Possible formation of high-temperature superconductor in the early stage of heavy-ion collisions (Postprint)

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Abstract

We investigate the effect of inverse magnetic catalysis (IMC) on charged meson condensation at finite temperature in the framework of the Nambu-Jona-Lasinio model, where mesons are calculated to the leading order of $1/N_c$ expansion by summing over infinite quark loops. The IMC effect on chiral condensate has been considered in three different ways, i.e., fitting Lattice data, using the running coupling constant, and introducing the chiral chemical potential, respectively. It is observed that, without including the IMC effect, the critical magnetic field eB_c for charged condensation increases monotonically with temperature. However, including IMC substantially affects the polarized charged condensation around the critical temperature T_c of the chiral phase transition; the critical magnetic field eB_c for charged condensation first decreases with temperature, reaches a minimum value around T_c, and then increases with temperature. Our calculations indicate that charged condensation can exist in the temperature region of $1-1.5T_c$ with a critical magnetic field $eB_c \sim 0.15-0.3\text{GeV}^2$, which suggests that high temperature superconductors might be created through non-central heavy ion collisions at LHC energies. We also show that the growing electric conductivity in the early stage of non-central heavy-ion collisions substantially delays the decay of the strong magnetic field, which facilitates the formation of the high temperature superconductor.

Full Text

Preamble

We investigate the effect of inverse magnetic catalysis (IMC) on charged meson condensation at finite temperature within the framework of the Nambu-Jona-Lasinio model, where mesons are calculated to leading order in the 1/Nc expansion by summing infinite quark loops. IMC for the chiral condensate has been considered through three different approaches: fitting lattice data, using a running coupling constant, and introducing a chiral chemical potential. It is observed that, without including the IMC effect, the critical magnetic field eBc for charged condensation increases monotonically with temperature. However, including IMC substantially affects the polarized charged condensation around the critical temperature Tc of the chiral phase transition: the critical magnetic field eBc for charged condensation first decreases with temperature, reaches a minimum value around Tc, and then increases with temperature. Our calculation indicates that charged condensation can exist in the temperature region of 1–1.5Tc with critical magnetic field eBc 0.3 GeV^2, which suggests that a high-temperature superconductor might be created through non-central heavy-ion collisions at LHC energies. We also show that growing electric conductivity in the early stage of non-central heavy-ion collisions substantially delays the decay of strong magnetic fields, which is helpful for the formation of high-temperature superconductors.

Introduction

Quantum Chromodynamics (QCD) is widely believed to be the fundamental theory of strong interactions. Investigations of its rich vacuum structure and how the QCD vacuum can be modified in extreme environments represent major theoretical challenges in modern physics. Such extreme environments include high temperature, finite baryonic chemical potential, and, more recently, strong magnetic fields [1-3]. On the surface of magnetars, magnetic fields can reach 10^1 G, and in the inner core of magnetars they could be as high as 10^1 G [4]. Strong magnetic fields with strength of 10^1 -10^2 G [corresponding to eB $(1.0\ GeV)^2$] can be generated in the laboratory through non-central heavy-ion collisions [5, 6] at the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC). Hence, heavy-ion collision experiments provide an excellent platform to investigate QCD vacuum and matter under strong magnetic fields. Strong magnetic fields cause many interesting effects on quark matter properties, such as the chiral magnetic effect (CME) [7–9], chiral vortical effect (CVE) [10], magnetic catalysis (MC) [11–13] and inverse magnetic catalysis (IMC) [14–16], and the formation of vacuum superconductors [17, 18].

Spontaneous breaking of chiral symmetry is one of the most important properties of strong interactions. It is well known that at zero chemical potential, chiral symmetry is broken in the low-temperature phase of QCD and restored at high temperature. The influence of external magnetic fields on chiral symmetry breaking and restoration has been a subject of intensive study for several years. At low temperature, background magnetic fields are known to enhance chiral symmetry breaking by increasing the value of the chiral condensate [11–13]. This phenomenon, known as magnetic catalysis, has been confirmed in various effective low-energy models of QCD [11–13, 19–30] as well as in numerical simulations of lattice QCD [31–35]. Since at low temperature the background magnetic field and thermal fluctuations have opposite effects on chiral symmetry

—respectively increasing and decreasing the chiral condensate—one might naturally assume that increasing the background magnetic field should lead to an increase in the critical temperature Tc of chiral symmetry restoration. Early analytical calculations in the simplest effective QCD models [22–30] and lattice simulations with heavy quarks [31, 34] suggested that magnetic catalysis should also be realized at high temperature. However, detailed QCD calculations with realistic quark masses have revealed that the chiral transition temperature Tc is, unexpectedly, a decreasing function of the background magnetic field [14–16]. This phenomenon is now known as inverse magnetic catalysis (IMC).

Theoretical understanding of the IMC mechanism has sparked significant activity and further controversies. At present, there is no unified consensus on the physical mechanism underlying IMC. For example, in Ref. [36] IMC was suggested to be caused by a magnetic inhibition effect related to the anisotropy of neutral meson propagation. In Ref. [37], however, magnetic inhibition was shown to be ineffective for explaining IMC, and it was recently demonstrated in [38] that IMC might be induced by neutral pion fluctuations if Pauli-Villars regularization is used. In Ref. [39] IMC was proposed to be related to particularities of infrared contributions to the quark mass gap, while in Ref. [40] IMC is explained by different effects of the magnetic field on low quark modes—responsible for chiral symmetry breaking—coming from valence and sea quarks. Alternatively, functional renormalization group studies in Ref. [41] have shown that the physics underlying IMC at high temperature cannot be captured by the standard Nambu–Jona-Lasinio (NJL) model, which is one of the most successful models of the chiral sector of low-energy QCD. IMC may, however, be reproduced if one introduces a phenomenological dependence of the NJL coupling on the magnetic field [42].

