i)} + \beta^{(i)} m_i}{\omega_i}$, $i = 1, 2$. $V_v(r)$ and $V_s(r)$ are the Fourier transformations of $V_v(k^2)$ and $V_s(k^2)$, respectively.

The instantaneous Bethe-Salpeter equation as an integral equation is now equivalent to a less complicated differential equation as in Eq. (7), but still it is hard to solve. The wave function $\phi(r)$ has two spinor indices, i.e., $4 \times 4 = 16$ components, making it difficult to solve an eigenequation with so many components.

For heavy-light systems, since the heavy quark Q is sufficiently heavy compared to the light antiquark $\bar{q}$, it is reasonable to consider the nonrelativistic expansion of the heavy quark, i.e., the $1/m_Q$ expansion, and one can reduce the eigenequation by calculating the interactions of the heavy-light quark-antiquark systems order by order. The reduction can be achieved in a systematic way by using the Foldy-Wouthuysen transformation [32, 33].

Here we designate the index 1 and 2 in Eq. (7) as the heavy quark Q and the light antiquark $\bar{q}$, respectively. Thus we perform the transformation on the heavy quark with the superscript "1". The representation of the Dirac matrix we choose to reduce the Hamiltonian is the Dirac representation, where

$$\alpha = \begin{pmatrix} 0 & \sigma \\ \sigma & 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}.$$

If the original Hamiltonian is written in the form $H = \beta m + \mathcal{E} + \mathcal{O}$, where $\mathcal{O}$ is the "odd" operator (typical examples are the matrices α and γ), while $\mathcal{E}$ is the "even" operator (examples of this class of operators are 1, β and Σ). According to the Foldy-Wouthuysen transformation, the transformed Hamiltonian reads

$$\tilde{H} = U_F^{-1} H U_F = \beta m + \mathcal{E} + \frac{1}{2m}\beta\mathcal{O}^2 + \frac{1}{8m^2}[[\mathcal{O}, \mathcal{E}], \mathcal{O}] - \frac{1}{8m^3}\{\mathcal{O}^4, \beta\} + \cdots$$

Now we return to the reduction of the Hamiltonian for the heavy-light system. Eq. (7) can be written as (the wave function in the equation is omitted):

$$h_1 E + \Delta\omega_1 + \omega_2 + (h_1 + h_2) U (h_1 + h_2) = 0, \tag{17}$$

where $\Delta\omega_1 = \omega_1 - E$. Now we consider performing the Foldy-Wouthuysen transformation on Eq. (17); the left side of it can be rewritten as $(1 + \tilde{H}')\phi = 0$, where $\tilde{H}'$ is the transformed Hamiltonian. (The reason is explained in detail in the next section.) Then we get:

$$\tilde{H}' = \tilde{H}'_a + \tilde{H}'_{\beta^{(1)} E},$$

where

$$\tilde{H}'_a = \frac{\beta^{(1)} E + \Delta\omega_1 + \omega_2 + (h_1 + h_2) U (h_1 + h_2)}{\omega_1}$$

and

$$\tilde{H}'_{\beta^{(1)}E} = \frac{\beta^{(1)}E + \Delta\omega_1 + \omega_2 + (h_1 + h_2)U(h_1 + h_2)}{\omega_1}.$$

The odd and even operators in the above equation are:

$$\mathcal{O} = \frac{\beta^{(1)}\alpha^{(1)} \cdot \mathbf{p}}{\omega_1} + \frac{\beta^{(1)} + h_2}{\omega_1}U(h_1 + h_2) + \text{h.c.}, \tag{20}$$

$$\mathcal{E} = \frac{\beta^{(1)}E + \Delta\omega_1 + \omega_2}{\omega_1} + \frac{\beta^{(1)} + h_2}{\omega_1}U_1\frac{\beta^{(1)} + h_2}{\omega_1} + \text{h.c.}, \tag{21}$$

where h.c. stands for Hermitian conjugate.

By applying the new form of Eq. (16)

$$\tilde{H} = \beta m + \mathcal{E} + \frac{1}{2E}\beta\mathcal{O}^2 + \frac{1}{8E^2}[[\mathcal{O}, \mathcal{E}], \mathcal{O}] + \cdots,$$

we insert the odd and even operators in Eqs. (21) and (20) into Eq. (22) and expand Eq. (17) to order $(1/E)^2$. After the simplification of the Hamiltonian, we take the substitution $\beta^{(1)} \to 1$ in the Foldy-Wouthuysen transformation if $\beta^{(1)}$ appears on the left or right side of the Hamiltonian, then we obtain:

$$\tilde{H} = \tilde{H}_0 + \tilde{H}',$$

where

$$\tilde{H}_0 = \omega_1 + \omega_2 + (1 + h_2)U_1(1 + h_2). \tag{25}$$

The perturbative term $\tilde{H}'$ consists of many different terms of order $1/m_Q$ and $1/m_Q^2$. Since the corrections are calculated as a perturbation to $\tilde{H}_0$, we can take the substitution $h_2 \to 1$ if h_2 appears on the left or right side of U_1 and U_2:

$$U_1(r) = V_v(r) + \beta^{(1)}\beta^{(2)}V_s(r),$$

$$U_2(r) = [\alpha^{(1)} \cdot \alpha^{(2)} + (\alpha^{(1)} \cdot \hat{r})(\alpha^{(2)} \cdot \hat{r})]V_v(r).$$

