

Characterization of Quantum Phase Transition using Holographic Entanglement Entropy (Post-print)

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Abstract

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Full Text

Preamble

Characterization of Quantum Phase Transition using Holographic Entanglement Entropy

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Abstract

Entanglement exhibits extremal or singular behavior near quantum critical points (QCPs) in many condensed matter models. These intriguing phenomena, however, still call for a widely accepted understanding. In this letter we study this issue within a holographic framework. We investigate the connection between holographic entanglement entropy (HEE) and quantum phase transition (QPT) in a lattice-deformed Einstein-Maxwell-Dilaton theory. Novel backgrounds exhibiting metal-insulator transitions (MIT) have been constructed in which both metallic and insulating phases have vanishing entropy density in the zero temperature limit. We find that the first-order derivative of HEE with respect to lattice parameters exhibits extremal behavior near QCPs. We propose that it would be a universal feature that HEE or its derivatives with respect to system parameters can characterize QPT in a generic holographic system. Our work opens a window for understanding the relation between entanglement and QPT from a holographic perspective.

Introduction

The relation between entanglement and quantum phase transition (QPT) has been extensively investigated in condensed matter physics (a recent review on this progress can be found for instance in [?]). In particular, it has been proposed that entanglement may be used to diagnose QPT because it exhibits extremal or singular behavior near the quantum critical point (QCP) in many models [?, ?, ?, ?, ?, ?, ?]. Especially, as an important quantity measuring non-local correlation, the entanglement entropy (EE) or its first derivative characterizes QPT with local extremes near QCPs [?, ?]. Theoretically, any entanglement measure can be written as a functional of the first derivatives of the ground state energy [?, ?, ?], which probably provides more insight into this issue since QPT is associated with the drastic modification of the ground state [?]. Nevertheless, a widely accepted understanding of how and why entanglement characterizes QPTs is still missing.

Usually QPT involves strong correlation physics and is hard to analyze; in quantum field theory EE is also difficult to compute. Gauge/gravity duality [?, ?] as a novel approach has provided powerful tools for both understanding strongly

correlated systems and computing EE, which stimulates us to study the deep relation between entanglement and QPT from a holographic perspective. Recently, the metal-insulator transition (MIT), as a prominent example of QPT, has been implemented in holographic approaches [?, ?, ?, ?, ?, ?, ?, ?]. We expect the holographic description of EE [?, ?], namely the holographic entanglement entropy (HEE), will also furnish a dramatic signature of QPT in holographic models. Motivated by this, a connection between HEE and QCPs of the MIT has first been revealed in [?], which is reminiscent of the relation between EE and QCP found in condensed matter theory [?]. Nonetheless, in [?] the ground state entropy density is vanishing for the insulating phase, while non-vanishing for the metallic phase since its near-horizon geometry is $\text{AdS}_2 \times \mathbb{R}^2$. The artifact of AdS_2 is a well-known open problem in AdS/CMT duality since any ordinary condensed matter system in reality has vanishing entropy density in ground states; otherwise the system would suffer from instability.

Hence in [?] a crucial issue is whether the relation between HEE and QPT originates from the artifact of AdS_2 , and whether this finally prevents the relation from linking to realistic condensed matter systems. To provide a definite answer to this issue, one direct way is to construct holographic models with MIT in the absence of AdS_2 . This is a challenging task since in all previous holographic realizations of MIT, the instability of AdS_2 characterized by its BF bound plays an essential role in generating a transition to new IR fixed points. Therefore, to implement MIT from a metallic phase with vanishing entropy density to an insulating phase, a novel mechanism for inducing instability is needed rather than violating the BF bound of AdS_2 . In this letter we construct a novel holographic model which exhibits an MIT, and both the metallic phase and the insulating phase have vanishing ground state entropy density. Remarkably, we find that the location of QCPs can be captured by the local maximum of the first-order derivative of HEE with respect to system parameters. Combined with [?], the phenomena in condensed matter theory that EE or its first-order derivatives [?, ?] characterize QPT are reproduced in a holographic framework.

