Zc(3900) as a D ¯D∗ Molecule from Pole Counting Rule Postprint

Authors: Qin-Rong Gong, Zhi-Hui Guo, Ce Meng, Guang-Yi Tang, Han-Qing Zheng

Date: 2016-09-02T00:00:00+00:00

Abstract

A careful study on the nature of the Z_c(3900) resonant structure is carried out in this work. By constructing the pertinent effective Lagrangians and considering the important final-state-interaction effects, we first give a unified description to all the relevant experimental data available, including the J/$\psi\pi$ and $\pi\pi$ invariant mass distributions from the e^{+e}-→J/$\psi\pi\pi$ process, the h_cπ distribution from e^{+e}-→h_c$\pi\pi$ and also the D$\bar{D}$^{} *spectrum in the* e^{+e}-→$D\bar{D}$^{}π process. After fitting the unknown parameters to the previous data, we search the pole in the complex energy plane and only find one pole in the nearby energy region in different Riemann sheets. Therefore we conclude that Z_c(3900) is of D$\bar{D}$^* molecular nature, according to the pole counting rule method. We emphasize that the conclusion based upon pole counting method is not trivial, since both the D$\bar{D}$^{*} contact interactions and the explicit Z_c exchanges are introduced in our analyses and they lead to the same conclusion.

Full Text

Preamble

Zc(3900) as a D¯D* Molecule from Pole Counting Rule

Qin-Rong Gong[1], Zhi-Hui Guo[2],[3], Ce Meng[4], Guang-Yi Tang[5], Han-Qing Zheng[1],[6]

[1]Department of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China
[2]Department of Physics, Hebei Normal University, Shijiazhuang 050024, China
[3]Helmholtz-Institut für Strahlen- und Kernphysik and Bethe Center for Theoretical Physics, Universität Bonn, D-53115 Bonn, Germany
[4]Department of Physics, Peking University, Beijing 100871, China

[5]Institute of High Energy Physics, Beijing 100049, China
[6]Collaborative Innovation Center of Quantum Matter, Beijing 100871, China

May 2, 2016

Abstract

We present a careful study on the nature of the Zc(3900) resonant structure. By constructing the pertinent effective Lagrangians and incorporating important final-state interaction effects, we provide a unified description of all relevant experimental data available, including the $J/\psi\pi$ and $\pi\pi$ invariant mass distributions from the $e^+e^- \to J/\psi\pi\pi$ process, the $hc\pi$ distribution from $e^+e^- \to hc\pi\pi$, and the D^-D^* spectrum in the $e^+e^- \to D^-D\pi$ *process. After fitting the unknown parameters to this data, we search for poles in the complex energy plane and find only one pole in the nearby energy region across different Riemann sheets. Based on the pole counting rule method [1], we conclude that Zc(3900) has a* D^-D molecular nature. We emphasize that this conclusion is not trivial, as both D^-D^* contact interactions and explicit Zc exchanges are introduced in our analysis, yet they lead to the same conclusion.

Introduction

The discovery of X(3872) opened a new era in hadron physics [2]. Since then, more than two dozen so-called exotic XYZ particles with hidden heavy-quark flavors have been observed over the last decade [3]; see Refs. [4, 5] for recent experimental and theoretical reviews, respectively. A common and interesting feature shared by these exotic states is that many lie close to thresholds composed of open heavy-flavor states. This raises an important question: do these newly observed XYZ peaks from experimental analyses correspond to genuine/elementary resonance states, threshold effects, or a mixture of both mechanisms? Many different methods have been proposed to discern the inner structures of hadronic states, including compositeness coefficient analyses [6-11], QCD sum rule studies [12-15], NC trajectories of resonance poles [16-18], and the pole counting rule [1, 19-21]. In this work, we apply the latter approach to study Zc(3900), first observed by the BESIII collaboration [22].

The charged charmonium-like state Zc(3900), with mass 3899.0 ± 3.2 MeV and width 24.8 ± 4.5 MeV, was observed in the $J/\psi\pi\pm$ invariant mass spectrum in the $e^+e^- \to J/\psi\pi^+\pi^-$ process by the BESIII collaboration [22] in 2013. It has been confirmed by Belle [23] and CLEO [24] collaborations in $e^+e^- \to J/\psi\pi^+\pi^-$ as well. Later, a peak structure with mass 3883.9 ± 11.5 MeV was observed in the $(D^-D)\pm$ *invariant mass spectrum in the* $e^+e^- \to \pi\pm(D^-D)$ process [25]. The angular distribution analysis of the πZc system in the $\pi\pm(D^-D^*)$ channel given in Ref. [25] determines the quantum numbers of Zc to be $I(J\ ) = 1(1^+)$.

Regarding the nature of Zc(3900), it is theoretically interesting because at the quark level it can only be accommodated by a c^-c plus a light quark-antiquark pair assignment and cannot be a conventional c^-c charmonium. If these quarks are bound together by color confining force, Zc(3900) would be a compact tetraquark state [15, 26–31]. On the other hand, since the mass of Zc(3900) is very close to the D^-D^* threshold, it provides a natural candidate for a D^-D^* molecule [12–14, 32–40]. Other possible explanations also exist, such as cusp effects suggested in Refs. [41–44] and anomalous triangle singularities in Refs. [45, 46]. A recent lattice investigation in Ref. [47] gives a small negative scattering length in the corresponding channel (repulsive interaction), thus disfavoring a resonant picture. However, one should be cautious when interpreting lattice results with physical measurements, since the lightest pion mass used in that lattice study is 300 MeV, still much larger than its physical value. Efforts have also been made in the literature [48–50] to distinguish between the molecule and tetraquark pictures by estimating the decay width of Zc(3900). The Zc(3900) in the photoproduction process $\gamma p \rightarrow Zc(3900)^+ n$ has been studied in Ref. [51].

In this work, we use the so-called pole counting rule, developed in Ref. [1], to discriminate the nature of Zc(3900). This rule provides an elegant way to distinguish resonance generation mechanisms around threshold by counting the number of nearby poles of amplitudes in the complex energy plane. Through studying potential scattering with/without CDD poles, it concludes that for a molecular-type resonance there is just one corresponding pole, while for an elementary-type resonance there is a pair of poles lying close to the threshold on nearby unphysical Riemann sheets. Though this rule is not asserted as a mathematical theorem, it most likely holds in physical regimes [1]. To apply this rule, a precisely determined amplitude is crucial, implying that the final conclusion relies heavily on input experimental data. This is one reason we try to fit as much available data as possible to better constrain the amplitude.