Another interesting opportunity to explain IMC comes from the fact that the theory of strong interactions has a nontrivial topological structure due to the existence of certain gluon configurations: instantons [43–45] and their thermal cousins, sphalerons [46–51], which describe transitions of the QCD vacuum between different topological sectors. The chirality imbalance induced by sphaleron transitions [52] or instanton-anti-instanton molecule pairing [53] can be invoked to explain IMC around high temperature. In continuation of these studies, the effect of the chiral chemical potential µ —which characterizes (local) imbalance between fermions with different chirality—on the chiral phase transition in the NJL model with different regularization schemes was investigated in Ref. [54].

Another interesting effect that may be realized in strong magnetic fields is the electromagnetic superconductivity of the QCD vacuum [17, 18]. The idea is based on the simple fact that the energy levels of a free pointlike charged particle in a static uniform external magnetic field B are $^2 = p_z^2 + m^2 + (2n + s_z + 1)|qB|$, where q is the electric charge of the particle, n 0 is the nonnegative integer characterizing Landau levels, s_z is the projection of the particle' s spin on the magnetic field axis z, and p_z is the particle' s momentum along the magnetic field. The ground-state mass of a charged ± meson with unit spin s = 1 corresponds to the lowest energy level with quantum numbers p_z = 0, n = 0 and s_z = sign(q). As the magnetic field increases, the ± mass decreases to zero, implying instability of the ground state toward condensation of charged mesons at the critical magnetic field eBc = m²_ .

The idea of magnetic-field-induced vacuum superconductivity remains under debate as different approaches yield controversial results. The NJL model approach [18, 55, 56] and independent non-perturbative holographic AdS/QCD techniques [57, 58] indicate that the mass of the -meson excitation should indeed vanish at a certain value of the magnetic field. On the other hand, relativistic Hamiltonian techniques [59] and Dyson-Schwinger equations [60] lead to the conclusion that the mass of charged mesons decreases at small magnetic fields and then increases as the magnetic field becomes larger, never dropping to zero. Another approach based on hidden local symmetry [61] at $O(p^2)$ and O(p) orders favors the vanishing of the -meson mass, supporting the conclusion obtained in -meson electrodynamics [17], while O(p) results are inconclusive. In contrast, existing numerical calculations in quenched lattice QCD without dynamical quarks [62, 63] show that the mass of ± does not vanish, while calculations in lattice QCD with light dynamical quarks have not yet been performed.

In Ref. [63] it was argued that -meson condensation is forbidden by the Vafa-Witten (VW) theorem because the VW theorem implies the absence of any massless Nambu-Goldstone (NG) bosons in QCD except for pions (the latter are associated with chiral symmetry breaking). According to Ref. [63], if -mesons were condensed in QCD they would lead to appearance of an NG boson, which is forbidden by the VW theorem. This argument was refuted in Ref. [64], where it was demonstrated that -meson condensation in an external magnetic field takes place not in QCD but in QCD QED, and consequently the would-be NG bosons associated with -meson condensation are absorbed into the longitudinal component of the electromagnetic gauge field in a Higgs-like mechanism. Consequently, no NG bosons appear due to -meson condensation, in agreement with the VW theorem. A similar conclusion about the consistency of condensation with the VW theorem was put forward in Ref. [65]. Moreover, mass calculations in quenched lattice QCD [62, 63] are consistent with a crossover transition to the superconducting phase [66].

In Ref. [56], by calculating the meson polarization function to leading order in the 1/Nc expansion in the NJL model, we obtained the critical magnetic field eBc 0.2 GeV², which is only 1/3 of the results from the point-particle calculation in [17]. Furthermore, in Ref. [67] it was observed that in the temperature range 0.2–0.6 GeV, the critical magnetic field for charged condensation is in the range of 0.2–0.6 GeV², which indicates that a high-temperature electromagnetic superconductor could be created at the LHC. However, in Ref. [67] we used MC for the chiral condensate to produce the constituent quark mass. In this work, we consider the effect of IMC on the charged meson condensate.

This paper is organized as follows: In Sec. II, by taking the quark propagator in the Landau level representation, we provide a general description of the two-flavor NJL model including the effective four-quark interaction in the vector channel, and derive the vector meson mass under magnetic fields at finite temperature and chemical potential. We introduce three different mechanisms for IMC in Sec. III and investigate how IMC affects charged meson condensation. In Sec. IV, we show the effect of large electric conductivity of high-temperature conductors on the decay of strong magnetic fields. Sec. V presents discussion and conclusions.

II. NJL Model and Meson Construction

A. The SU(2) Magnetized NJL Model

We investigate charged meson condensation in a magnetic field background at nonzero temperature within the framework of a two-flavor NJL model. The Lagrangian density of the NJL model takes the following form [68–73]:

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - \hat{m})\psi + G_S[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2] - G_V[(\bar{\psi}\gamma^\mu\tau^a\psi)^2 + (\bar{\psi}\gamma^\mu\gamma_5\tau^a\psi)^2] - \frac{1}{4}F_{\mu\nu}F^{\mu\nu},$$

where $\psi = (u, d)^T$ is the doublet of the two light quark flavors u and d with masses given by the diagonal current mass matrix $\hat{m} = \mathrm{diag}(m_u, m_d)$ and $\tau^a = (I, \vec{\tau})$ with $\vec{\tau} = (\tau^1, \tau^2, \tau^3)$ are the isospin Pauli matrices. G_S and G_V are the coupling constants for scalar (pseudoscalar) and vector (axial-vector) channels, respectively. The covariant derivative $D_\mu = \partial_\mu - iq_f A_\mu^{\mathrm{ext}}$ couples quarks to an external magnetic field $\mathbf{B} = (0, 0, B)$ along the positive z direction via a background field $A_\mu^{\mathrm{ext}} = (0, 0, Bx_1, 0)$. The electric charges of the (u, d) quark fields are $q_f = (2e/3, -e/3)$ and the field strength tensor is $F_{\mu\nu} = \partial_{[\mu}A_{\nu]}^{\mathrm{ext}}$.