We can reduce the above equations by inserting $(\gamma_5)^2 = 1$ between two odd operators of the heavy quark. By using the relations $\{\gamma_5, \beta\} = 0$, $[\gamma_5, \alpha] = 0$, $\gamma_5\alpha = \Sigma$, and the substitutions $\beta^{(1)} \to 1$, $\alpha^{(1)} \to \sigma^{(1)}$, we obtain the final Hamiltonian:

$$H_0 = \omega_1 + \omega_2 + (1 + h_2)U_1(1 + h_2), \tag{30}$$

$$H'_{1/m_Q} = \frac{1}{2m_1}\{\sigma^{(1)} \cdot \mathbf{p}, U_1\} + \text{h.c.}, \tag{31}$$

$$H'_{1/m_Q^2} = H'_a + H'_b,$$

where U_1 and U_2 are defined as:

$$U_1(r) = V_v(r) + \beta^{(2)}V_s(r),$$

$$U_2(r) = [\sigma^{(1)} \cdot \alpha^{(2)} + (\sigma^{(1)} \cdot \hat{r})(\alpha^{(2)} \cdot \hat{r})]V_v(r).$$

The zeroth order Hamiltonian H_0 we obtain for the heavy-light quark-antiquark system in Eq. (30) is not Dirac-like as in Refs. [34, 35]. Its form is more like the form used in relativized quark models [16, 36, 37]. As for double-heavy systems, we have $h_2 \to 1$ and $\beta^{(2)} \to 1$, then H_0 can be reduced to:

$$H_0 = \omega_1 + \omega_2 + V_v(r) + V_s(r),$$

which is the Schrödinger formalism extensively used in nonrelativistic or semirelativistic quark models.

III Solution of the Wave Equation

In this section, we solve the eigenequation of the zeroth order Hamiltonian H_0 in Eq. (30). Before doing this, we would like to discuss the properties of the solution of the eigenequation associated with H_0.

The eigenequation of H_0 can be written as:

$$\omega_1 + \omega_2 + (1 + h_2)U_1(1 + h_2)\psi = 0. \tag{39}$$

The above equation is equivalent to:

$$\omega_1 + \omega_2 + (1 + h_2)U_1(1 + h_2)\psi = 0, \tag{40}$$

and it is easy to verify that the eigenfunction set of the new Hamiltonian includes both subsets: $h_2\psi = \psi$ and $h_2\psi = -\psi$, where the subset $h_2\psi = \psi$ is identical to the eigenfunction set associated with the original Hamiltonian H_0 in Eq. (39).

Now we turn to solving the eigenequation associated with the new Hamiltonian H_0, that is:

$$\omega_1 + \omega_2 + \{h_2, U_1\}\psi = 0. \tag{41}$$

It is easy to verify that the operators $\{j^2, j_z, K, S_z\}$ are a set of mutually commuting operators which commute with the new Hamiltonian H_0, where $\mathbf{j} = \mathbf{L} + \mathbf{S}^{(2)}$, $\mathbf{S}^{(2)} = \frac{1}{2}\sigma^{(1)}$. Then the eigenstates of H_0 can be labeled by the corresponding set of quantum numbers $\{n, j, m_j, k, s\}$. The eigenequation associated with H_0 can be written as:

$$H_0 \Psi^{(0)}_{n,k,j,m_j,s}(\mathbf{r}) = E^{(0)}_{n,k,j} \Psi^{(0)}_{n,k,j,m_j,s}(\mathbf{r}), \tag{46}$$

where

$$\Psi^{(0)}_{n,k,j,m_j,s}(\mathbf{r}) = \begin{pmatrix} g_{n,l,j}(r) y^{m_j}_{j,l}(\theta,\phi) \\ i f_{n,l,j}(r) y^{m_j}_{j,2j-l}(\theta,\phi) \end{pmatrix}, \tag{50}$$

and $y^m_{j,l}$ is the spherical spinor. For $j = l \pm 1/2$, we have:

$$y^m_{j,l} = \begin{pmatrix} \pm\sqrt{\frac{l \pm m + 1/2}{2l+1}} Y_l^{m-1/2} \\ \sqrt{\frac{l \mp m + 1/2}{2l+1}} Y_l^{m+1/2} \end{pmatrix},$$

where Y_l^m is the spherical harmonics. For $k = \pm(j + 1/2)$, the eigenfunction set of H_0 is NOT complete.

A complete set is needed to construct the identity operator $\mathbf{1} = \sum_i |i\rangle\langle i|$ in order to calculate the perturbative correction of H'. Thus we construct a new Hamiltonian. Inspired by the relation $h_2\psi = \psi$, we transform the potential term in Eq. (39) as:

$$(1 + h_2)U_1(1 + h_2) = \frac{1}{2}\{h_2, U_1\} + \frac{1}{2}[h_2, U_1] + \frac{1}{2}U_1 h_2 + \frac{1}{2}h_2 U_1.$$

Then the new Hamiltonian we construct can be written as:

$$H_0 = \omega_1 + \omega_2 + \{h_2, U_1\}. \tag{46}$$

The normalization condition of Eq. (50) is:

$$\int dr\, r^2 (f^2_{n,l,j} + g^2_{n,l,j}) = 1. \tag{54}$$

For a state with quantum number j, the operator K can have two opposite eigenvalues $\pm(j + 1/2)$. With $H_0 = \omega_1 + \omega_2 + \{h_2, U_1\}$, we have:

$$K\Psi^{(0)}_{n,k,j,m_j,s}(\mathbf{r}) = k\Psi^{(0)}_{n,k,j,m_j,s}(\mathbf{r}),$$

where the quantum number k is defined as $k = \pm(j + 1/2)$.