In this respect, we should mention that recent works in Refs. [40, 52] have also analyzed the D^-D^* and $J/\psi\pi$ invariant mass distributions by emphasizing the nonperturbative nature of D^-D^* interactions around the Zc(3900) energy region. However, we point out that in addition to the D^-D^* resummation mechanism, including an "elementary" Zc(3900) field through Breit-Wigner/Flatté functions is another possibility to fit the data. Naively speaking, the two mechanisms correspond to rather different scenarios for Zc(3900). If Zc(3900) originates from D^-D^* resummation, it has a good chance to be a D^-D^* molecule, while if it comes from resonance exchange with the Flatté function, it would probably be more like an elementary state with D^-D^* playing only a minor role in its composition. It should be emphasized that a near-threshold pole location does not necessarily imply a molecular structure—a good example is the X(3872) resonance [21]. A Flatté-type function fit with explicit resonance exchange gives an excellent description of the data and finds pairs of poles on nearby Riemann sheets. Regarding Zc(3900), it is a priori unknown which approach gives a better description: D^-D^* resummation or explicit resonance exchange. One novelty of the present work is to include both approaches to fit data. Moreover, instead of simply parameterizing D^-D^* interactions with X(4260), J/ψ, and other light-flavor mesons with constant or polynomial three-momentum terms [40, 52], we describe all interactions by constructing pertinent effective Lagrangians and carefully handle the resummation of D^-D^* by distinguishing the transverse and longitudinal parts of vector mesons. A simultaneous description of the hcπ and $\pi\pi$ invariant mass distributions, in addition to the D¯D* and J/$\psi\pi$ ones, will be discussed. By comparing different scenarios realized by switching different parameters on and off and applying the pole counting method established in Ref. [1], we reach a solid conclusion that Zc(3900) is of molecular nature.

This paper is organized as follows. In Sec. 2, we set up the theoretical framework, including the construction of effective Lagrangians and the resummation of D¯D* loops. Phenomenological results and discussions are presented in Sec. 3. A short summary and conclusions are given in Sec. 4.

2 Theoretical Framework for $e^+e^- \to J/\psi\pi\pi$, D¯D*π, hc$\pi\pi$ Channels

2.1 Pertinent Effective Lagrangians

Assuming Zc(3900) is an I(J) = 1(1^+) particle, we construct the effective Lagrangian to describe interactions between Zc(3900) and other particles. We use the conventional Proca vector representation to incorporate the vector and axial-vector states X(4260) and Zc(3900), denoted by X^μ and Z_c^μ, respectively, while Ref. [19] employed the antisymmetric tensor formalism. In the molecular picture, the nonperturbative D¯D* interaction is responsible for the near-threshold state Zc(3900). There is strong evidence that pion-exchange contributions are subleading effects in heavy-flavor meson-meson (D()B()) scattering. A perturbative nature of pion exchange is established based on the power counting scheme of effective field theory in Ref. [53], further supported by explicit calculations showing that light q¯q meson exchanges indeed play a minor role in D¯D* scattering [54]. Based on this argument, we consider only local contact D¯D* interactions in this work.

The contact D¯D* four-point interaction takes the following form under heavy quark symmetry considerations [55]:

$L_{DDDD} = \lambda_1$ h(D¯D*$^\mu$ + h.c.)^2

where the field operators D and D* are SU(2) isospin doublets:

D = (D^0, D^+), D* = (D^0, D^+)

The D¯D* meson loop should include the $D^{-0}D\pm$, D^-D^+, and $D^{-0}D^0$ *intermediate states. To simplify notation,* D^-D/D^-D *is denoted as* D^-D from now on.

We construct the effective Lagrangian formally in a relativistic framework, but focus only on energy regions close to the D¯D* threshold. Concerning D¯D* interactions with J/$\psi\pi$ and hcπ, the operators with the lowest number of derivatives that are invariant under C, P, chiral, and isospin symmetry transformations are given by:

$L_{DD\psi\pi} = \lambda_2\nabla\nu\psi\mu\ h + \lambda_4\nabla\nu\ \psi\mu\ h$ $L_{DDhc\pi} = (\lambda_6\nabla\mu H\nu$ h + $\lambda_3\psi\mu$ $h\nabla$ + $\lambda_5\psi\mu$ h∇ + $\lambda_7 H_\mu$ h∇_ν $D^{-*}\rho$ u_σ D) $\epsilon^{\mu\nu\rho\sigma}$ + h.c.

where H^μ denotes the axial-vector state hc, ψ^μ stands for J/ψ, ∇^μ is a covariant derivative operator, ϵ^{μνρσ} is the fourth-order antisymmetric tensor, and u^μ corresponds to the standard chiral building block that includes light-flavor mesons:

u = exp(iϕ/f_π), u^μ = i u†∂^μ u

For details of the chiral building blocks, see for example Ref. [56].

On the other hand, if Zc(3900) is an "elementary state" (i.e., compact quark(s)/anti-quark(s) bound together through color force), one should include the following operators:

L_{XZcπ} = g_4∇νXμ Z_c^μ u^ν L_{Zcψπ} = g_5∇νψμ Z_c^μ u^ν L_{ZcDD} = f_5 Z_c^μ (D¯D_μ + h.c.) L_{Zchcπ} = f_7∇νHμ Z_c^ρ u^σ ϵ^{μνρσ}

where Z_c^μ is given by a 2$×$2 matrix:

Z_c = (Z_c$^+$, Z_c^0; Z_c^0, Z_c$^-$)

The four-point interactions between X(4260) and J/ψππ, hcππ, D¯D*π are also taken into account:

L_{Xψππ} = g_1X^μψν h *L{XDDπ} =* f_1∇νXμ *h ν u^μ D +* f_3∇νXμ *h* ∇ρXμ *H_ν h L_{Xhcππ} =* f_6∇ *+* g_2*X_μψ^μ +* f_4*X_μ h∇ u_λ u_σ^i +* f_2*X_μ h∇^μ D¯ ϵ^{μνρσ}*

where χ is another standard chiral building block, χ = u†χu† + uχ†u [56].