The four-point interactions in Eq. (1) can be semi-bosonized via Gaussian transformations, and the NJL Lagrangian can be rewritten as:

$$\mathcal{L}_{sb} = \bar{\psi}(x)(i\gamma^\mu D_\mu - \hat{m})\psi(x) - \bar{\psi}(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi} + \gamma^\mu V_\mu^a\tau^a + \gamma^\mu\gamma_5 A_\mu^a\tau^a)\psi - \frac{\sigma^2 + \vec{\pi}^2}{4G_S} - \frac{V_\mu^a V^{\mu a} + A_\mu^a A^{\mu a}}{4G_V},$$

where the Euler-Lagrange equations of motion for the auxiliary fields lead to the constraints:

$$\begin{aligned} \sigma(x) &= -2G_S\langle\bar{\psi}(x)\psi(x)\rangle, \\ \vec{\pi}(x) &= -2G_S\langle\bar{\psi}(x)i\gamma_5\vec{\tau}\psi(x)\rangle, \\ V_\mu^a(x) &= -2G_V\langle\bar{\psi}(x)\gamma_\mu\tau^a\psi(x)\rangle, \\ A_\mu^a(x) &= -2G_V\langle\bar{\psi}(x)\gamma_\mu\gamma_5\tau^a\psi(x)\rangle. \end{aligned}$$

Assuming the masses of u and d quarks are equal ($m_u = m_d = m_0$), the constituent quark mass M of these light quarks is given by:

$$M = m_0 - 2G_S\langle\bar{\psi}\psi\rangle.$$

In the NJL model, at least in the mean-field approximation, the chiral condensate increases with magnetic field at zero temperature as well as at finite temperature. We will introduce inverse magnetic catalysis for the quark condensate around the critical temperature through three different approaches in Sec. III.

B. Charged Meson Construction Using the Landau Level Representation

In the NJL model framework, the meson can be expressed as an infinite sum of quark-loop chains using the random phase approximation. The -meson propagator $D^{ab}_{\mu\nu}(q^2)$ can be obtained from the one-quark-loop polarization function $\Pi^{ab}_{\mu\nu}(q^2)$ via the Schwinger-Dyson equation:

$$D^{ab}_{\mu\nu}(q^2) = [-2iG_V\delta^{ab}g_{\mu\nu}] + [-2iG_V\delta^{ac}g_{\mu\lambda}][i\Pi^{\lambda\sigma,cd}(q^2)][D^{db}_{\sigma\nu}(q^2)],$$

where a, b, c, d are isospin indices and $\mu, \nu, \lambda, \sigma$ are Lorentz indices. The one-quark-loop polarization function is given by:

$$\Pi^{ab}_{\mu\nu}(x, x') = i\mathrm{Tr}[\gamma_\mu\tau^a S_Q(x, x')\gamma_\nu\tau^b S_Q(x', x)].$$

Here we use the Landau level representation of the quark propagator $S_Q(x, x')$ [13, 75]:

$$S_Q(x, y) = \exp\left[i\Phi(x_\perp, y_\perp)\right]\int \frac{d^4k}{(2\pi)^4} e^{-ik(x-y)}\tilde{S}_Q(k),$$

with the Fourier transform:

$$\tilde{S}_Q(k) = i\exp\left(-\frac{k_\perp^2}{|QeB|}\right)\sum_{n=0}^{\infty}\frac{D_n(QeB, k)}{k_0^2 - k_3^2 - M^2 - 2n|QeB| + i\epsilon},$$

where:

$$D_n(QeB, k) = (k_0\gamma^0 - k_3\gamma^3 + M)[(1 + i\gamma^1\gamma^2\mathrm{sign}(QeB))L_n(2k_\perp^2/|QeB|) - (1 - i\gamma^1\gamma^2\mathrm{sign}(QeB))L_{n-1}(2k_\perp^2/|QeB|)] +$$

and L_n^α are generalized Laguerre polynomials with $L_n \equiv L_n^0$ and $L_{-1} = 0$. Note that Q is the diagonal matrix in flavor space.

For charged mesons, the magnetic field violates Lorentz invariance, so $\Pi^{ab}_{\mu\nu}(x, x')$ should be written as:

$$\Pi^{ab}_{\mu\nu}(x, x') = e^{i\Phi(x_\perp, x'_\perp)} \int \frac{d^4q}{(2\pi)^4} e^{-iq(x-x')} \Pi^{ab}_{\mu\nu}(q_\parallel, q_\perp),$$

where $\Phi(x_\perp, x'_\perp)$ is the Schwinger phase. If we consider the gauge $A^{\text{ext}}_\mu = \delta_{\mu 1} B x_2$ and assume $eB > 0$, the Schwinger phase is:

$$\Phi(x_\perp, x'_\perp) = -\frac{qB}{2}(x_1 + x'_1)(x_2 - x'_2).$$

The isospin Pauli matrices $\tau^a = \tau^\pm$ and $\tau^b = \tau^\mp$ represent the charged $\pm$ mesons respectively, with $\tau^\pm = (\tau^1 \pm i\tau^2)/2$ and q is the electric charge of the meson. Equation (6) can thus be rewritten as:

$$\Pi^{\rho^\pm}_{\mu\nu}(x, x') = i\text{Tr}[\gamma_\mu \tau^\pm S_Q(x, x') \gamma_\nu \tau^\mp S_Q(x', x)].$$

Considering momentum conservation ($p = k + q$), we can present this in momentum space:

$$\Pi^{\rho^\pm}_{\mu\nu}(q_\parallel, q_\perp) = N_c \int \frac{d^4k}{(2\pi)^4} \text{tr}_{\text{cs}}[S(k+q)\gamma_\mu S(k)\gamma_\nu],$$

where the trace is over color and spinor indices.