II. Holographic Setup

We start with an action in the framework of Einstein-Maxwell-Dilaton theory, which contains two $U(1)$ gauge fields A and B , one dilaton field Φ_1 , and an additional complex scalar field Φ . The Lagrangian reads,

$$\mathcal{L} = R - \partial_a \Phi^* \partial^a \Phi - \partial_a \Phi_1 \partial^a \Phi_1 + \mathcal{L}_2 |\Phi|^2 - \mathcal{L}_2 \cosh(\Phi_1),$$

where $F = dA$, $G = dB$ are the curvatures of the two gauge fields and the complex field Φ simulates the Q-lattice structure [?, ?]. In the absence of gauge field B and the Q-lattice, this theory allows a well-known black brane solution with vanishing ground state entropy density, which is now called the Gubser-Rocha solution [?]. In the current work, the motivation for introducing the Q-lattice

structure is twofold. One is to break the translational symmetry of the background so as to obtain a finite DC conductivity, which has been extensively investigated in [?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?]. More importantly, we expect to obtain an MIT which, from the viewpoint of renormalization group flow, requires that the Q-lattice deformation must be relevant so that the gravitational system can run from the original IR fixed point associated with a metallic phase to a new IR fixed point associated with an insulating phase. It can be proved that Q-lattices cannot induce such phase transitions over the Gubser-Rocha background in the zero temperature limit if only a single gauge field is present¹. Therefore, we introduce a second gauge field B and treat it as the Maxwell field. We expect MIT associated with this gauge field may take place for ground states in this new framework, and we will justify our expectation in subsequent numerical analysis. Previously this strategy has been adopted in the construction of holographic Mott-like insulators [?].

We consider the following ansatz for a black brane solution:

$$ds^2 = \frac{1-z}{(1+Qz)^{1/2}} \left[-f(z)U(z)dt^2 + \frac{dz^2}{f(z)} + V_1(z)dx^2 + V_2(z)dy^2 \right],$$

$$A_t(z) = \mu a(z), \quad B_t(z) = -\mu b(z),$$

$$\Phi(z) = e^{ikx} z^{3-\Delta} \phi, \quad \Phi_1(z) = \ln(1 + Qz\phi_1),$$

where $f(z) = (1-z)p(z)/g(z)$ with $p(z) = 1 + (1+3Q)z + (1+3Q(1+Q))z^2$ and $g(z) = (1+Qz)^{3/2}$, while $\mu = L\sqrt{3Q(1+Q)}$. All the functions ($U, V_1, V_2, a, b, \phi, \phi_1$) depend only on the radial direction z . The Hawking temperature of the black brane is given by $\hat{T} = (1+Q)U(1)/(4\pi)$. When we set ($U = 1, V_1 = V_2 = g(z), a = 1, b = 0, \phi = 0, \phi_1 = 1$), the present ansatz reduces to the Gubser-Rocha solution presented in [?, ?]. In particular, the Gubser-Rocha solution in the zero temperature limit can be obtained as $Q \rightarrow \infty$, with dimensionless temperature $T \equiv \hat{T}/\mu = 3/(4\pi Q)$ and dimensionless entropy density $s \equiv \hat{s}/\mu^2 = 1/(3Q)$, thus the entropy density vanishes linearly with temperature.

Next we numerically solve the background equations of motion based on the ansatz in Eq.(2). The requirement that the UV region ($z = 0$) be asymptotically AdS₄ leads to $V_1(0) = V_2(0) = U(0) = 1$ and the scaling dimension of Φ is $\Delta = 3/2 + \sqrt{9/4 + m^2}$. For definiteness we fix $L = 1, m^2 = -2$, thus $\Delta = 2$. In addition, we set the boundary conditions for the remaining functions as $a(0) = 1, b(0) \equiv b_0, \phi(0) = \hat{\lambda}$ and $\phi_1(0) = 1$. Within this setting the chemical potential of the dual field corresponding to A is μ and we will use it as the unit of scaling throughout this letter. As a result, the family of background solutions based on ansatz (2) can be parameterized by four dimensionless quantities,

¹We thank Zhuoyu Xian for discussion on this issue.

namely $\{\hat{T}/\mu, \hat{\lambda}/\mu, \hat{k}/\mu, b_0\}$. For simplicity we abbreviate these dimensionless quantities as $\{T, \lambda, k, b_0\}$. The parameters (λ, k) related to the Q-lattice Φ can be interpreted as the lattice strength and the lattice wave vector.

Without loss of generality we will concentrate on the family of black brane solutions with fixed $b_0 = 0.1$. We have also worked with other values of b_0 , and qualitatively similar phenomena can be observed. It is worthwhile to point out that for large b_0 insulating phases exist only for large values of λ at which the numerical work becomes very costly.