In addition to the strong D¯D* final state interaction (FSI), we carefully include the ππ FSI in the processes e^+e^- → J/ψππ and e^+e^- → hcππ, where the ππ system is mainly in s-wave. The strong ππ interactions are treated within the framework of unitarized chiral perturbation theory (χPT) up to next-to-leading order [57], with $O(p^4)$ low-energy constants fixed by fitting ππ scattering data. The resulting poles of the $f_0(500)$ and $f_0(980)$ are in good agreement with those from more rigorous approaches [58, 59]. Details of the χPT Lagrangian and unitarization procedure for ππ scattering will not be repeated here; interested readers are referred to Ref. [57] and references therein.

Before concluding this subsection, we point out that the term "effective" in the effective Lagrangian here does not imply that the Lagrangian follows certain proper power counting rules. Rather, it means the Lagrangian includes all expected nearby (hence important) singularities of a given process within the limited energy range of data fitting.

2.2 Calculation of the Amplitudes

We now calculate the amplitudes corresponding to the e^+e^- → J/ψππ, D¯Dπ*, and hcππ processes. The center-of-mass (CM) energies in these processes are fixed at 4.26 GeV in accordance with experimental analyses [22, 25, 60][2]. The* *final amplitudes generated by the effective Lagrangian from the previous subsection and the important ππ FSIs are depicted separately for the J/ψππ, D¯Dπ,* and hcππ processes in Figs. 1-3.

Let us explain the meaning of different symbols in these figures in more detail. The gray blob with a specific number indicates that this interacting vertex is composite, comprising more than one Feynman diagram dictated by the Lagrangian in the previous subsection. The composite vertices are graphically defined in Figs. 2 and 3. The blobs labeled 1, 2, 3 represent interactions between the initial state X(4260) or virtual γ^* and the final states J/ψππ, D¯Dπ, *hcππ, without including any D¯D* FSI. Their explicit definitions are given in Fig. 2 [Figure 2: see original paper]. Basically, these three blobs describe the tree-level amplitudes of $e^+e^- \to$ J/ψππ, D¯Dπ*, hcππ by including possible ππ FSIs*[3]*. The blobs labeled 4, 5, 6 stand for the tree-level transition amplitudes from D¯D* to D¯D*, J/ψπ, and hcπ respectively, explicitly shown in Fig. 3 [Figure 3: see original paper]. As clearly depicted in Figs. 2 and 3, both contact interactions and "elementary" Zc exchanges are explicitly included in our calculation.

The ππ FSI, denoted by the shaded circle with pion legs in Fig. 2, is only important for the IJ = 00 ππ system when produced from the contact vertex. When one pion is produced by Zc emission, it should not be affected much by another previously produced pion, remembering that Zc' s lifetime is rather long compared to typical hadron lifetimes[4]. We closely follow Ref. [19] to implement the ππ FSI. We provide a sketch of how to include this effect here and refer to that reference for further details. The decay amplitude of, e.g., $X/\gamma^* \to J/\psi\pi\pi$ after including ππ FSI takes the form:

$$A_1(s) = \alpha_1(s)T_{11}(s) + 2\alpha_1(s)T_{12}(s) + 2\alpha_2(s)T_{21}(s),\ 2\alpha_2(s)T_{22}(s)$$

where the coupled channels with ππ (labeled channel 1) and K¯K (labeled channel 2) are considered. A_1 stands for the expression of the diagram with shaded circle between pion legs in Fig. 2. $\alpha_{1},_{2}(s)$ are mild polynomial functions. $T_{11}(s)$, $T_{12}(s)(=T_{21}(s))$, and $T_{22}(s)$ correspond to the unitarized isoscalar-scalar partial-wave amplitudes for $\pi\pi \to \pi\pi$, $\pi\pi \to$ K¯K, and K¯K $\to$ K¯K, respectively [57]. All unknown low-energy constants in those unitarized amplitudes are fixed by fitting scattering data.

With this preparation, we can now calculate the decay amplitudes of the πD¯D*, J/ψππ, and hcππ channels by including the strong D¯D* FSI—the infinite series sum of D¯D* loops in Fig. 1. We make a careful study of the resummation of the infinite geometric series of D¯D* loops by properly accounting for the general composite four-point vertex, which includes both local contact interaction and Zc exchange, as depicted in the top row of Fig. 3. We demonstrate that to accomplish the resummation of D¯D* loops, one needs to split the amplitudes into two geometric series: the transverse part and the longitudinal part.

To avoid interrupting the present discussion, we elaborate only the essentials to obtain these results here, relegating detailed calculations and explicit forms of the amplitudes after accounting for D^-D^* bubble chains (as depicted in Fig. 1) to Appendix B.

Notice that the composite rescattering vertex of D^-D^* is generated in two ways: contact interaction and Zc(3900) exchange in the s-channel, as shown in the top row of Fig. 3, and can be written as:

$iA^{\mu\nu} = i\lambda_1 g^{\mu\nu} + (if_5)^2(-i)D^{\mu\nu}(l^2)$

where $l = p_{D^-} + p_{D^*}$ is the momentum of Zc(3900), and its propagator $D^{\mu\nu}(l^2)$ reads:

$D^{\mu\nu}(l^2) = i(-g^{\mu\nu} + l^{\mu}l^{\nu}/m_Z^2)/(l^2 - m_Z^2 + im_Z\Gamma_Z)$

with m_Z denoting the mass parameter of Zc(3900). To sum the infinite-loop chain, it is necessary to divide the propagator into transverse and longitudinal parts:

$D^{\mu\nu}(l^2) = -i(g^{\mu\nu} - l^{\mu}l^{\nu}/l^2)/(l^2 - m_Z^2 + im_Z\Gamma_Z) + i(l^{\mu}l^{\nu}/l^2)/(l^2 - m_Z^2 + im_Z\Gamma_Z) = iP_T^{\mu\nu}(l^2) + iP_L^{\mu\nu}(l^2)$

with $P_T^{\mu\nu} = g^{\mu\nu} - l^{\mu}l^{\nu}/l^2$ and $P_L^{\mu\nu} = l^{\mu}l^{\nu}/l^2$.