In the rest frame of the $\pm$ meson, we can decompose $\Pi_{\mu\nu}$ in Lorentz indices as:

$$\Pi^{\rho^\pm}_{\mu\nu}(q) = \Pi_T(q^2) P^T_{\mu\nu} + \Pi_L(q^2) L_{\mu\nu} + \Pi_{00}(q^2) u_\mu u_\nu,$$

where $u_\mu = (1, 0, 0, 0)$ is the 4-velocity in the rest frame. The operators $P^T_{\mu\nu}$ and $L_{\mu\nu}$ are spin-projection operators on the polarization states of mesons with projections $s_z = \pm 1$ and $s_z = 0$, respectively:

$$P^T_{\mu\nu} = \sum_{s_z = \pm 1} \epsilon^{(s_z)}_\mu \epsilon^{(s_z)*}_\nu, \quad L_{\mu\nu} = b_\mu b_\nu,$$

where $b_\mu = (0, 0, 0, 1)$ is a unit vector along the external magnetic field axis, and $\epsilon^{(1)}_\mu = (0, 1, i, 0)/\sqrt{2}$ and $\epsilon^{(2)}_\mu = (0, 1, -i, 0)/\sqrt{2}$ are the right- and left-handed polarization vectors.

Consequently, the meson propagator takes the form:

$$D^{ab}_{\mu\nu}(q) = \left[\frac{P^T_{\mu\nu}}{1+2G_V\Pi_T(q^2)} + \frac{L_{\mu\nu}}{1+2G_V\Pi_L(q^2)} + \frac{u_\mu u_\nu}{1+2G_V\Pi_{00}(q^2)}\right]\delta^{ab}.$$

The poles of these propagators give the gap equations:

$$1+2G_V\Pi_T(M^2_{\rho^\pm}) = 0,$$
$$1+2G_V\Pi_L(M^2_{\rho^\pm}) = 0,$$

which determine the masses of   mesons with corresponding spin projections.

In the rest frame of the ± meson, the matrix elements are given by:

$$\Pi_{11} = \Pi_{22} = a, \quad \Pi_{12} = -\Pi_{21} = ib, \quad \Pi_{33} = c.$$

The relationship between these matrix elements and Π_i is:

$$\Pi_T = \frac{a+b}{2}, \quad \Pi_L = c, \quad \Pi_{00} = \frac{a-b}{2}.$$

The matrix elements Π_{11} and Π_{12} in Eq. (14) were calculated in vacuum for the   meson in Ref. [56]. At finite temperature, these become:

$$\Pi_{11} = iN_cN_f\sum_{p,k}\int\frac{d^3k}{(2\pi)^3}\exp\left(-\frac{k_\perp^2+p_\perp^2}{|q_feB|}\right)\frac{1}{\omega_{u,k}\omega_{d,p}}\times\left[(I_1+I_1')\left(L^0_pL^0_k+L^0_{p-1}L^0_{k-1}\right)+I_2\left(L^0_pL^0_{k-1}+L^0_{p-1}L^0_k\right)\right]$$

$$\Pi_{12} = iN_cN_f\sum_{p,k}\int\frac{d^3k}{(2\pi)^3}\exp\left(-\frac{k_\perp^2+p_\perp^2}{|q_feB|}\right)\frac{1}{\omega_{u,k}\omega_{d,p}}\times\left[(I_1+I_1')\left(L^0_pL^0_k-L^0_{p-1}L^0_{k-1}\right)+I_2\left(L^0_pL^0_{k-1}-L^0_{p-1}L^0_k\right)\right]$$

where I_1, I_1', and I_2 are given by Matsubara summation, and $\omega_{u,k} = \sqrt{k_3^2+M^2+2k|q_ueB|}$, $\omega_{d,p} = \sqrt{p_3^2+M^2+2p|q_deB|}$.

III. The Effect of IMC on Charged   Condensation

In Ref. [67], we used MC for the chiral condensate to produce the constituent quark mass. In this section, we consider the effect of IMC on charged   meson condensation. We introduce three different mechanisms for IMC: fitting lattice data, using a running coupling constant $G_S(eB)$, and introducing a chiral chemical potential $\mu_5(eB) = 0.5\sqrt{eB}$.

For numerical calculations, we use the soft cutoff function [78]:

$$\Lambda(k) = \Lambda^{10}/(\Lambda^{10} + k^{10}) \quad \text{for } B = 0,$$
$$\Lambda_{eB}(k) = \Lambda^{10}/(\Lambda^{10} + (k_3^2 + 2|q_f eB|k)^5) \quad \text{for } B \neq 0.$$

We sum up to 30 Landau levels and the results are saturated. As in Ref. [74], we obtain the parameters $\Lambda = 582$ MeV, $G_S\Lambda^2 = 2.388$, and $G_V\Lambda^2 = 1.73$ by reproducing the pion decay constant $f_\pi = 95$ MeV, pion mass $m_\pi = 140$ MeV, and mass $M_\rho = 768$ MeV in vacuum. The vacuum quark mass is $M = 458$ MeV and the current quark mass is $m_0 = 5$ MeV.

