On uplimit of accurate measurement of tau mass (postprint)

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Abstract

The tau lepton, as one of the three elementary leptons in nature, has been subject to continuous mass measurement since its discovery. The current relative accuracy has already reached a level better than 10^{-4}, with ongoing efforts to further improve the precision. However, analysis of available techniques and expected luminosity from e^+e^- collider indicates that the precision upper limit for the tau mass has nearly been attained, necessitating the exploration of novel approaches if significant improvement is desired.

Full Text

On the Upper Limit of Accurate Measurement of the Mass

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Abstract

The lepton, as one of the three elementary leptons in nature, has had its mass measured since its discovery. The present relative accuracy stands at 4×10^{-4}, with ongoing efforts pushing for improvements beyond the 10^{-5} level. However, analysis of available techniques and expectable luminosity from e^+e^- colliders indicates that the precision upper limit of mass measurement is nearly reached, meaning that brand new approaches must be sought if significant further improvement is desired.

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Introduction

The history of   mass (m_τ) measurement spans forty years. In the first experimental paper on the   lepton [?], m_τ was estimated to lie in the range 1.6–2.0 GeV. Since then, numerous experiments have measured m_τ [?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?], with results displayed in Fig. 1 [Figure 1: see original paper].

The m_τ measurements from the 21st century are summarized in Table 1 , where two results were obtained using the pseudo-mass method while the others employed threshold scanning. For the pseudo-mass method, the huge datasets from B-factories yield good statistical accuracy, but large systematic uncertainty arises primarily from absolute calibration of particle momentum. For the threshold-scan method, m_τ is extracted from the dependence of the production cross section on beam energy. In the KEDR experiment [?], both resonant depolarization and Compton backscattering techniques [?] were used to determine the beam energy, greatly reducing the beam energy uncertainty.

Although the accuracies from both methods are comparable, systematic errors already dominate for the pseudo-mass method, making further luminosity increases unappealing. For the threshold-scan method, both statistical and systematic errors appear to have room for further improvement. Therefore, subsequent discussions will focus exclusively on the threshold-scan method.

2 Statistical Considerations

Through optimization theory [?], the relationship between the absolute error of m_τ (denoted by u) and total luminosity (denoted by L) is given by:

$$u = \frac{A}{\sqrt{L}}$$

where A is a constant depending on the cross section, its derivative with respect to m_τ, and other quantities [?].

An effective error variation called **L-profit** is introduced and defined as:

$$\nu \equiv -\frac{du}{dL} = \frac{A}{2L^{3/2}}$$

The minus sign reflects the fact that u always decreases as L increases. This quantity reveals the variation of u per unit luminosity. While error reduction with accumulating luminosity is expected, L-profit becomes small for large L. As shown in Fig. 2 [Figure 2: see original paper], when u is relatively large, increasing L effectively improves u significantly. However, step by step, when u is already sufficiently accurate, enormous luminosity must be consumed to reduce the error while improvement remains limited—an uneconomical proposition.

This analysis is incomplete, however. In practice, increasing L requires more machine time, more data processing, and more analysis effort. Moreover, improving u demands extensive systematic uncertainty analysis, which necessitates generator improvements. Numerous details must be considered, sometimes requiring software upgrades. Intuitively, the work required for error improvement increases exponentially, becoming even more pronounced when the error is already small. Under this consideration, a **work-factor** D is designated as:

$$D = e^{B/u} = e^{C\sqrt{L}}$$

where $C = B/A$ and constant B relates to concrete analysis procedures. Similar to Eq. (2), another effective error variation called **D-profit** is introduced:

$$\xi \equiv -\frac{du}{dD}$$

As evident in Fig. 2, ξ decreases dramatically with L, indicating that luminosity increases have diminishing effects on m_τ improvement.

There is no rigorous model or proof for this exponential assumption, as work for a given target is difficult to quantify. However, an interesting fact may be instructive for understanding this exponential growth. In the Standard Model, the anomalous magnetic moment a_e receives contributions from electromagnetic, hadronic, and electroweak interactions:

$$a_e = a_e(\mathrm{QED}) + a_e(\mathrm{hadronic}) + a_e(\mathrm{electroweak})$$

The QED contribution can be evaluated through perturbative expansion in α/π:

$$a_e(\mathrm{QED}) = \sum_{n=1}^{\infty} \left(\frac{\alpha}{\pi}\right)^n a^{(2n)}$$

where $a^{(2n)}$ is finite due to QED renormalizability and may be written to show mass dependence explicitly. The $A_1^{(2n)}$ coefficients can be calculated perturbatively with increasing order n. Notably, the number of Feynman diagrams grows even faster than exponentially with n. The comparison is striking [**?**]:

Order	$A_1^{(2n)}$	Number of Feynman Diagrams
$n = 1$	$A_1^{(2)}$	1
$n = 5$	$A_1^{(10)}$	891

The number of diagrams follows $e^{2(n-1)}$ approximately. In other words, the error improvement of m_τ in e^+e^- colliders is approaching its upper limit.

To concretize this analysis, we compare two pedagogical experiments: the first reduces error from 2 MeV to 1 MeV, while the second from 0.2 MeV to 0.1 MeV. Coefficients A and B are set to 1 (in MeV units). Ratios are defined as:

$$R^i_X \equiv \frac{X^i_a}{X^i_b}$$

where superscript i indicates the first ($i = 1$) or second ($i = 2$) experiment, subscript a denotes after-experiment values and b before-experiment values, and X represents L, ν, D, and ξ respectively. Evaluations are presented in Table 2 .