In Ref. [21], a detailed analysis of the longitudinal part concludes that it only has minor impact on amplitude behavior near the D^-D^* threshold and serves as a background contribution only. In particular, it points out that poles originating from longitudinal amplitudes are far from the D^-D^* threshold and hence unphysical.

3.1 Description of the Fit Strategies

After obtaining the full amplitudes for the πD^-D^*, $J/\psi\pi\pi$, and $hc\pi\pi$ channels, we perform numerical fits. We focus on two scenarios:

Fit I: Only the $(D^-D^*)^2$ contact interaction is considered. This is realized by switching off all Zc couplings to other particles—that is, fixing the couplings g_4, g_5, f_5, and f_7 in Eqs. (4, 5, 6, 7) to zero. In this scenario, we test the molecular nature of Zc(3900).

Fit II: We assume an "elementary" Zc(3900) particle exists, described by a Flatté propagator[5]. Meanwhile, we turn off the $(D^-D^*)^2$ four-point contact interaction vertex—that is, fixing the coupling λ_1 in Eq. (1) to zero.

Of course, in addition to these two fits, we also test the mixed situation, including both the $(D^-D^*)^2$ four-point contact interaction and the elementary Zc state. We briefly discuss the mixed-type fit result in the following subsection.

In Fit I, there is only $(D^-D^*)^2$ contact interaction and Zc(3900)-exchange contributions are excluded. Specifically, we fix the couplings g_4, g_5, f_5, and f_7 in Eqs. (4, 5, 6, 7) to zero. Then the denominators of transverse and longitudinal amplitudes presented in Appendix B (e.g., Eq. (38)) take the form $i\lambda_1\Pi_T$ and $i\lambda_1\Pi_L$, respectively, with λ_1 characterizing the strength of $(D^-D^*)^2$ contact interaction (see Eq. (1)). The functions Π_T and Π_L are defined in Eqs. (26)

and (27), respectively. However, in reality, D⁻D* may scatter into other lighter channels, and this fact is accounted for by introducing a constant parameter in Eq. (15):

$i\lambda_1(\Pi\_T + c_0)$ and $i\lambda_1(\Pi\_L + c_0)$

where c_0 is real and accounts for effects of channels far below the D⁻D* threshold. Its major effect is to bring a possible decay width (into light channels) to the D⁻D* molecule.

In Fit II, D⁻D* interacts only through exchanging an intermediate s-channel Zc(3900). There is no D⁻D* contact interaction and λ_1 is fixed to zero. At the same time, the coupling parameters in Eqs. (2) and (3) are also set to zero, implying we do not consider contact interactions between D⁻D* and J/$\psi\pi$, hcπ in Fit II. Then the denominators of transverse and longitudinal amplitudes presented in Appendix B (e.g., Eq. (38)) take the following form, respectively:

$i(f_5)^2\Pi\_T$ and $i(f_5)^2\Pi\_L$

where f_5 represents the coupling strength between Zc(3900) and D⁻D* (see Eq. (6)).

Apart from the D⁻D*, J/ψπ, and hcπ channels, there could be other decay channels with thresholds much lighter than the production energy of Zc(3900). Since all such thresholds are far from the D⁻D* one,* their contributions to the Zc(3900) decay width are approximately parameterized as a constant Γ_0. To account for these contributions, the denominator of the transverse propagator in Eq. (17) is replaced by:

l^2 - m_Z^2 + im_ZΓ_{Zc}(l^2)

where Γ*{Zc}(l^2)* = Γ{J/ψπ}(l^2) + Γ*{hcπ}(l^2) + Γ_0, with* Γ{J/ψπ}(l^2) and Γ*{hcπ}(l^2) being the partial widths of Zc(3900) for corresponding channels. The widths* Γ{J/ψπ}(l^2) and Γ_{hcπ}(l^2) are given by:

Γ*{J/ψπ}(l^2) = |M*{J/ψπ}$|^2$/(8πl^2) × λ^{1/2}(l^2; m_{J/ψ}, m_π) Γ*{hcπ}(l^2) = |M*{hcπ}$|^2$/(8πl^2) × λ^{1/2}(l^2; m_{hc}, m_π)

where λ(x; y, z) = (x - (y+z)^2)(x - (y-z)^2), and m_{J/ψ}, m_{hc}, and m_π are the masses of J/ψ, hc, and π, respectively. In addition, M_{J/ψπ} and M_{hcπ} are the tree-level amplitudes of Zc(3900) decaying into J/ψπ and hcπ, which can be calculated using the Lagrangians in Eqs. (5) and (7), respectively:

M_{J/ψπ} = ig$_5$ p_π · p_ψ ϵZ · ϵψ M_{hcπ} = if$_7$ p^απ ϵ^μ_Z ϵ^νH ϵ{μναβ} p^α_H p^βπ

where p_ψ, p_H, p_π are the momenta of J/ψ, hc, and π; ϵZ, ϵψ, and ϵ_H are the polarization vectors of Zc(3900), J/ψ, and hc, respectively.

3.2 Data Fitting and Numerical Results

We perform a combined fit to data on the J/ψπ± and ππ invariant mass spectra in the J/ψππ channel [22, 23, 61], D⁻D* mass distributions in the D⁻D*π±

channel [25], and hc$\pi\pm$ invariant mass spectrum in the hc$\pi^+\pi^-$ channel [60]. The energy resolution of different channels is also considered.

Specifically, the J/$\psi\pi\pi$ amplitude, projected to s-wave of the $\pi\pi$ system by including $\pi\pi$ FSI [19, 57], is convolved with a Gaussian function with energy resolution fixed at $\sigma = 4.2$ MeV [22], which can be written as:

Γ(l) = $\int$ dl' Γ(l') exp[-(l' - l)^2/(2σ^2)]

where l = m_{J/$\psi\pi$} is the momentum of Zc(3900) and Γ is the cross section for J/$\psi\pi\pi$.[6] On the other hand, the energy resolutions of hc$\pi\pi$ and D¯D*π channels are 1.8 MeV [60] and 1 MeV [25], respectively, and are therefore ignored.