A. Fitting the Lattice Data

In Refs. [14, 15], the lattice group confirmed magnetic catalysis at low temperature as predicted by most QCD model calculations. Moreover, they first observed inverse magnetic catalysis around the critical temperature T_c of chiral symmetry restoration. The dimensionless quantity $\Sigma_{u,d}(B,T) = [\langle\bar{\psi}\psi\rangle_{u,d}(B,T)/\langle\bar{\psi}\psi\rangle_{u,d}(0,0)] + 1$ was defined in Ref. [15]. We combine the lattice data for $(\Sigma_u + \Sigma_d)/2$ and $\Sigma_u - \Sigma_d$ with different T and eB from [15] and our parameters to reproduce the quark condensation. The magnetic field dependence of quark condensation is shown in Fig. 1 [Figure 1: see original paper]. In Fig. 2 [Figure 2: see original paper], we present the eB dependence of critical temperature T_c from lattice calculations, demonstrating that the IMC scenario around T_c has been reproduced.

Using the quark condensation from Fig. 1 and solving the gap equation Eq. (22), we obtain the critical magnetic field eB_c for polarized charged $\pm$ meson condensation. The temperature dependence of eB_c is shown in Fig. 3 [Figure 3: see original paper]. At zero temperature, eB_c for polarized charged $\pm$ meson condensation with IMC effect from fitting lattice data is slightly larger than its value without IMC, because the quark condensate at zero temperature from lattice calculations is slightly larger than in the NJL model [67]. As shown by the dashed line, if no IMC mechanism is included, the critical magnetic field eB_c for charged condensation increases monotonically with T. However, when IMC is considered (shown by dots), eB_c first decreases with temperature T, reaches a minimum around T_c, then increases with T. The IMC substantially affects polarized charged condensation around T_c. With IMC, the critical magnetic field eB_c for charged condensation drops to 0.13 GeV² at T_c, and eB_c in the temperature region $T < 1.3T_c$ is smaller than its value at $T = 0$.

B. Using the Running Coupling Constant $G_S(eB)$

In this approach, we introduce a running scalar coupling constant $G_S(eB)$ into the NJL model to calculate the constituent quark mass. Following Ref. [42], we fit $G_S(eB)$ to reproduce $T_c/T_c(eB = 0)$ obtained in Ref. [14].

We present the critical temperature used to fit $G_S(eB)$ in Fig. 4 [Figure 4: see original paper]. We assume:

$$\frac{G_S(\xi)}{G_S(0)} = \frac{1 + a\xi^2 + b\xi^3}{1 + c\xi^2 + d\xi^4},$$

where $G_S(0) = G_S$, $\xi = eB/\Lambda_{\mathrm{QCD}}^2$, and $\Lambda_{\mathrm{QCD}} = 300$ MeV. The fitted function $G_S(eB)$ is shown in Fig. 5 [Figure 5: see original paper] with parameters $a = 0.014056$, $b = 0.00532074$, $c = 0.0281766$, $d = 0.00161148$. The fitted function $G_S(eB)$ must decrease with magnetic field to produce IMC.

We then solve for the dynamical quark mass in the NJL model with the running coupling constant $G_S(eB)$. The one-loop effective potential is:

$$\Omega = \frac{(M - m_0)^2}{4G_S(eB)} - N_c \sum_f \frac{|q_f eB|}{2\pi} \sum_{p=0}^{\infty} \alpha_p \int \frac{dk_3}{2\pi} \left[E_q + \frac{1}{\beta} \ln(1 + e^{-\beta(E_q+\mu)}) + \frac{1}{\beta} \ln(1 + e^{-\beta(E_q-\mu)}) \right],$$

where $\beta = 1/T$ and $\alpha_p = 2 - \delta_{p0}$ is the spin degeneracy factor. Minimizing the effective potential determines the dynamical quark mass. Using Eq. (3), we obtain the quark condensation shown in Fig. 6 [Figure 6: see original paper] and the critical magnetic field $eB_c(T)$ in Fig. 7 [Figure 7: see original paper].

The critical magnetic field as a function of temperature $eB_c(T)$ shows similar behavior to the previous subsection. IMC substantially affects polarized charged condensation around T_c. Without IMC, eB_c increases monotonically with T (dashed line). With IMC, eB_c first decreases with T, reaches a minimum around T_c, then increases. With IMC, eB_c drops to 0.17 GeV² around T_c, and eB_c in the region $T < 1.3T_c$ is smaller than when no IMC is considered. Notably, eB_c with IMC below $1.2T_c$ is lower than in the original NJL model [67] without IMC, because the coupling constant decreases with magnetic field (Fig. 5).

C. Introducing the Chiral Chemical Potential $\mu_5(eB) = 0.5\sqrt{eB}$

In Refs. [52, 53], a chiral chemical potential $\mu_5(eB)$ is introduced to reproduce IMC. We use $\mu_5(eB) = 0.5\sqrt{eB}$ to produce IMC and the corresponding quark mass. The eB dependence of quark condensation is shown in Fig. 8 [Figure 8: see original paper] and the critical temperature as a function of eB is shown in Fig. 9 [Figure 9: see original paper], clearly reproducing IMC around T_c.

We then solve for the critical magnetic field eB_c for charged condensation using gap equation Eq. (22), with $eB_c(T)$ shown in Fig. 10 [Figure 10: see original paper]. The behavior is similar to the previous cases: IMC substantially affects polarized charged condensation around T_c. Without IMC, eB_c increases monotonically with T (dashed line). With IMC, eB_c first decreases with T, reaches a minimum of 0.16 GeV² around T_c, then increases. The critical magnetic field in the region $T < 1.3T_c$ is smaller than when no IMC is considered.