III. MIT and Ground State Entropy Density

In this section we investigate the conductivity behavior of the current dual to gauge field B and the entropy density of the dual system in the zero temperature limit.

At zero temperature an insulator has vanishing direct-current conductivity σ_{DC} , while a metal has non-vanishing σ_{DC} . At finite temperature T , we practically distinguish the metallic phase and insulating phase by $\sigma_{DC}(T) > 0$, therefore the critical line is identified as $\sigma'_{DC}(T) = 0$. In this model the σ_{DC} can be computed with the data on the horizon:

$$\sigma_{DC} = \sqrt{\frac{V_2}{V_1} \frac{2k^2(1+Q)\phi}{k^2(1+Q)\phi + 2b^2}} \Big|_{z=1}.$$

First, we demonstrate that an MIT takes place at low temperature as we change the parameters (λ, k) of the black brane background. We plot the phase diagram over the phase space (λ, k) at $T = 0.02$ in Fig. 1 (the left plot). In general, it is noticed that the insulating phase is usually formed with large λ but small k . Although our phase structure is obtained at a nonzero temperature, we show that an MIT could occur in the zero temperature limit. As illustrated in the right plot of Fig. 1 [Figure 1: see original paper] (blue curve), k_c changes with temperature in an almost linear manner, implying that a non-zero critical point k_{c0} may be reached at $T = 0$.

Now we address the entropy density of the black brane in the zero temperature limit. We intend to numerically test the scaling behavior of the entropy density with temperature, and then justify that our model exhibits vanishing ground state entropy density indeed. Fig. 2 [Figure 2: see original paper] is a plot of Ts'/s versus T for both metallic and insulating phases, where prime denotes the derivative with respect to T . It is clearly seen from this figure that down to an extremely low temperature $T \sim 10^{-2}$, Ts'/s converges to a nonzero constant, implying that for both phases the entropy density has a power law relation with temperature $s \sim T^\alpha$. Therefore, the entropy density is expected to be vanishing in the zero temperature limit indeed. Moreover, we notice that the power coefficient α is always very close to one for the metallic phase, while

smaller than one for the insulating phase, strongly implying that the RG flow runs to different IR fixed points for a metallic phase and an insulating phase at zero temperature.

IV. HEE Close to QCPs

In this section, we study the relation between HEE and QPTs. In a bipartite system composed of subsystems A and B , the EE of A is defined as $S_A \equiv -\text{Tr}_A(\rho_A \ln \rho_A)$, where $\rho_A \equiv \text{Tr}_B(\rho_{\text{total}})$. In AdS/CFT correspondence, the EE for a region A of the boundary system is obtained from the gravity side as the area of the minimal surface γ_A in the bulk which ends at ∂A [?, ?], i.e.,

$$S_A = \frac{\text{Area}(\gamma_A)}{4G_N},$$

where G_N is the bulk Newton constant. We consider the HEE for a strip geometry on the boundary with infinite length L_y in the y -direction, while with finite width \hat{l} in the x -direction. Since the metric components of the bulk are functions of z only, the minimal surface bounded by the strip can be specified by the location z_* of the bottom of the minimal surface in the z -direction. Therefore, the HEE $\hat{S} \equiv S_A/(L_y/4G_N)$, where we will omit the common prefactor and treat \hat{S} as the HEE, can be written as

$$\hat{S} = \int_0^{z_*} dz \chi(z) \sqrt{V_2(z) \left[V_1(z) - \frac{z^4 V_1(z_*) V_2(z_*)}{z_*^4 V_2(z)} \right]},$$

$$\hat{l} = 2 \int_0^{z_*} dz \frac{z^2 \sqrt{V_1(z_*) V_2(z_*)}}{z_*^2 \sqrt{V_2(z) \left[V_1(z) - \frac{z^4 V_1(z_*) V_2(z_*)}{z_*^4 V_2(z)} \right]}},$$

where $\chi \equiv [(1-z)p(z)U(z)\xi/g(z)]^{-1/2}$ with $\xi \equiv z^4$ and $l = \hat{l}\mu$. Note that we have subtracted out the vacuum contribution to HEE by adding a counter term $-1/z^2$ into the integrand of S .