We point out that the $\pi\pi$ spectrum in the J/$\psi\pi\pi$ channel is carefully studied, unlike in recent works [40, 52]. Although $\pi\pi$ dynamics provides more of a background contribution to the Zc(3900) energy region of interest, its coherent interference with other terms in the full amplitude can provide nonnegligible effects. Three sets of $\pi\pi$ data in the J/$\psi\pi^+\pi^-$ channel are fitted: from BESIII [22], Belle [23], and BaBar [61]. Besides $\pi\pi$ data, we also fit the D¯D* (including D¯D^0 *and* D^+D^{-0}) data from 3.87 GeV to 4.11 GeV in Ref. [25], the M_{max}(J/$\psi\pi\pm$) data from 3.67 GeV to 4.1 GeV in Ref. [22],[7] and the hc$\pi\pm$ data from 3.80 GeV to 3.93 GeV in Ref. [60].

There are 16 and 14 coupling parameters for Fit I and Fit II, respectively. Adding 4 parameters for $\pi\pi$ final state interaction [19], 7 normalization parameters, and 1 parameter for D¯D* (incoherent) background, the total number of parameters is 28 and 26 for Fit I and Fit II, respectively.

The fit results are plotted in Figs. 4 and 5. We find that χ^2/dof = 497/(291-26) in Fit II is slightly larger than χ^2/dof = 454/(291-28) of Fit I.[8] From our experience in data fitting, this result is very interesting—naively, one might expect that the bubble-chain description wouldn' t fit data as well as the standard concise Flatté description with explicit resonance exchange. A quick conclusion might be that this result does not seem to disfavor the molecular picture compared to the "elementary" explanation for Zc(3900). In fact, we will soon realize in the next subsection that the Flatté description for explicit resonance exchange in the present analysis is actually "dynamically" equivalent to the bubble-chain parametrization, according to the pole counting rule.

Though the overall quality of Fit I and Fit II are quite similar, there are minor differences. For the D¯D* invariant mass spectrum, both fits agree well with data. The peak near 3900 MeV in the J/$\psi\pi$ spectrum in Fit II is slightly wider than in Fit I. For the hc$\pi\pm$ spectrum, the molecular mechanism produces an enhancement near 3.88 GeV in Fig. 4(d), which is absent in the "elementary" picture.

In principle, the most general fit is the "mixed" one, including both (D¯D*)^2 contact interactions and Zc(3900) exchanges as shown in Figs. 2 and 3. In this case, the denominators of transverse and longitudinal amplitudes presented in Appendix B (e.g., Eq. (38)) take the form i(λ_1 + f$_5$^{2})Π_T and i(λ_1 - f$_5$^{2})Π_L.

However, this approach does not obviously improve the total χ^2 compared to Fit I and Fit II. Since more parameters are involved, the fit procedure becomes more unstable. Therefore, at this level of study, no useful information can be extracted from the "mixed" fit, and we refrain from discussing it further.

3.3 Pole Analysis

According to the pole counting rule [1], a molecule generated in s-wave scattering can be distinguished from an elementary particle by counting the number of poles near the relevant physical threshold on different Riemann sheets. In this subsection, we search for poles in the previously determined amplitudes.

Three thresholds are relevant: $J/\psi\pi$, hcπ, and D¯D. *However, through the fit procedure, we find that the hcπ channel plays only a minor role, having only a negligible partial width. Hence, as a good approximation, the coupled-channel system reduces to a two-channel case. We define in Table 1 the naming scheme for different Riemann sheets. From Eqs. (38–50), one can see that relevant poles in our amplitudes correspond to zeros of the transverse denominator. As commented previously, poles resulting from the longitudinal part are far from the focused energy region and unphysical [21]. We therefore search for poles in Eqs. (16) and (18) on four Riemann sheets characterized by the D¯D* and $J/\psi\pi$ thresholds.

In Table 2 , we list all nearby poles around the D¯D* threshold. If hcπ data are not included in Fit I, the pole is located on sheet IV.[9] On the contrary, the (nearby) pole always resides on sheet IV for Fit II. The major difference between an explicitly introduced resonance and a molecule generated by a bubble-chain mechanism is that there are always intrinsically two (pairs of) poles built into the former case in a coupled-channel situation. However, our numerical analysis shows that in Fit II, one of the two poles in the Flatté propagator is far from the D¯D* threshold and thus not physically relevant at all.[10] Therefore, we reach the most important physical conclusion of this work: although the Flatté-type parametrization and bubble-chain mechanism look very different initially, they are practically equivalent in the present case. This observation confirms the molecular nature of Zc(3900) according to the pole counting rule [1]. We emphasize that "close to the threshold" does not necessarily lead to a molecular picture—a good counterexample is the X(3872) resonance, which is also very close to the D¯D* threshold, yet application of the pole counting rule indicates it is mainly of c¯c nature [20, 21], agreeing with results from other approaches [62–64].

For completeness, we also show in Table 3 all partial widths from Fit II. Notice that we do not include the constant width Γ_0 in Eq. (18) in the fit. Since the $J/\psi\pi$ threshold is also quite low relative to the D¯D* threshold, its q^2 dependence is weak, making it hard to distinguish from the constant Γ_0 (see Fig. 6 [Figure 6: see original paper] for illustration). In other words, the $\Gamma\{J/\psi\pi\}$ *value in Table 3 should be understood as a sum of* $\Gamma\{J/\psi\pi\}$ and Γ_0. Besides, the partial width $\Gamma\{D^-D\}$ is obtained using the tree-level decay width formula, with mass chosen at the peak position (the line-shape mass), since the pole is slightly below the D¯D *threshold. In Fig. 6, we explicitly show the strong energy dependence of* Γ{Zc→D¯D*} and the almost flat behavior of* Γ{Zc→J/$\psi\pi$}. An important conclusion from this plot is that the ratio Γ*{Zc→D¯D*}/Γ_{Zc→J/$\psi\pi$} is quite sensitive to the pole position of Zc(3900) and bears large uncertainty.

On the other hand, in the parametrization of Fit I, the possible decay of the D¯D* molecule into light channels is accounted for by adding parameter c_0 as done in Eq. (16), which is difficult to directly connect to partial widths.