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IV. The Decay of the Strong Magnetic Field

In the previous section, we investigated the temperature dependence of eB_c by considering inverse magnetic catalysis for the chiral condensate. Using three different IMC mechanisms, we find that $eB_c(T)$ shows the same behavior around T_c: eB_c decreases with temperature, drops to its minimum around T_c, then increases with temperature. Notably, in the region $T < 1.3T_c$, the critical magnetic field eB_c is smaller than when no IMC is considered. Our calculation shows that charged condensation can exist in the temperature region of 1–1.5T_c with critical magnetic field $eB_c \sim 0.3$ GeV², suggesting that a high-temperature superconductor might be created in non-central heavy-ion collisions at LHC energies. We also show that growing electric conductivity in the early stage of non-central heavy-ion collisions substantially delays the decay of strong magnetic fields, which is helpful for forming high-temperature superconductors.

If a high-temperature superconductor is formed in the early stage of non-central heavy-ion collisions at LHC energies, it should have rather large electric conductivity. It has been shown in Refs. [79, 80] that finite electric conductivity substantially delays the decay of strong magnetic fields. Therefore, we investigate how high-temperature superconductors affect the decay of strong magnetic fields created in non-central heavy-ion collisions.

Following Ref. [79], we estimate the magnetic field in the center-of-mass frame. The y-component of the magnetic field is:

$$eB_y(\tau,\eta,x_\perp,\phi) = \alpha\sinh(Y_b)\int d^2x'\,\frac{(x-x')\sinh(Y_b-\eta)-\Delta\cos\phi'}{[(x-x')^2+\Delta^2]^{3/2}}\sigma_E(\tau,\eta,x'_\perp),$$

where σ_E is electric conductivity, $\alpha = e^2/(4\pi)$ is the electromagnetic coupling, $Y_b = \mathrm{arctanh}(\beta)$ is the rapidity, and $\Delta = \tau\sinh(Y_b-\eta) + x'\sinh(Y_b)\sqrt{\Delta}\cos(\phi-\phi')$.

Since high-temperature superconductors, if formed, are created after the non-central collision, we assume the electrical conductivity of the QGP σ_E is time-dependent:

$$\sigma_E(\tau) = \begin{cases}\sigma_{\max}(1-e^{-k\tau}) & \tau < \tau_{\text{form}}\\ \sigma_{\max} & \tau \geq \tau_{\text{form}}\end{cases}$$

where k is the growth rate and $\tau_{\text{form}} = \sigma_{\max}/k$ gives the formation time for the high-temperature superconductor.

With a simple assumption about proton distribution, the total magnetic field produced by all spectators in a collision with impact parameter b is:

$$eB_{y,s} = eB_y^-(\tau,\eta,x_\perp,\phi) + eB_y^+(\tau,\eta,x_\perp,\phi),$$

where $eB_y^{\pm}$ are defined by integration over crescent-shaped regions containing only positive- or negative-moving spectators.

At LHC energies, assuming $Y_b = Y_0 = 8$, $R = 7$ fm, $b = 7$ fm, $\phi = 0$, $\eta = 0$, and $Z = 82$, a strong magnetic field with magnitude $eB_y \sim 0.4$ GeV² is created. Numerical results for the decay of $eB_y(\tau)$ with different $\sigma_{\max}$ are shown in Fig. 11 [Figure 11: see original paper].

With the time-dependent electric conductivity assumption, we find that a conducting medium with σ_E effectively delays magnetic field decay. At later stages, larger σ_E leads to slower decay. Notably, if one chooses constant σ_E starting from $\tau = 0$ as in [79], even small conductivity makes the magnetic field at $\tau = 0$ drop to a small value. Our results suggest that growing electric conductivity starting from zero at $\tau = 0$ substantially delays magnetic field decay, which is helpful for high-temperature superconductor formation.

V. Conclusion

In this paper, we have investigated charged condensation in an external magnetic field at finite temperature in the NJL model. When the mass decreases with magnetic field and becomes zero, it indicates charged condensation exists. The magnetic field at which charged becomes massless is called the critical magnetic field eB_c.

In the NJL model, mesons are constructed as an infinite sum of quark-loop chains using the random phase approximation, and we calculate to leading order in the 1/Nc expansion for the meson.

We studied the temperature dependence of eB_c by considering IMC for the quark chiral condensate. We used three different approaches to reproduce IMC: fitting lattice data, using a running scalar coupling constant $G_S(eB)$, and considering chirality imbalance $\mu_5(eB) = 0.5\sqrt{eB}$. In all three cases, IMC substantially affects polarized charged condensation around the critical temperature T_c of the chiral phase transition. Without IMC, eB_c for charged condensation increases monotonically with temperature. However, with IMC, eB_c first decreases with temperature, reaches a minimum around T_c, then increases. The critical magnetic field with IMC in the region $T < 1.2T_c$ is smaller than without IMC. Our calculation shows that charged condensation can exist in the temperature region of 1–1.5T_c with critical magnetic field $eB_c \sim 0.3$ GeV², which can be regarded as a high-temperature superconductor that might form in non-central heavy-ion collisions at LHC energies. We also show that growing electric conductivity in the early stage of non-central heavy-ion collisions substantially delays strong magnetic field decay, which is helpful for high-temperature superconductor formation.

It is worth mentioning that our research was done in a background without condensation. It would be interesting to investigate the conductivity of charged condensation by introducing the charged polarized $\pm$ condensate in the Lagrangian in future work.