We rewrite the spinor part of Eq. (50) as:

$$\Psi(\mathbf{r}) = \begin{pmatrix} \Psi_A \\ \Psi_B \end{pmatrix},$$

where the superscripts "A" and "B" stand for the upper and lower parts of Eq. (57). By using the formula (here σ stands for $\sigma^{(2)}$):

$$\mathbf{p} = i(\sigma \cdot \hat{r}) \left(\frac{\partial}{\partial r} + \frac{1}{r} - \frac{\sigma \cdot \mathbf{L}}{r} \right),$$

we have:

$$\mathbf{p} g(r) y^{m_j}_{j,l_B} = i \left(\frac{\partial}{\partial r} + \frac{k+1}{r} \right) g(r) y^{m_j}_{j,l_A},$$

$$\mathbf{p} f(r) y^{m_j}_{j,l_A} = i \left(\frac{\partial}{\partial r} - \frac{k-1}{r} \right) f(r) y^{m_j}_{j,l_B},$$

where the relations $\sigma \cdot \hat{r}\, y^{m_j}_{j,l_B} = -y^{m_j}_{j,l_A}$ and $\sigma \cdot \hat{r}\, y^{m_j}_{j,l_A} = -y^{m_j}_{j,l_B}$ are used.

Inserting Eq. (8) and (10) into Eq. (46), we can rewrite H_0 in the matrix form:

$$H_0 = \begin{pmatrix} \omega_1 + \omega_2 + H_a & H_b \\ H_c & \omega_1 + \omega_2 + H_d \end{pmatrix},$$

where

$$H_a = (V_v + V_s),$$

$$H_b = (V_v - V_s),$$

$$H_c = (V_v + V_s),$$

$$H_d = (V_s - V_v).$$

For a bound state of two particles, when the separation between them is large enough, the wave function drops dramatically. The wave function will effectively vanish at a typically large distance L, and the quark and antiquark can be treated as if they are restricted in a limited space, $0 < r < L$. Thus the unsolved functions $f(r)$ and $g(r)$ can be expanded in terms of the spherical Bessel function in the limited space. We write the basis as:

$$\psi_i^A(r) = N_n j_{l_A}(k_n r) Y_{l_A m}(\hat{r}),$$

$$\psi_\alpha^B(r) = N_n j_{l_B}(k_n r) Y_{l_B m}(\hat{r}),$$

with $k_n = a_n/L$, where a_n is the n-th root of the spherical Bessel function $j_l(x) = 0$. N_n is added in accordance with the normalization condition in Eq. (54). In the limited space, the momentum k is discrete, and we can have the relevance $p \to k$.

The eigenfunction of H_0 can be expanded in the orthonormalized basis $\{\psi_i^A, \psi_\alpha^B\}$. In the numerical calculation, the infinite summation can be truncated at a large integer N:

$$\Psi_A = \sum_{i=1}^{N} g_i \psi_i^A, \quad \Psi_B = \sum_{\alpha=1}^{N} f_\alpha \psi_\alpha^B. \tag{72}$$

According to Eq. (63), we can rewrite the eigenequation of H_0 in the representation of $\{\psi_i^A, \psi_\alpha^B\}$. In this representation, the operator H_0 can be written in its matrix form:

$$H_0 = \begin{pmatrix} \langle \omega_1 + \omega_2 \rangle_{ij} + \langle H_a \rangle_{ij} & \langle \omega_1 + \omega_2 \rangle_{i\beta} + \langle H_b \rangle_{i\beta} \\ \langle \omega_1 + \omega_2 \rangle_{\alpha j} + \langle H_c \rangle_{\alpha j} & \langle \omega_1 + \omega_2 \rangle_{\alpha\beta} + \langle H_d \rangle_{\alpha\beta} \end{pmatrix} + \text{h.c.},$$

where the matrix elements of H_0 can be calculated by applying Eq. (59), Eq. (60) and the eigenequation of $\Omega(p)$, which is derived in the previous work [23]:

$$\Omega(p) j_l(kr) Y_{lm}(\hat{r}) = \Omega(k) j_l(kr) Y_{lm}(\hat{r}),$$

where $\Omega(p)$ is a pseudo-differential operator function and $\Omega(k)$ is a normal function, p and k stand for the modules of momentum operator $\mathbf{p}$ and momentum k, respectively.

With the normalization condition Eq. (53), we can get the matrix elements of H_0 easily:

$$\langle \omega_1 + \omega_2 \rangle_{ij} = \int dr\, r^2 j_{l_A}(k_i r)(\omega_1(p) + \omega_2(p)) j_{l_A}(k_j r) = (\omega_1(k_i) + \omega_2(k_i))\delta_{ij},$$

$$\langle\omega_1+\omega_2\rangle_{\alpha\beta} = (\omega_1(k_\alpha)+\omega_2(k_\alpha))\delta_{\alpha\beta}.$$

Here we define a symbolic notation:

$$\langle\phi\rangle_{i,l_A;j,l_B} = \int dr\, r^2 j_{l_A}(k_i r)\phi(r) j_{l_B}(k_j r).$$