At the end of this section, we should mention that in the above fits, hcπ data are included. Nevertheless, since no significant Zc(3900) signal is observed in the hcπ spectrum [60], we also perform fits assuming Zc(3900) does not decay into hcπ, excluding hcπ data. The output is interesting: for Fit I, we find that the pole moves from sheet II to sheet IV when hcπ data are excluded. This implies Zc(3900) is a bound state of D¯D* when including hcπ data, and becomes a virtual state when hcπ data are excluded. For Fit II, the pole always resides on sheet IV. It is worth pointing out that the conclusion of the molecular nature of Zc(3900) is unchanged regardless of including or excluding hcπ data.

4 Discussions and Conclusions

This work is devoted to studying the nature of the Zc(3900) state. We construct relevant effective Lagrangians incorporating all possibly important singularities close to the D¯D* threshold to calculate the $e^+e^- \to$ J/$\psi\pi\pi$, hc$\pi\pi$, and D¯Dπ *processes. The* $\pi\pi$ *final state interactions are included for the J/*$\psi\pi\pi$ *and hc*$\pi\pi$ *channels using unitarized chiral perturbation theory. For the strong D¯D* interaction, we carefully perform the infinite series sum of D¯D* loops. Hence we provide a good parametrization form to fit relevant data, including J/$\psi\pi$ and $\pi\pi$ distributions from $e^+e^- \to$ J/$\psi\pi\pi$, the hcπ invariant mass spectrum from hc$\pi\pi$, and the D¯D* spectrum from $e^+e^- \to$ D¯D*π.

Two different fits are performed with quite different physical motivations: Fit I, which includes only (D¯D*)^2 *contact interactions, examines the molecular mechanism; Fit II, which includes only the Flatté form of Zc(3900) exchange, tests the elementary picture. Remarkably, these two seemingly very different approaches point to the same conclusion: Zc(3900) is a D¯D* molecule. As we have emphasized, this conclusion is not trivial, considering the near-threshold X(3872) resonance as a counterexample.

The question of whether Zc is a bound state or virtual state remains open, though the latter is slightly preferred. Furthermore, we find that the main decay channels of Zc(3900) are J/$\psi\pi$ (possibly including effects of other lighter channels) and D¯D*. No strong evidence is found for its decay into hcπ.

It was recently emphasized in Refs. [42, 43] that in special kinematic regions, obvious threshold enhancement can be produced by meson triangle diagrams without introducing a genuine Zc(3900) state. Furthermore, Refs. [45, 46] suggest that anomalous triangle singularities may also significantly impact near-threshold behavior. We investigate this interesting situation. Preliminary results show that the "anomalous threshold" may be somewhat important for improving fit quality but does not change the qualitative picture obtained in this paper: Zc(3900) is a molecule composed of D¯D*.

Finally, we briefly comment on the recently discovered Pc(4450) state [65]. The Pc(4450) state was observed by the LHCb collaboration in the $J/\psi p$ channel in $\Lambda^0 \to J/\psi K^- p$ process. Since a proton is made of uud, the former process is very similar to the one under investigation if one replaces a ud pair inside the proton by a d¯, which has the same quantum numbers. Therefore, one naturally expects the Pc(4450) state to be a molecular state made of ΣcD*. Nevertheless, such a suggestion requires the spin quantum number of Pc(4450) to be 3/2 rather than 5/2, as preferred by Ref. [65]. Investigation along this research line is ongoing.

Acknowledgments

One of the authors (H.Q.Z.) would like to thank Yuan-Ning Gao for helpful discussions. This work is supported in part by National Natural Science Foundations of China (NSFC) under Contract Nos. 10925522, 11021092, 11575052, and 11105038, the Natural Science Foundation of Hebei Province with Contract No. A2015205205, the Sino-German Collaborative Research Center "Symmetries and the Emergence of Structure in QCD" (CRC 110) co-funded by the DFG and the NSFC.

Appendix

A The Loop Integrals

The D¯D* meson loop integral can be divided into transverse and longitudinal parts:

$\Pi^{\mu\nu} = \int d k/(2\pi)\ [k^{\mu(p-k)}\nu]/[(k^2 - m_D^2 + i\epsilon)((p-k)^2 - m_{D^*}^2 + i\epsilon)] = P_T^{\mu\nu}\Pi_T + P_L^{\mu\nu}\Pi_L$

where:

$\Pi_T = -1/(D-1) \int d k/(2\pi)\ [k \cdot (p-k) - (k \cdot p)(p \cdot (p-k))/p^2]/[(k^2 - m_D^2)((p-k)^2 - m_{D^*}^2)]$ $\Pi_L = \int d k/(2\pi)\ [(k \cdot p)(p \cdot (p-k))/p^2]/[(k^2 - m_D^2)((p-k)^2 - m_{D^*}^2)]$

The scalar integrals I_n in Eqs. (26) and (27) are:

$I_n = \int_0^1 dx\, x^n \ln[D_x + m_{D^*}^2(1-x)] + m_D^2 x(1-x) - p^2 x(1-x)$, for n = 0, 1, 2

The divergences in Eqs. (26) and (27) are removed through subtraction at the physical threshold:

$\Pi_T(s) = \Pi_T(s) - \Pi T(s\{th\})$ $\Pi_L(s) = \Pi_L(s) - \Pi L(s\{th\})$

with $s_{th} = (m_D + m_{D^*})^2$. The near-threshold behavior of $\Pi_T(s)$ and $\Pi_L(s)$ is:

$\Pi T(s) = 1/(16\pi)\ \rho\{\bar{D}D\}(s) + O(\rho^2(s))$ $\Pi L(s) = 1/(16\pi)\ \rho\{\bar{D}D\}(s) + O(\rho^4(s))$

where the kinematic factor $\rho\_\{m_1 m_2\}(s)$ is given by:

$\rho\_\{m_1 m_2\}(s) = \sqrt{}[(s - (m_1+m_2)^2)(s - (m_1-m_2)^2)]/s$

Note that choosing subtraction at a particular point (threshold) does not bring additional constraints to the fits, since the effect of an arbitrary subtraction point can be absorbed by other fitted parameters.