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References

[1] J. O. Andersen, W. R. Naylor and A. Tranberg, arXiv:1411.7176 [hep-ph]. [2] V. A. Miransky and I. A. Shovkovy, Phys. Rept. 576, 1 (2015) doi:10.1016/j.physrep.2015.02.003 [arXiv:1503.00732 [hep-ph]]. [3] X. G. Huang, arXiv:1509.04073 [nucl-th]. [4] T. A. Mihara et al., Nature (London) 346, 250 (1990); R. C. Duncan and C. Thompson Astrophys. J. 392 L9 (1992); Lyne A and Graham-Smith F Pulsar Astronomy (Cambridge: Cambridge University Press) (2005). [5] V. Skokov, A. Y. Illarionov and V. Toneev, Int. J. Mod. Phys. A 24, 5925 (2009) [arXiv:0907.1396 [nucl-th]]. [6] W. T. Deng and X. G. Huang, Phys. Rev. C 85, 044907 (2012) [arXiv:1201.5108 [nucl-th]]. [7] D. Kharzeev and A. Zhitnitsky, Nucl. Phys. A 797, 67 (2007) [arXiv:0706.1026 [hep-ph]]. [8] D. E. Kharzeev, L. D. McLerran and H. J. Warringa, Nucl. Phys. A 803, 227 (2008) [arXiv:0711.0950 [hep-ph]]. [9] K. Fukushima, D. E. Kharzeev and H. J. Warringa, Phys. Rev. D 78, 074033 (2008) [arXiv:0808.3382 [hep-ph]]. [10] D. E. Kharzeev and D. T. Son, Phys. Rev. Lett. 106, 062301 (2011) [arXiv:1010.0038 [hep-ph]]. [11] S. P. Klevansky and R. H. Lemmer, Phys. Rev. D 39, 3478 (1989). [12] K. G. Klimenko, Theor. Math. Phys. 89, 1161 (1992) [Teor. Mat. Fiz. 89, 211 (1991)]. [13] V. P. Gusynin, V. A. Miransky and I. A. Shovkovy, Nucl. Phys. B 462, 249 (1996); Nucl. Phys. B 563, 361 (1999). [14] G. S. Bali, F. Bruckmann, G. Endrodi, Z. Fodor, S. D. Katz, S. Krieg, A. Schafer and K. K. Szabo, JHEP 1202, 044 (2012) [arXiv:1111.4956 [hep-lat]]. [15] G. S. Bali, F. Bruckmann, G. Endrodi, Z. Fodor, S. D. Katz and A. Schafer, Phys. Rev. D 86, 071502 (2012) [arXiv:1206.4205 [hep-lat]]. [16] G. S. Bali, F. Bruckmann, G. Endrodi, F. Gruber and A. Schaefer, JHEP 1304, 130 (2013) [arXiv:1303.1328 [hep-lat]]. [17] M. N. Chernodub, Phys. Rev. D 82, 085011 (2010) [arXiv:1008.1055 [hep-ph]]. [18] M. N. Chernodub, Phys. Rev. Lett. 106, 142003 (2011) [arXiv:1101.0117 [hep-ph]]. [19] I. A. Shushpanov and A. V. Smilga, Phys. Lett. B 402, 351 (1997) [hep-ph/9703201]. [20] N. O. Agasian and I. A. Shushpanov, Phys. Lett. B 472, 143 (2000) [hep-ph/9911254]. [21] J. Alexandre, K. Farakos and G. Koutsoumbas, Phys. Rev. D 63, 065015 (2001) [hep-th/0010211]. [22] N. O. Agasian, Phys. Atom. Nucl. 64, 554 (2001) [Yad. Fiz. 64, 608 (2001)] [hep-ph/0112341]. [23] T. D. Cohen, D. A. McGady and E. S. Werbos, Phys. Rev. C 76, 055201 (2007) [arXiv:0706.3208 [hep-ph]]. [24]