Then we have:

$$\langle H_a\rangle_{ij} = \frac{k_i+1}{2\omega_2(k_i)}\langle V_v+V_s\rangle_{i,l_A;j,l_A},$$

$$\langle H_b\rangle_{i\beta} = \frac{1}{2\omega_2(k_i)}\langle(V_v-V_s)\rangle_{i,l_A;\beta,l_B},$$

$$\langle H_c\rangle_{\alpha j} = \frac{1}{2\omega_2(k_\alpha)}\langle V_s-V_v\rangle_{\alpha,l_B;j,l_A},$$

$$\langle H_d\rangle_{\alpha\beta} = \frac{k_\alpha-1}{2\omega_2(k_\alpha)}\langle V_v+V_s\rangle_{\alpha,l_B;\beta,l_B}.$$

Diagonalizing the Hermitian matrix of H_0, we can get the eigenenergy of H_0 and the coefficients g_i, f_α, which are defined in Eq. (72). Then the eigenequation associated with H_0 is solved and the eigenfunction is obtained.

IV 1/m^2_Q Correction

Now we discuss the perturbative corrections of H'. The perturbative term H' does not commute with any of the operators introduced in solving the eigenequation of H_0, but still it commutes with $\{J^2, J_z, P\}$, where $\mathbf{J}=\mathbf{j}+\mathbf{S}$ and P is the parity operator. Thus the eigenstates of the total Hamiltonian $H=H_0+H'$ can be labeled by the set of quantum numbers $\{n, J, M_J, P\}$.

With the help of Clebsch-Gordan coefficients, we can compose the basis states with the quantum numbers set $\{n, k, j, J, M_J\}$ by combining the eigenstates of H_0:

$$\Psi_{n,k,j;J,M_J}(r) = \sum_{m_j,s} C^{J,M_J}_{j,m_j;1/2,s}\Psi^{(0)}_{n,k,j,m_j,s}(r),$$

then calculate the corrections and mixings in the new basis.

The $1/m_Q$ and $1/m_Q^2$ perturbative terms are given in Eqs. (32)-(34). Due to the properties of the eigenfunction of H_0, the $(1+h_2)/2$ term on both left and right sides of H' can be taken to 1 in this work. The properties of the eigenfunctions can be of great help in calculating the perturbative correction of the $1/m_Q^2$ term H_b' in Eq. (34), which can be rewritten as:

$$H_b' = \frac{1}{2m_1 E^{(0)}}\left[\frac{1+h_2}{2}U_1\frac{1-h_2}{2}U_1\frac{1+h_2}{2} + \text{h.c.}\right]. \tag{81}$$

From Eq. (81), the correction of H_b' for ψ_n^+ can be written as:

$$\delta E^{(b)}_{n,l,j,J} = \frac{1}{2m_1 E^{(0)}_{n,l,j}}\sum_m \langle\psi_n^+|\frac{1+h_2}{2}U_1\frac{1-h_2}{2}|\psi_m^-\rangle\langle\psi_m^-|U_1\frac{1+h_2}{2}|\psi_n^+\rangle + \text{h.c.}$$

$$= \frac{1}{2m_1 E^{(0)}_{n,l,j}}\sum_m A_{nm}B_{mn},$$

where

$$A_{nm} = \langle\psi_n^+|\frac{1+h_2}{2}U_1\frac{1-h_2}{2}|\psi_m^-\rangle,$$

$$B_{mn} = \langle\psi_m^-|U_1\frac{1+h_2}{2}|\psi_n^+\rangle.$$

The formulas for calculating $1/m_Q$ correction have been given in the earlier work [23], where the mixing between states with $j = l \pm 1/2$ is considered. Here we focus on the $1/m_Q^2$ corrections. With the wave function solved in the last section we can calculate the $1/m_Q^2$ corrections by applying the above formulas. For different J^P states, the $1/m_Q^2$ corrections are given below:

(1) $J^P = 0^-$: Only state with $j = 1/2$, $l = 0$ can construct the $J^P = 0^-$ state. We have:

$$\delta E^{(a)}_{n,0,1/2,0} = \int dr\left[(V_v+V_s)(4f^2+4rff'+g(2g'+rg'')) + (V_v-V_s)r^2 f(2f'+rf'')\right]dr,$$

$$A_{nm} = \int dr\, i(V_v-V_s)r^2\tilde{f}g\,dr,$$

$$B_{mn} = \int dr\, i(V_v+V_s)r(2f\tilde{f}+r\tilde{f}f'+g(2\tilde{g}+r\tilde{g}'))\,dr,$$

where f and g stand for $f_{n,0,1/2}$ and $g_{n,0,1/2}$, and $\tilde{f}$ and $\tilde{g}$ stand for $f_{m,1,1/2}$ and $g_{m,1,1/2}$, respectively.

(2) $J^P = 0^+$: Only state with $j = 1/2$, $l = 1$ can construct the $J^P = 0^+$ state. We have:

$$\delta E^{(a)}_{n,1,1/2,0} = \int dr \left[(V_v + V_s)(2g^2 + r^2 {f'}^2 + rg(2g' + rg'')) + (V_v - V_s)r^2 f(2f' + rf'')\right] dr,$$

where f and g stand for $f_{n,1,1/2}$ and $g_{n,1,1/2}$. The matrix A and B for $\delta E^{(b)}_{n,1,1/2,0}$ are the Hermitian conjugate of A in Eq. (86) and B in Eq. (87), respectively.

(3) $J^P = 1^-$: Both states with $j = 1/2$, $l = 0$ and $j = 3/2$, $l = 2$ can construct the $J^P = 1^-$ state.