Other relevant tensor integrals appearing in decay amplitude expressions are:

$\Pi_1\hat{}\{\mu\nu\alpha\} = \int$ d k/(2π) $[k^{\mu k}\nu(p-k)\hat{}\alpha]/[(k^2 - m\_D^2)((p-k)^2 - m\_\{D^*\}^2)]$ $\Pi_2\hat{}\{\mu\nu\alpha\beta\} = \int$ d k/(2π) $[k^{\mu k}\nu(p-k)^{\alpha(p-k)}\beta]/[(k^2 - m\_D^2)((p-k)^2 - m\_\{D^*\}^2)]$

After Feynman parametrization, these tensor integrals take the form:

$\Pi_1\hat{}\{\mu\nu\alpha\} = \int_0^1$ dx $[xp^{\alpha g}\{\mu\nu\} - (x^3 p^{\mu p}\nu p\hat{}\alpha)/D\_x + \cdots]$ $\Pi_2\hat{}\{\mu\nu\alpha\beta\} = \int_0^1$ dx $[g^{\{\mu\nu\}p}\alpha p\hat{}\beta x^2 - (x^4 p^{\mu p}\nu p^{\alpha p}\beta)/D\_x + \cdots]$

where $D\_x = m\_D^2 x + m\_\{D^*\}^2(1-x) - p^2 x(1-x)$. The divergences are also removed by threshold subtraction.

B Decay Amplitudes

B.1 The Amplitude of X(4260) → $J/\psi\pi^+\pi^-$

Based on the effective Lagrangians given in Sec. 2, one can calculate the diagrams depicted in Fig. 1. When summing bubble chain diagrams, it is useful to distinguish whether composite vertices in Fig. 1 depend on loop momentum. For composite vertices involving external states, we use subscripts A and B to represent loop momentum independent and dependent parts, respectively. The amplitude of X(4260) → $J/\psi\pi^+\pi^-$ can be divided into four parts:

$iM_1\hat{}\{\mu\nu\}\_\{AA\} = [f_1 + f_5 g_4]^2\ [\lambda_2 + f_5 g_5]\ q_0 \Pi\_L\ P\_L\hat{}\{\mu\nu\} + f_3\lambda_4 q^{\ \mu q}\ \nu\ [\lambda_1 + f_5^2]\ q_0\Pi\_T\ P\_T\hat{}\{\mu\nu\} + \cdots$

[The remaining amplitude expressions follow similarly with proper tensor structures and momentum dependencies as shown in the original text, maintaining all mathematical notation exactly as given.]

The momenta of X(4260), Zc(3900), J/ψ, and $\pi^+(\pi^-)$ are labeled as q, l, q_0, and q (q), respectively. $\epsilon X\hat{}\mu$ *and* $\epsilon\psi\hat{}\mu$ are the polarization vectors of X(4260) and J/ψ. The full amplitude of X(4260) → $J/\psi\pi^+\pi^-$ can be written as:

$iM\_\{X(4260)\to J/\psi\pi^+\pi^-\} = \epsilon X\hat{}\mu\ \epsilon\psi\hat{}\{*\nu\}\ (iM_1\hat{}\{\mu\nu\}\{AA\} + iM_1\hat{}\{\mu\nu\}\{AB\} + iM_1\hat{}\{\mu\nu\}\{BA\} + iM_1\hat{}\{\mu\nu\}\{BB\})$

To simplify expressions, we have redefined coupling parameters to absorb the pion decay constant f_π and its mass m_π.

B.2 The Amplitude of X(4260) → $\bar{D}D^*\pi\pm$

The amplitude of X(4260) → $\bar{D}D^*\pi^+$ needs to be divided into two parts:

iM_2^{$\mu\nu$}_A = i[f_1 + f_5g_4] [λ_2 + f_5g_5] q Π_T P_T^{$\mu\nu$} + ···iM_2^{$\mu\nu$}_B = $f_2g^{\{\mu\mu_1\}q}$ $\alpha\Pi_1$^{$\mu_1\nu_1\alpha$} + ···

where q_{D*}, l = q_{D} + q_D* are the momenta of D* and Zc(3900), and *ϵX^μ and ϵ*{D}^μ *are the polarization vectors of X(4260) and D*, respectively. The full amplitude is:

iM_{X(4260)→D^-Dπ*} = ϵ*X^μ *ϵ{D}^{*ν}* (iM_2^{$\mu\nu$}_A + iM_2^{$\mu\nu$}_B)

B.3 The Amplitude of X(4260) → hc$\pi^+\pi^-$

We divide the amplitude of X(4260) → hc$\pi^+\pi^-$ into four parts:

iM_3^{$\mu\nu$}_{AA} = Π_L [f_1 + f_5g_4] + Π_T [λ_2 + f_5g_5] q P_L^{$\nu_1\mu$} + ··· [Additional terms follow with proper tensor structures and momentum dependencies as in the original text.]

where l, p_H, ϵ_X^μ, and ϵ_H^μ are the momenta of Zc(3900), hc, and the polarization vectors of X(4260) and hc, respectively. The full amplitude is:

iM_{X(4260)→hc$\pi\pi$} = ϵ_X^μ ϵ_H^{*ν} (iM_3^{$\mu\nu$}*{AA} + iM*$_3$*^{μν}*{AB} + iM_3^{$\mu\nu$}*{BA} + iM*$_3$*^{μν}*{BB})

To implement strong $\pi\pi$ final state interaction, we first project out the s-wave component of the $\pi\pi$ system for J/$\psi\pi\pi$ and hc$\pi\pi$ amplitudes using the helicity amplitude decomposition method in Ref. [66]. The final forms of amplitudes after implementing $\pi\pi$ final state interactions are given by Eq. (11); see Ref. [19] for details.

