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Abstract

Careful observation of the experimental spectra of heavy-light mesons tells us that heavy-light mesons with the same angular momentum L are almost degenerate. The estimate is given how much this degeneracy is broken in our relativistic potential model, and it is analytically shown that expectation values of a commutator between the lowest order Hamiltonian and L 2 are of the order of 1/mQ with a heavy quark mass mQ. It turns out that nonrelativistic approximation of heavy quark system has a rotational symmetry and hence degeneracy among states with the same L. This feature can be tested by measuring higher orbitally and radially excited heavy-light meson spectra for D/Ds/B/Bs in LHCb and forthcoming BelleII.

Full Text

Preamble

Approximate Degeneracy of Heavy-Light Mesons with the Same L

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Careful observation of the experimental spectra of heavy-light mesons reveals that heavy-light mesons with the same orbital angular momentum L are almost

degenerate. We estimate the degree to which this degeneracy is broken in our relativistic potential model and analytically show that expectation values of the commutator between the lowest-order Hamiltonian and $\tilde{L}^2$ are of order $1/m_Q$, where m_Q is the heavy quark mass. It turns out that the nonrelativistic approximation of the heavy quark system possesses rotational symmetry and hence yields degeneracy among states with the same L. This feature can be tested by measuring higher orbitally and radially excited heavy-light meson spectra for D/Ds/B/Bs at LHCb and the forthcoming Belle II.

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Introduction

Ever since the discovery of X(3872), Ds0(2317), and Ds1(2460) in 2003, many more XYZ particles as well as higher radially and orbitally excited states have been found at Belle, BESII, BESIII, BaBar, and LHCb [?]. There are several problems associated with these particles. First, most of them appear near thresholds, suggesting possible kinematical explanations. Second, some of them should be multiquark states because they cannot be explained as higher excited states of ordinary quarkonium due to the existence of charged states.

When focusing on higher orbital excitations of the heavy-light system, we observe a spectroscopic tendency that has not yet been explained by heavy quark symmetry. The problem is described as follows: even though the orbital angular momentum L is not a good quantum number in the heavy quark system, the masses of states with the same L appear to be close to each other even for heavy-light systems. To explain this approximate degeneracy among heavy-light mesons with the same L observed in experiments, we must demonstrate—at least analytically or numerically—how small the matrix elements of the resultant difference operator are.

One powerful quark model is the relativized Godfrey-Isgur (GI) model [?, ?], in which the lowest-order Hamiltonian commutes with $\tilde{L}$ even in their relativistic formulation. Hence, it is not surprising within their framework that masses with the same L are similar. However, when calculating the commutator of the lowest-order Hamiltonian with $\tilde{L}$ in our relativistic potential model [?, ?], we obtain a nonvanishing result. The difference between the GI and our models lies in our treatment of the light quark as a four-component Dirac spinor, which causes a nonvanishing commutator as shown below, whereas the GI model treats it as a two-component spinor.

In recent decades, the heavy-light meson families have become a rich structure as seen in the PDG [?]. Even though it does not explicitly incorporate heavy quark symmetry, the GI model [?, ?] has been successful in reproducing and predicting low-lying hadrons and heavy-light mesons except for DsJ. This model respects angular momentum conservation at lowest order, so states with the same angular momentum L are degenerate without spin-orbit interactions.

Let us examine numerical results from various models for D mesons (containing a heavy charm quark) and compare them with each other and with experimental data in Table I. The model in the second column [?, ?, ?] is the GI model itself, while the model in the seventh column [?, ?] is a nonrelativistic potential model including one-loop computation of the heavy-quark interaction. Those in the third column [?, ?] use the Bethe-Salpeter formulation expanded in $1/m_Q$, while our model in the sixth column [?] employs the Foldy-Wouthuysen-Tani transformation to obtain the equation of motion for a $Q\bar{q}$ bound system and is essentially the same formulation as that of Ref. [?]. Hence, the arguments given in Sect. II can also be derived from Refs. [?, ?]. Finally, Ref. [?] uses a quasipotential approach whose details are given in their paper. Similar tables for Ds/B/Bs mesons can be easily obtained and show tendencies similar to Table I. Because we wish to extract and demonstrate the essence of our claim, we omit them in this article.

It is not surprising that states with the same L in the GI model have similar mass values because the model respects L. However, it is remarkable that even models respecting heavy quark symmetry produce results similar to the GI model, as seen in Table I. States in Table I are assigned definite values of ${}^{2S+1}L_J$ in the first column. Even though our relativistic wave function is not an eigenstate of L in our formulation [?], we can still assign ${}^{2S+1}L_J$ to each state in the nonrelativistic limit.