For $J^P = 1^-$ state with $j = 1/2$, $l = 0$, we have:

$$\delta E^{(a)}_{n,0,1/2,1} = \int dr \left[(V_v + V_s)(4f^2 + 3r^2 {g'}^2 + g(2g' + rg'')) + (V_v - V_s)(6f^2 + 4rff' + 3r(r{f'}^2 + rf(2f' + rf''))\right.$$

$$A_{nm} = \int dr\, V_v r^2(\tilde{f}g + 3f\tilde{g})\, dr,$$

$$B_{mn} = \int dr\, (V_v + V_s)r(2f\tilde{f} + r\tilde{f}f' + g(2\tilde{g} + r\tilde{g}'))\, dr,$$

where f and g stand for $f_{n,0,1/2}$ and $g_{n,0,1/2}$, and $\tilde{f}$ and $\tilde{g}$ stand for $f_{m,1,1/2}$ and $g_{m,1,1/2}$, respectively.

For $J^P = 1^-$ state with $j = 3/2$, $l = 2$, we have:

$$\delta E^{(a)}_{n,2,3/2,1} = \int dr \left[(V_v + V_s)(11f^2 + 18g^2 + 10rff' + 3r(r{f'}^2 + 3(3g + rg')^2 + g(2g' + rg''))) + (V_v - V_s)(6f^2\right.$$

$$A_{nm} = \int dr\, V_v r^2(\tilde{f}g + 3f\tilde{g})\, dr,$$

$$B_{mn} = \int dr\, (V_v + V_s)r(r\tilde{f}f' + f\tilde{f} + g(r\tilde{g}' + 3\tilde{g}))\, dr,$$

where f and g stand for $f_{n,2,3/2}$ and $g_{n,2,3/2}$, and $\tilde{f}$ and $\tilde{g}$ stand for $f_{m,1,3/2}$ and $g_{m,1,3/2}$, respectively.

(4) $J^P = 1^+$: Both states with $j = 1/2$, $l = 1$ and $j = 3/2$, $l = 1$ can construct the $J^P = 1^+$ state.

For $J^P = 1^+$ state with $j = 1/2$, $l = 1$, we have:

$$\delta E^{(a)}_{n,1,1/2,1} = \int dr \left[(V_v + V_s)(6g^2 + 3r^2 {f'}^2 + 3rg(2g' + rg'')) + (V_v - V_s)(4g^2 + 4rgg' + rf(2f' + rf''))\right] dr,$$

where f and g stand for $f_{n,1,1/2}$ and $g_{n,1,1/2}$. The matrix A and B for $\delta E^{(b)}_{n,1,1/2,1}$ are the Hermitian conjugate of A in Eq. (90) and B in Eq. (91), respectively.

For $J^P = 1^+$ state with $j = 3/2$, $l = 1$, we have:

$$\delta E^{(a)}_{n,1,3/2,1} = \int dr \left[(V_v + V_s)(27f^2 + 6g^2 + 18rff' + 3r^2 {f'}^2 + rg'(10g + 3rg') + 3rg(2g' + rg'')) + (V_v - V_s)(18\right.$$

where f and g stand for $f_{n,1,3/2}$ and $g_{n,1,3/2}$. The matrix A and B for $\delta E^{(b)}_{n,1,3/2,1}$ are the Hermitian conjugate of A in Eq. (93) and B in Eq. (94), respectively.

(5) $J^P = 2^-$: Both states with $j = 3/2$, $l = 2$ and $j = 5/2$, $l = 2$ can construct the $J^P = 2^-$ state.

For $J^P = 2^-$ state with $j = 3/2$, $l = 2$, we have:

$$\delta E^{(a)}_{n,2,3/2,2} = \int dr \left[(V_v + V_s)(5f^2 + 30g^2 + 5r^2 {f'}^2 + rg'(5rg' + 28g) + 5rg(2g' + rg'')) + (V_v - V_s)(10f^2 + 21g^2\right.$$

where f and g stand for $f_{n,2,3/2}$ and $g_{n,2,3/2}$. The matrix A and B for $\delta E^{(b)}_{n,2,3/2,2}$ are the Hermitian conjugate of A in Eq. (98) and B in Eq. (99), respectively.

For $J^P = 2^-$ state with $j = 5/2$, $l = 2$, we have:

$$\delta E^{(a)}_{n,2,5/2,2} = \int dr \left[(V_v + V_s)(90f^2 + 30g^2 + 40rff' + 5r^2 {f'}^2 + rg'(5rg' + 28g) + 5rg(2g' + rg'')) + (V_v - V_s)(6\right.$$

$$A_{nm} = \int dr\, V_v r^2 (2\tilde{f} g + f\tilde{g})\, dr,$$

$$B_{mn} = \int dr\, (V_v + V_s) r (r\tilde{f} f' + 4f\tilde{f} + g(r\tilde{g}' + 4\tilde{g}))\, dr,$$

where f and g stand for $f_{n,2,5/2}$ and $g_{n,2,5/2}$, and $\tilde{f}$ and $\tilde{g}$ stand for $f_{m,3,5/2}$ and $g_{m,3,5/2}$, respectively.

(6) $J^P = 2^+$: Both states with $j = 3/2$, $l = 1$ and $j = 5/2$, $l = 3$ can construct the $J^P = 2^+$ state.