We study HEE itself as well as the derivatives of HEE with respect to parameters (λ, k) . Remarkably, we find in this model it is the first-order derivative of HEE, $\partial_\lambda S$ and $\partial_k S$, rather than HEE itself that exhibits extremal behavior close to critical points. We demonstrate this phenomenon by a plot of $\partial_\lambda S$ versus k in Fig. 3. From this figure we notice that the location of peaks is independent of the width l when it is relatively large, from the region $Tl \ll 1$ to $Tl \rightarrow \infty$.

Furthermore, we demonstrate that the above phenomenon is valid in the zero temperature limit. In the right plot of Fig. 1 we show the locations of the peaks of $\partial_\lambda S$ for different temperatures in the phase diagram (k, T) . We find that the peaks always coincide with the critical line when temperature goes down,

indicating that the extremal behavior of the derivative of HEE at critical points is independent of T in the low temperature region. This result allows us to infer safely that this extremal behavior close to QCPs can be observed in the zero temperature limit.

In parallel, a similar phenomenon can be observed when we change the parameter λ but fix k . As a result, we show the contour plot of $\partial_\lambda S$ over the phase space in Fig. 4 [Figure 4: see original paper] with $T = 0.02$, $l = 6.319$. It can be seen that the ridge of the contour plot matches very well with the critical line which characterizes the occurrence of MIT. In addition, we remark that a similar contour plot can be drawn for $\partial_k S$, and the only difference is that the ridge is replaced by a valley.

In summary, we conclude that in this holographic model the first-order derivative of HEE with respect to system parameters characterizes QPT. Previously, the phenomenon that local extremes of derivatives of EE diagnose QPT has been observed in condensed matter theory literature [?]. Our holographic model reproduces this significant feature of strongly correlated systems in condensed matter physics.

Finally, we give some remarks on our results in the limit $l \rightarrow \infty$ where HEE is dominated by thermal entropy $S(l) \sim s \cdot l$. We find that the thermal entropy density s does capture the locations of critical points as well as S with finite l at low temperature in our numerical analysis. In the zero temperature limit, however, s vanishes and therefore is not essential in linking HEE and QPT. As an attempt, it is instructive to define a quantity $f \equiv S(l) - s \cdot l$ for large l at finite temperature, which roughly speaking subtracts out the contribution of pure thermal entropy $s \cdot l$ from the total HEE $S(l)$. As a matter of fact, it is shown in [?, ?] that in the limit $lT \gg 1$ the $S(l)$ could be separated as thermal contribution $s \cdot l$ and another finite contribution S_{finite} . The quantity f defined above will become S_{finite} in the large l limit. Numerically, we find that f becomes independent of l for large l , and $\partial_\lambda f$ also characterizes the location of QPTs indeed. Moreover, the ridge of the contour plot of $\partial_\lambda f$ becomes steeper with the decrease of temperature, and potentially diverges in the zero temperature limit. Therefore, we expect that this quantity would be helpful for us to investigate the scaling behavior of HEE near QCPs in the zero temperature limit but with $lT \gg 1$. We leave this for future investigation.

V. Discussion

In this letter we have constructed a novel holographic model which contains black brane solutions with vanishing entropy density in the zero temperature limit, and have demonstrated that a novel MIT can take place in the absence of AdS₂ IR geometry. A significant improvement to our previous work [?] is that an interesting connection between HEE and QPT still exists in this framework, which means that HEE characterizing QPT is not an artifact of AdS₂. Therefore, our present work has paved a bridge linking AdS/CMT duality to realistic

condensed matter systems. More importantly, it is the first time to find that the derivatives of HEE with respect to parameters diagnose QPT, which not only coincides with the phenomenon observed in condensed matter physics, but also enriches our understanding of the nature of HEE itself. Given the observations in [?] and this letter, we conjecture that it would be a universal feature that HEE or the derivatives of HEE could characterize QPT in a generic holographic framework. This conjecture reflects the fact that in condensed matter theory the higher-order derivatives of entanglement measures diagnose QPT [?].

Our current work has opened a window for understanding the relation between HEE and QPT from a holographic perspective. It is intriguing to further investigate the role of HEE in QPT with the following proposals. First, it is reasonable to expect that not only HEE or its first-order derivative, but also higher-order derivatives of HEE, can characterize QPTs in holographic models. We have gained further evidence to support this conjecture and the progress will be reported elsewhere [?]. Second, it is crucial to understand what determines the order of derivative of HEE in diagnosing QPT, which is also an open question for ordinary EE in condensed matter physics. Further investigation on this issue should be valuable for disclosing the nature of both HEE and QPT.

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