R. Gatto and M. Ruggieri, Phys. Rev. D 82, 054027 (2010) [arXiv:1007.0790 [hep-ph]]. [25] R. Gatto and M. Ruggieri, Phys. Rev. D 83, 034016 (2011) [arXiv:1012.1291 [hep-ph]]. [26] A. J. Mizher, M. N. Chernodub and E. S. Fraga, Phys. Rev. D 82, 105016 (2010) [arXiv:1004.2712 [hep-ph]]. [27] K. Kashiwa, Phys. Rev. D 83, 117901 (2011) [arXiv:1104.5167 [hep-ph]]. [28] S. S. Avancini, D. P. Menezes, M. B. Pinto and C. Providencia, Phys. Rev. D 85, 091901 (2012) [arXiv:1202.5641 [hep-ph]]. [29] J. O. Andersen, Phys. Rev. D 86, 025020 (2012) [arXiv:1202.2051 [hep-ph]]. [30] D. D. Scherer and H. Gies, Phys. Rev. B 85, 195417 (2012) [arXiv:1201.3746 [cond-mat.str-el]]. [31] P. V. Buividovich, M. N. Chernodub, E. V. Luschevskaya and M. I. Polikarpov, Phys. Lett. B 682, 484 (2010) [arXiv:0812.1740 [hep-lat]]. [32] V. V. Braguta, P. V. Buividovich, T. Kalaydzhyan, S. V. Kuznetsov and M. I. Polikarpov, Phys. Atom. Nucl. 75, 488 (2012) [arXiv:1011.3795 [hep-lat]]. [33] M. D' Elia, S. Mukherjee and F. Sanfilippo, Phys. Rev. D 82, 051501 (2010) [arXiv:1005.5365 [hep-lat]]. [34] M. D' Elia and F. Negro, Phys. Rev. D 83, 114028 (2011) [arXiv:1103.2080 [hep-lat]]. [35] E.-M. Ilgenfritz, M. Kalinowski, M. Muller-Preussker, B. Petersson and A. Schreiber, Phys. Rev. D 85, 114504 (2012) [arXiv:1203.3360 [hep-lat]]. [36] K. Fukushima and Y. Hidaka, Phys. Rev. Lett. 110, no. 3, 031601 (2013) [arXiv:1209.1319 [hep-ph]]. [37] K. Kamikado and T. Kanazawa, JHEP 1403, 009 (2014) [arXiv:1312.3124 [hep-ph]]. [38] S. Mao, arXiv:1602.06503 [hep-ph]. [39] T. Kojo and N. Su, Phys. Lett. B 720, 192 (2013) [arXiv:1211.7318 [hep-ph]]. [40] F. Bruckmann, G. Endrodi and T. G. Kovacs, JHEP 1304, 112 (2013) [arXiv:1303.3972 [hep-lat]]. [41] N. Mueller and J. M. Pawlowski, Phys. Rev. D 91, no. 11, 116010 (2015) doi:10.1103/PhysRevD.91.116010 [arXiv:1502.08011 [hep-ph]]. [42] M. Ferreira, P. Costa, O. Lourenço, T. Frederico and C. Providência, Phys. Rev. D 89, no. 11, 116011 (2014) [arXiv:1404.5577 [hep-ph]]. [43] A. A. Belavin, A. M. Polyakov, A. S. Schwartz and Y. S. Tyupkin, Phys. Lett. B 59, 85 (1975). [44] G. ' t Hooft, Phys. Rev. Lett. 37, 8 (1976). [45] G. ' t Hooft, Phys. Rev. D 14, 3432 (1976) [Phys. Rev. D 18, 2199 (1978)]. [46] N. S. Manton, Phys. Rev. D 28, 2019 (1983). [47] F. R. Klinkhamer and N. S. Manton, Phys. Rev. D 30, 2212 (1984). [48] V. A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B 155, 36 (1985). [49] P. B. Arnold and L. D. McLerran, Phys. Rev. D 36, 581 (1987). [50] S. Y. Khlebnikov and M. E. Shaposhnikov, Nucl. Phys. B 308, 885 (1988). [51] P. B. Arnold and L. D. McLerran, Phys. Rev. D 37, 1020 (1988). [52] J. Chao, P. Chu and M. Huang, Phys. Rev. D 88, 054009 (2013) [arXiv:1305.1100 [hep-ph]]. [53] L. Yu, H. Liu and M. Huang, Phys. Rev. D 90, no. 7, 074009 (2014) [arXiv:1404.6969 [hep-ph]]. [54] L. Yu, H. Liu and M. Huang, arXiv:1511.03073 [hep-ph]. [55] M. Frasca, JHEP 1311, 099 (2013) [arXiv:1309.3966 [hep-ph]]. [56] H. Liu, L. Yu and M. Huang, Phys. Rev. D 91, no. 1, 014017 (2015) [arXiv:1408.1318 [hep-ph]]. [57] N. Callebaut, D. Dudal and H. Verschelde, PoS FACESQCD, 046 (2010) [arXiv:1102.3103 [hep-ph]]. [58] M. Ammon, J. Erdmenger, P. Kerner and M. Strydom, Phys. Lett. B 706, 94 (2011) [arXiv:1106.4551 [hep-th]]. [59] M. A. Andreichikov, B. O. Kerbikov, V. D. Orlovsky and Y. A. Simonov, Phys. Rev. D 87, no. 9, 094029 (2013) [arXiv:1304.2533 [hep-ph]]. [60] Kunlun Wang, PhD thesis from Peking University. [61] M. Kawaguchi and S. Matsuzaki, arXiv:1511.06990 [hep-ph]. [62] O. Larina, E. Luschevskaya, O. Kochetkov and O. V. Teryaev, PoS LATTICE 2014, 120 (2014) [arXiv:1411.0730 [hep-lat]]; JETP Lett. 101, no. 10, 674 (2015). [63] Y. Hidaka and A. Yamamoto, Phys. Rev. D 87, no. 9, 094502 (2013) [arXiv:1209.0007 [hep-ph]]. [64] M. N. Chernodub, Phys. Rev. D 86, 107703 (2012) [arXiv:1209.3587 [hep-ph]]. [65] C. Li and Q. Wang, Phys. Lett. B 721, 141 (2013) [arXiv:1301.7009 [hep-th]]. [66] M. N. Chernodub, Phys. Rev. D 89, no. 1, 018501 (2014) [arXiv:1309.4071 [hep-ph]]. [67] H. Liu, L. Yu and M. Huang, arXiv:1507.05809 [hep-ph]. [68] Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122, 345 (1961). [69] Y. Nambu and G. Jona-Lasinio, Phys. Rev. 124, 246 (1961). [70] S. Klimt, M. F. M. Lutz, U. Vogl and W. Weise, Nucl. Phys. A 516, 429 (1990). [71] U. Vogl and W. Weise, Prog. Part. Nucl. Phys. 27, 195 (1991). [72] S. P. Klevansky, Rev. Mod. Phys. 64, 649 (1992). [73] T. Hatsuda and T. Kunihiro, Phys. Rept. 247, 221 (1994) [hep-ph/9401310]. [74] Y. B. He, J. Hufner, S. P. Klevansky and P. Rehberg, Nucl. Phys. A 630, 719 (1998) [nucl-th/9712051]. [75] A. Chodos, K. Everding and D. A. Owen, Phys. Rev. D 42 (1990) 2881. doi:10.1103/PhysRevD.42.2881 [76] A. K. Das and M. B. Hott, Mod. Phys. Lett. A 9 (1994) 3383 [hep-ph/9406426]. [77] P. Rehberg and S. P. Klevansky, Annals Phys. 252, 422 (1996) doi:10.1006/aphy.1996.0140 [hep-ph/9510221]. [78] M. Frasca and M. Ruggieri, Phys. Rev. D 83, 094024 (2011) [arXiv:1103.1194 [hep-ph]]. [79] U. Gursoy, D. Kharzeev and K. Rajagopal, Phys. Rev. C 89, no. 5, 054905 (2014) doi:10.1103/PhysRevC.89.054905 [arXiv:1401.3805 [hep-ph]]. [80] H. Li, X. l. Sheng and Q. Wang, arXiv:1602.02223 [nucl-th].

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