For $J^P = 2^+$ state with $j = 3/2$, $l = 1$, we have:

$$\delta E^{(a)}_{n,1,3/2,2} = \int dr\,\Big[(V_v + V_s)(21f^2 + 10g^2 + 5r^2 {f'}^2 + rg'(5rg' + 2g) + 5rg(2g' + rg'')) + (V_v - V_s)(30f^2 + 3r^2 g$$

where f and g stand for $f_{n,1,3/2}$ and $g_{n,1,3/2}$. The matrix A and B for $\delta E^{(b)}_{n,1,3/2,2}$ are the Hermitian conjugate of A in Eq. (98) and B in Eq. (99), respectively.

For $J^P = 2^+$ state with $j = 5/2$, $l = 3$, we have:

$$\delta E^{(a)}_{n,3,5/2,2} = \int dr\,\Big[(V_v + V_s)(44f^2 + 60g^2 + 28rff' + 5r^2 {f'}^2 + 5(4g + rg')^2 + 5rg(2g' + rg'')) + (V_v - V_s)(30f$$

where f and g stand for $f_{n,3,5/2}$ and $g_{n,3,5/2}$. The matrix A and B for $\delta E^{(b)}_{n,3,5/2,2}$ are the Hermitian conjugate of A in Eq. (101) and B in Eq. (102), respectively.

(7) $J^P = 3^-$: Both states with $j = 5/2$, $l = 2$ and $j = 7/2$, $l = 4$ can construct the $J^P = 3^-$ state.

For $J^P = 3^-$ state with $j = 5/2$, $l = 2$, we have:

$$\delta E^{(a)}_{n,2,5/2,3} = \int dr\,\Big[(V_v + V_s)(64f^2 + 42g^2 + 40rff' + 7r^2 {f'}^2 + 7(rg' + 5g)^2 + 7rg(2g' + rg'')) + (V_v - V_s)(84f$$

$$A_{nm} = \int dr\, V_v r^2 (3\tilde{f} g + 5 f \tilde{g})\, dr,$$

$$B_{mn} = \int dr\, (V_v + V_s) r (r \tilde{f} f' + 3 f \tilde{f} + g(r\tilde{g}' + 3\tilde{g}))\, dr,$$

where f and g stand for $f_{n,2,5/2}$ and $g_{n,2,5/2}$, and $\tilde{f}$ and $\tilde{g}$ stand for $f_{m,3,5/2}$ and $g_{m,3,5/2}$, respectively.

For $J^P = 3^-$ state with $j = 7/2$, $l = 4$, we have:

$$\delta E^{(a)}_{n,4,7/2,3} = \int dr\,\Big[(V_v + V_s)(111f^2 + 140g^2 + 54rff' + 7r^2 {f'}^2 + 7(rg' + 5g)^2 + 7rg(2g' + rg'')) + (V_v - V_s)(8$$

$$A_{nm} = \int dr\, V_v r^2 (3\tilde{f} g + 5 f \tilde{g})\, dr,$$

$$B_{mn} = \int dr\,(V_v + V_s) r(r\tilde{f}f' + 3f\tilde{f} + g(r\tilde{g}' + 5\tilde{g}))\,dr,$$

where f and g stand for $f_{n,4,7/2}$ and $g_{n,4,7/2}$, and $\tilde{f}$ and $\tilde{g}$ stand for $f_{m,3,7/2}$ and $g_{m,3,7/2}$, respectively.

(8) $J^P = 3^+$: Both states with $j = 5/2$, $l = 3$ and $j = 7/2$, $l = 3$ can construct the $J^P = 3^+$ state.

For $J^P = 3^+$ state with $j = 5/2$, $l = 3$, we have:

$$\delta E^{(a)}_{n,3,5/2,3} = \int dr\,\Big[(V_v + V_s)(28f^2 + 84g^2 + 28rff' + 7r^2{f'}^2 + rg'(7rg' + 2g) + 7rg(2g' + rg'')) + (V_v - V_s)(42$$

where f and g stand for $f_{n,3,5/2}$ and $g_{n,3,5/2}$. The matrix A and B for $\delta E^{(b)}_{n,3,5/2,3}$ are the Hermitian conjugate of A in Eq. (106) and B in Eq. (107), respectively.

For $J^P = 3^+$ state with $j = 7/2$, $l = 3$, we have:

$$\delta E^{(a)}_{n,3,7/2,3} = \int dr\,\Big[(V_v + V_s)(175f^2 + 84g^2 + 70rff' + 7r^2{f'}^2 + rg'(7rg' + 2g) + 7rg(2g' + rg'')) + (V_v - V_s)($$

where f and g stand for $f_{n,3,7/2}$ and $g_{n,3,7/2}$. The matrix A and B for $\delta E^{(b)}_{n,3,7/2,3}$ are the Hermitian conjugate of A in Eq. (109) and B in Eq. (110), respectively.

V Numerical Result and Discussion

The vector and scalar potentials we choose have the simple form:

$$V_v(r) = \frac{4\alpha_s(r)}{3r}, \tag{113}$$

$$V_s(r) = br + c. \tag{114}$$

The potentials are chosen to have a Coulombic behavior at short distance and a linear confining behavior at long distance.