Based on Table 2, improving error by a factor of two requires fourfold luminosity, yielding an L-profit of one-eighth. According to L and ν, as long as the improvement proportion is identical, R_L and R_ν remain the same, implying these quantities cannot reflect realistic working strength or profit. Turning to R_D and R_ξ, the situation differs dramatically. Although the improvement proportion is identical, the second experiment requires ninety times more work than the first, while its D-profit is only one-ninetieth of the first! This confirms that when u is already sufficiently accurate, further luminosity increases become uneconomical and ineffective.

A remark is in order. During BESIII' s m_τ scan studies, methodical and meticulous optimization was performed to ensure effective luminosity usage [**?**, **?**, **?**, **?**, **?**]. Nevertheless, this effort does not resolve the low R_ξ issue.

3 Systematic Error Limitations

Before BESIII's actual m_τ scan, extensive studies were conducted, including systematic uncertainty estimation [**?**] as listed in Table 3 . The energy scale error dominates. To reduce this uncertainty, a high-accuracy beam energy measurement system (BEMS) was designed, constructed, and commissioned at BEPC-II' s north crossing point starting in 2007, becoming operational by late 2010 [**?**, **?**, **?**, **?**]. The system performed excellently, requiring only two days for a ψ' resonance scan. The mass difference between the PDG2010 value [**?**] and BEMS measurement is 36 keV, indicating BEMS relative accuracy at the 2.5×10^{-5} level [**?**].

During actual data taking, BEMS measurement accuracy was found sensitive to accelerator running status [**?**]. At the end of 2011, a test scan near the -pair threshold was performed, collecting 23.26 pb^{-1} integrated luminosity, plus 1.5 pb^{-1} and 7.5 pb^{-1} datasets for J/ψ and ψ' resonance scans respectively. These enabled determination of systematic uncertainty from beam energy. The lepton mass from maximum likelihood fit to -pair production cross-section is $m_\tau = (1776.91 \pm 0.13)$ MeV [**?**], where systematic uncertainty from energy scale again dominates, as shown in Table 4 . This difficult situation applies to all e^+e^- colliders using the threshold-scan method and cannot be circumvented even by the pseudo-mass method.

The crucial issue is energy scale uncertainty. For BESIII at BEPC-II, BEMS was established to control this uncertainty. The system works as follows [**?**]: a laser source provides the laser beam, optics focus and guide it for head-on collisions with electron (or positron) beams in the vacuum pipe, where Compton backscattering occurs; the backscattered high-energy photons are detected by an HPGe detector, whose detection capacity determines beam energy accuracy.

Currently, the HPGe detector's relative accuracy limit is at the 10^{-5} level, which in turn limits beam energy determination accuracy.

4 Prospects for Accuracy Improvement

The preceding analyses clearly indicate that efforts at e^+e^- colliders will not yield promising improvements in m_τ accuracy. Therefore, new approaches are indeed needed to significantly increase m_τ precision.

Without plausible precedent or theoretical implication, we examine relevant high-accuracy measurements to identify clues for future m_τ experiments.

4.1 Experimental Prospects

According to PDG2012 [**?**]:

$$m_e = 0.510998910 \pm 0.000000013 \text{ MeV} \quad (\delta m_e/m_e \approx 2.5 \times 10^{-8})$$

$$m_\mu = 105.658367 \pm 0.000004 \text{ MeV} \quad (\delta m_\mu/m_\mu \approx 3.8 \times 10^{-8})$$

Such precise electron mass determination comes from measuring the mass ratio to a nucleus, yielding results in atomic mass units (u). For example, by comparing cyclotron frequencies of electrons and single C^{6+} ions alternately confined in the same uniform magnetic field in a Penning trap [**?**], the electron mass is determined as $m_e = 0.0005485799111(12)$ u. A conversion factor 931.494013 ± 0.000037 MeV/u is then applied [**?**, **?**].

The muon mass is obtained from the muon-electron mass ratio via Zeeman transition frequency measurements in muonium (μ^+e^- atom). Unfortunately, the lepton' s lifetime is too short to form such an atom, preventing similar accurate mass determination.

Nevertheless, it is enlightening to find a relation between m_τ and another accurately measurable quantity, or to measure their ratio precisely. This may be the new direction for future accurate m_τ measurements.

4.2 Theoretical Prospects

Theoretically, if a certain relation for m_τ can be found, excellent accuracy may be expected. An interesting formula for the masses of three leptons [**?**, **?**] is:

$$m_e + m_\mu + m_\tau = (\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2$$

From which it follows that:

$$\sqrt{m_\tau} = 2(\sqrt{m_e} + \sqrt{m_\mu})$$

Using this formula with the electron and muon mass values gives:

$$m_\tau = 1776.968884 \pm 0.000058 \text{ MeV} \quad (\delta m_\tau / m_\tau \approx 3.3 \times 10^{-8})$$

Unfortunately, this formula is not derived from first principles of particle physics, so such efforts can only be considered as exploratory directions for the future.

5 Summary

This paper discusses the error upper limit of m_τ measurement from an unconventional perspective. The main conclusions are:

1. **Statistically**, luminosity increases have diminishing effects on m_τ accuracy improvement. Moreover, from an efficiency and profitability standpoint, the input-output ratio decreases considerably as m_τ accuracy increases.
2. **Systematically**, the 10^{-5} level upper limit of HPGe detector accuracy circumscribes the m_τ accuracy upper limit. For e^+e^- collider experiments, this represents an insurmountable systematic uncertainty barrier.
3. A completely distinctive approach from both theoretical and experimental perspectives is needed to seek new directions for improving m_τ accuracy.

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Note: Figure translations are in progress. See original paper for figures.