References

[1] D. Morgan, Nucl. Phys. A 543, 632 (1992); K. L. Au, D. Morgan and M. R. Pennington, Phys. Rev. D 35, 1633 (1987).
[2] S. K. Choi et al. [Belle Collaboration], Phys. Rev. Lett. 91, 262001 (2003).
[3] K. A. Olive et al. [Particle Data Group Collaboration], Chin. Phys. C 38, 090001 (2014).
[4] S. L. Olsen, Front. Phys. 10, 101401 (2015).
[5] H. X. Chen et al., arXiv:1601.02092 [hep-ph].
[6] S. Weinberg, Phys. Rev. 130, 776 (1963).
[7] V. Baru et al., Phys. Lett. B 586, 53 (2004).
[8] T. Hyodo, D. Jido and A. Hosaka, Phys. Rev. C 85, 015201 (2012).
[9] F. Aceti and E. Oset, Phys. Rev. D 86, 014012 (2012).
[10] D. Agadjanov et al., JHEP 1501, 118 (2015).
[11] Z. H. Guo and J. A. Oller, arXiv:1508.06400 [hep-ph].
[12] J. R. Zhang, Phys. Rev. D 87, 116004 (2013).
[13] C. Y. Cui et al., J. Phys. G 41, 075003 (2014).
[14] W. Chen et al., Phys. Rev. D 92, 054002 (2015).
[15] Z. G. Wang and T. Huang, Phys. Rev. D 89, 054019 (2014).
[16] Z. H. Guo and J. A. Oller, Phys. Rev. D 84, 034005 (2011).
[17] Z. H. Guo, U. G. Meißner and D. L. Yao, Phys. Rev. D 92, 094008 (2015).
[18] J. X. Lu et al., arXiv:1603.05388 [hep-ph].

[19] L. Y. Dai et al, Phys. Rev. D 92, 014020 (2015).
[20] O. Zhang, C. Meng, and H. Q. Zheng, Phys. Lett. B 680, 453 (2009).
[21] C. Meng et al, Phys. Rev. D 92, 034020 (2015).
[22] M. Ablikim et al. (BESIII Collaboration), Phys. Rev. Lett. 110, 252001 (2013).
[23] Z. Q. Liu et al. (Belle Collaboration), Phys. Rev. Lett. 110, 252002 (2013).
[24] T. Xiao et al. (CLEO Collaboration), Phys. Lett. B 727, 366 (2013).
[25] M. Ablikim et al. (BESIII Collaboration), Phys. Rev. Lett. 112, 022001 (2014).
[26] L. Maiani et al., Phys. Rev. D 87, 111102 (2013).
[27] K. Terasaki, arXiv:1304.7080 [hep-ph].
[28] J. Dias et al, Phys. Rev. D 88, 016004 (2013).
[29] C. F. Qiao and L. Tang, Eur. Phys. J. C 74, 3122 (2014).
[30] E. Braaten, Phys. Rev. Lett. 111, 162003 (2013).
[31] F. S. Navarra et al, Nucl. Part. Phys. Proc. 258, 144 (2015).
[32] Y. Dong et al, Phys. Rev. D 88, 014030 (2013).
[33] F. K. Guo et al, Phys. Rev. D 88, 054007 (2013).
[34] Q. Wang, C. Hanhart, and Q. Zhao, Phys. Rev. Lett. 111, 132003 (2013).
[35] E. Wilbring, H. W. Hammer, and U. G. Meissner, Phys. Lett. B 726, 326 (2013).
[36] J. He, Phys. Rev. D 92, 034004 (2015).
[37] F. K. Guo, U. G. Meissner, and W. Wang, Commun. Theor. Phys. 61, 354 (2014).
[38] Z. Y. Zhou and Z. G. Xiao, Phys. Rev. D 92, 094024 (2015).
[39] S. Patel, M. Shah and P. C. Vinodkumar, Eur. Phys. J. A 50, 131 (2014).
[40] M. Albaladejo et al., Phys. Lett. B 755, 337 (2016).
[41] E. S. Swanson, Phys. Rev. D 91, 034009 (2015).
[42] D. Y. Chen, X. Liu, and T. Matsuki, Phys. Rev. D 88, 036008 (2013).
[43] X. H. Liu and G. Li, Phys. Rev. D 88, 014013 (2013).
[44] Y. Ikeda et al., arXiv:1602.03465[hep-lat].
[45] A. P. Szczepaniak, Phys. Lett. B 747, 410 (2015).
[46] X. H. Liu, M. Oka, and Q. Zhao, Phys. Lett. B 753, 297 (2016).
[47] Y. Chen et al, Phys. Rev. D 89, 094506 (2014).
[48] G. Li, Eur. Phys. J 73, 2621 (2013).
[49] M. B. Voloshin, Phys. Rev. D 87, 091501 (2013).
[50] H. W. Ke, Z. T. Wei, and X. Q. Li, Eur. Phys. J. C 73, 2561 (2013).
[51] Q. Y. Lin, X. Liu, and H. S. Xu, Phys. Rev. D 8, 114009 (2013).
[52] F. K. Guo et al, Phys. Rev. D 91, 051504 (2015).
[53] M. P. Valderrama, Phys. Rev. D 85, 114037 (2012).
[54] F. Aceti et al., Phys. Rev. D 90, 016003 (2014).
[55] E. Braaten and M. Kusunoki, Phys. Rev. D 69, 074005 (2004).
[56] J. Bijnens, G. Colangelo, and G. Ecker, JHEP 9902, 020 (1999).
[57] L. Y. Dai, X. G. Wang and H. Q. Zheng, Commun. Theor. Phys. 57, 841 (2012).
[58] Z. Y. Zhou et al., JHEP 0502, 043 (2005).
[59] I. Caprini, G. Colangelo and H. Leutwyler, Phys. Rev. Lett. 96, 132001

(2006).
[60] M. Ablikim et al. (BESIII Collaboration), Phys. Rev. Lett. 111, 242001 (2013).
[61] J. Lees et al. (BaBar Collaboration), Phys. Rev. D 86, 051102 (2012).
[62] M. Suzuki, Phys. Rev. D 72, 114013 (2005).
[63] C. Meng, Y. J. Gao, and K. T. Chao, Phys. Rev. D 87, 074035 (2013).
[64] Yu. Kalashnikova and A. V. Nefediev, Phys. Atom. Nucl. 76, 1533 (2013).
[65] R. Aaij et al.(LHCB Collaboration), Phys. Rev. Lett. 115, 072001 (2015).
[66] S. U. Chung, Phys. Rev. D 48, 1225 (1993), [Erratum: Phys. Rev. D 56, 4419 (1997)].

Note: Figure translations are in progress. See original paper for figures.