The running coupling constant $\alpha_s(r)$ in Eq. (113) is obtained from the coupling constant in momentum space $\alpha_s(Q^2)$ after the Fourier transformation. It can be written in a more convenient form [16]:

$$\alpha_s(r) = \sum_{i=1}^{3} \alpha_i \frac{e^{-\gamma_i r}}{r}, \tag{115}$$

where α_i and γ_i are free parameters which can be fitted according to the behavior of the running coupling constant at short distance predicted by QCD. In this work, $\alpha_1 = 0.15$, $\alpha_2 = 0.15$, $\alpha_3 = 0.20$, and $\gamma_1 = 1/2$, $\gamma_2 = \sqrt{10}/2$, $\gamma_3 = \sqrt{1000}/2$ [15].

All the other parameters have the same values as in the previous paper [23], except for the masses of the light quarks u, d and s. The quark masses are $m_u = m_d = 0.280$ GeV, $m_s = 0.460$ GeV, $m_c = 1.478$ GeV, $m_b = 4.865$ GeV. The parameters of the linear potential in this work are quite different from the values in usual quark models. As discussed in the previous paper, the parameter b is responsible for the energy levels of states with higher quantum numbers n or l, but unlike the Dirac-like Hamiltonian, the influence of the confining potential $V_s(r)$ weakens as the mass of light quark decreases in H_0. In other words, the energy level is also sensitive to the mass of light quark. It suggests that the parameter b may depend on the masses of the quark or antiquark, especially the light quark or antiquark. With the above considerations, our fitting gives the following values: $b = 0.350$ GeV^2 for $(c\bar{q}, b\bar{q})$, $b = 0.260$ GeV^2 for $(c\bar{s}, b\bar{s})$, $c = -0.320$ GeV.

Numerical calculation shows that the solution of the wave equation is stable when $L > 5$ fm, $N > 35$. Here we take $L = 10$ fm, $N = 50$. The spectra of the heavy-light D, D_s, B, B_s mesons are fitted mainly based on the meson states presented in PDG [9]. The numerical results for the spectra of heavy-light mesons are presented in two tables. Table I is for D, D_s mesons and Table II for B, B_s mesons. The obtained spectra are in reasonable agreement with the experimental measurements.

Our results are compared to the results of two other relativistic models [35, 38], one derived by quasipotential approach and the other obtained by reducing the Bethe-Salpeter vertex function. In the previous work, we had $m_D = 167$ GeV, $m_{D_s} = 161$ GeV, while in this work we have $m_D = 136$ GeV, $m_{D_s} = 141$ GeV. The discrepancy from experimental data is decreased for D, D^*, D_s and D_s^* states as well as other states.

Theoretical deviations from experimental data mainly occur in the D_s meson sector, specifically for the $D_{s0}^*(2317)^0$ and $D_{s1}(2460)$ resonances. Our calculations for the two resonances are about 90 GeV higher than their masses measured in experiment. The discrepancy may be ascribed to the instantaneous approximation, the naive assumption of the kernel and the $\alpha_s^2(r)$ contributions, i.e., the loop corrections, but it is more likely to be explained beyond the naive quark model [39]. (Both models we cite in Table I cannot give the right masses.) As the mass difference between D_{s0}^* and D_0^* is about 0.6 GeV, the masses of the two resonances cannot be accounted for simultaneously in conventional quark models if their difference in the model is merely their light quark masses m_s and $m_{u,d}$. In this work the confinement parameter b is taken to be dependent on the light quark, but still it is not capable of explaining the discrepancy.

As the D_{s0}^* lies just below the DK threshold, while the D_{s1} lies just below the

DK^*, the authors in Ref. [40] have suggested that the two resonances may be $D^*_{s0}(DK)$ and $D_{s1}(DK^*)$ molecular states, while in Refs. [41-43], the D^*_{s0} and D_{s1} are considered as $c\bar{s}$ states which are heavily renormalized by mixing with the DK and DK^* continua. In Ref. [44], the authors suggest that the discrepancy of the calculated masses in quark models can be qualitatively understood as a consequence of self-energy effects due to strong coupled channels. In Refs. [45–47], the interpretation of the heavy $J^P(0^+, 1^+)$ spin multiplet as the parity partner of the groundstate $(0^-, 1^-)$ multiplet is proposed. Both theoretical and experimental efforts are required in order to fully understand the nature of the anomalous $D^*_{s0}(2317)$ and $D_{s1}(2460)$ states.

The highly excited meson states are also calculated in the spectra, and the newly observed charmed meson states are identified in our model. As for D mesons, several resonances are observed by the LHCb collaboration in the mass region between 2500 and 3000 MeV. We list their measured masses as follows [2]:

$$
\begin{aligned}
M(D_J(2580)^0) &= 2579.5 \pm 3.8 \text{ MeV}, \\
M(D^*_J(2650)^0) &= 2649.2 \pm 3.5 \pm 3.5 \text{ MeV}, \\
M(D_J(2740)^0) &= 2737.0 \pm 3.5 \pm 11.2 \text{ MeV}, \\
M(D^*_J(2760)^0) &= 2760.1 \pm 1.1 \pm 3.7 \text{ MeV}, \\
M(D^*_J(2760)^+) &= 2771.7 \pm 7.2 \text{ MeV}, \\
M(D_J(3000)^0) &= 2971.8 \pm 8.7 \text{ MeV}, \\
M(D^*_J(3000)^0) &= 3008.1 \pm 4.0 \text{ MeV}, \\
M(D^*_J(3000)^+) &= 3008.1 \text{ MeV}.
\end{aligned}
$$

The assignments of the above observed states are listed in Table I, where the resonances $D_J(2740)$, $D^*_J(2760)$, $D_J(3000)$ are identified as $n = 1$ states and the resonances $D_J(2580)$, $D^*_J(2650)$, $D^*_J(3000)$ are identified as radially excited states with $n = 2$.

In the calculated spectrum of D meson, $D_J(2740)$ and $D^*_J(2760)$ are identified as the $1^{5/2}D_2$ state with $J^P = 2^-$ and the $1^{5/2}D_3$ state with $J^P = 3^-$, respectively, which agrees with Ref. [48]. $D_J(3000)$ favors the natural parity and $D^*_J(3000)$ the unnatural parity, thus we assign $D_J(3000)$ as the $1^{7/2}F_3$ state with $J^P = 3^+$ and $D^*_J(3000)$ as the $1^{5/2}F_2$ state with $J^P = 2^+$, although our calculations for the two resonances are about 40 MeV lower than experimental data. The last two resonances $D_J(2580)$ and $D^*_J(2650)$ are identified as $2^{1/2}S_0$ and $2^{1/2}S_1$, respectively.

As for D_s mesons, several new D_s meson states have been observed besides the low-lying states; their masses and identifications are presented in Table I. Recently, the LHCb collaboration identified the resonance $D^*_{sJ}(2860)$ as an admixture of spin-1 and spin-3 resonances [7, 8], with measured masses $M(D^*_{s1}(2860)^-) = 2859 \pm 12 \pm 6 \pm 23$ MeV and $M(D^*_{s3}(2860)^-) = 2860.5 \pm 2.6 \pm 2.5 \pm 6.0$ MeV. In Refs. [35, 38] cited in Table I, their predictions do not favor this identification, with calculations generally 60 MeV higher than the measured masses. While our predictions for the masses of the $1^{3/2}D_1$ and $1^{5/2}D_3$ states are around 2860 MeV, our model appears to be able to interpret the two states as the $J^P = 1^-$ and 3^- members of the 1D family. Thus we assign $D^*_{sJ}(2860)$ as the $1^{3/2}D_1$ state and $D^*_{s3}(2860)^-$ as the $1^{5/2}D_3$ state.

The resonances $D_{sJ}(2632)$, $D^*_{s1}(2710)$ and $D_{sJ}(3040)$ are identified as radially excited states with $n = 2$ in our model. The $D_{sJ}(2632)$ was first observed in Ref. [3] at a mass of 2632.5 ± 1.7 MeV; it can be assigned as the $2^{1/2}S_0$ state with $J^P = 0^-$. For the $D^*_{s1}(2710)$, it is measured to be 2708^{+11}_{-10} MeV [5]; the $J^P = 1^-$ assignment is proposed in Refs. [49, 50]. Comparing the experimental mass of the $D^*_{s1}(2710)$ with our calculation for the $2^{1/2}S_1$ state, our prediction favors this assignment. The $D_{sJ}(3040)$ resonance is observed by the BABAR Collaboration in the D^*K mass spectrum at a mass of 3044^{+30}_{-5} MeV [6]. The assignment of this resonance is discussed in detail in Ref. [51]. Here we assign it as the $2^{3/2}P_1$ state with $J^P = 1^+$ in the mass spectrum for D_s meson.

As for the b-flavored meson sector, the calculated mass spectra in Table II are in good agreement with experimental results. Although experimental data for b-flavored mesons is limited for now, the predictions for the highly excited b-flavored meson states can be tested in further experiments.

Finally, the wave function of each bound state can be obtained simultaneously when solving the wave equation. The radial wave functions $g_{n,l,j}(r)$ and $f_{n,l,j}(r)$ for physical and unphysical D meson states are depicted as an example in Fig. 1 [Figure 1: see original paper] and Fig. 2 [Figure 2: see original paper], respectively. We stress that the solution of the eigenequation associated with the original Hamiltonian H_0 in Eq. (30) gives only the wavefunctions for physical states depicted in Fig. 1; our model is free from unphysical states problems. In Section III we constructed a new H_0 for heavy-light systems in Eq. (30), and the unphysical states depicted in Fig. 2 are due to the new Hamiltonian H_0 for which the original Hamiltonian is substituted.

VI Summary

We restudy the spectra of heavy-light mesons in a relativistic model derived by reducing the instantaneous Bethe-Salpeter equation. The kernel we choose is based on scalar confining and vector Coulomb potentials, and it shows a Coulombic behavior at short distance and a linear confining behavior at long distance. The Hamiltonian for the heavy-light quark-antiquark system is calculated up to order $1/m_Q^2$. We find that the parameter b in the confinement potential may depend on the masses of the quarks in the framework of the instantaneous Bethe-Salpeter equation. The spectra of D, D_s, B and B_s mesons are calculated in the relativistic model. Our results are in reasonable agreement with experimental measurements. Theoretical deviations from experimental data mainly appear in the D_s meson sector, especially for the $D^*_{s0}(2317)^0$ and $D_{s1}(2460)$ resonances. The discrepancy may be ascribed to the naive assumption of the kernel; kernels with different spin structure can be studied in further research. The predictions of the spectra and the assignments for the newly observed charmed mesons are presented. Our prediction favors the identification of the resonance $D^*_{sJ}(2860)$ as an admixture of spin-1 and spin-3 resonances. The wave functions can be obtained when solving the wave equation and can be used in the study of B and D decays.

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[TABLE:I]
[TABLE:II]

Note: Figure translations are in progress. See original paper for figures.