In the last two columns of Table I, we provide average values of experimental data within a spin doublet of the heavy-quark system and gap values between spin doublets. For instance, average values are 1938 MeV for the spin multiplet ($J^P = 0^-, 1^-$), 2394 MeV for the multiplet $(0^+, 1^+)$, 2443 MeV for the multiplet $(1^+, 2^+)$, 2763 MeV for the multiplet $(1^-, 2^-)$, etc. Gap values are given by differences of these values: 456 MeV between multiplets $(0^-, 1^-)$ with $L = 0$ and $(0^+, 1^+)$ with $L = 1$, 49 MeV between $(0^+, 1^+)$ and $(1^+, 2^+)$ with the same $L = 1$, 320 MeV between $(1^+, 2^+)$ with $L = 1$ and $(1^-, 2^-)$ with $L = 2$, etc. We can see that mass differences within a spin doublet and between doublets with the same L are very small compared with mass gaps between different multiplets with different L, which is nearly equal to the QCD scale $\Lambda_{\rm QCD} \sim 300$ MeV [?] for $n_f = 4$.

II. Analytical Analysis

We begin with the quantum numbers $\ell = \{k, j, m\}$, where j is the total angular momentum and m its z-component. The quantum number k is related to the angular momentum of the light quark j_ℓ and the parity P for a heavy-light meson as [?, ?]:

$$j_\ell = |k| - \frac{1}{2}, \quad P = (-1)^{|k|+1}.$$

The wave functions are defined as [?]:

$$\Psi_{jm}^{(k)}(r,\Omega) = \begin{pmatrix} u_k(r) y_{jm}^{(k)} \\ i v_k(r) y_{jm}^{(-k)} \end{pmatrix}.$$

Using heavy quark symmetry, the lowest-order Hamiltonian in our relativistic potential model [?, ?] is given by:

$$H_0 = \vec{\alpha}_q \cdot \vec{p} + m_q \beta_q,$$

whose commutation relation with $\vec{L} = \vec{r} \times \vec{p}$ is:

$$[H_0, L_i] = -i(\vec{\alpha}_q \times \vec{p})_i.$$

On the other hand, we have the following commutation relation:

$$\left[\vec{\alpha}_q \cdot \vec{p}, \vec{\alpha}_q \times \vec{r}\right] = 2i(\vec{\alpha}_q \times \vec{p})_i + 2i\epsilon_{ijk} p_j \Sigma_q^k,$$

with the light quark spin $\vec{\Sigma}_q/2$. Adding these equations, we obtain conservation of $\vec{j}_\ell = \vec{L} + \vec{\Sigma}_q/2$ of light-quark degrees of freedom as expected, $[H_0, \vec{j}_\ell] = 0$. Because matrices related to a heavy quark are not included in H_0, a heavy quark spin $\vec{\Sigma}_Q/2$ also commutes with H_0, $[H_0, \vec{\Sigma}_Q/2] = 0$, which means the total angular momentum $\vec{J} = \vec{L} + \vec{\Sigma}_q/2 + \vec{\Sigma}_Q/2$ is also conserved: $[H_0, \vec{J}] = 0$.

We would like to estimate the expectation value of $[H_0, \vec{L}^2]$, whose explicit form is given by:

$$M \equiv [H_0, \vec{L}^2] = i\alpha_q^j \left\{ i p_j + \frac{1}{r}\left[r_j p^2 - (\vec{r}\cdot\vec{p})p_j\right]\right\} \equiv \alpha_q^j f_j(\vec{r},\vec{p}).$$

There is a lemma stating that if we calculate the expectation value $\langle \Psi_\ell | [H_0, O] | \Psi_\ell \rangle$ and Ψ_ℓ is an eigenfunction of H_0 with real eigenvalue E_ℓ (i.e., $H_0 \Psi_\ell = E_\ell \Psi_\ell$), then $\langle \Psi_\ell | [H_0, O] | \Psi_\ell \rangle = 0$ for any operator O.

The actual wave function includes both positive- and negative-energy states of the heavy quark:

$$\psi_\ell = \sum_{\ell'} c_{\ell,\ell'}^{+} \Psi_{\ell'}^{+} + \sum_{\ell'} c_{\ell,\ell'}^{-} \Psi_{\ell'}^{-}.$$

For the case $k = -1$ ($j^P = (1/2)^-$), we obtain the following results up to first order in $1/m_Q$ [?, ?]:

$$\psi_\ell(0^-) = \Psi_{-1}^{+} + c_{-1,1}^{+} \Psi_1^{+} + c_{-1,2}^{+} \Psi_{-2}^{+} + O(1/m_Q^2),$$

$$\psi_\ell(1^-) = \Psi_{-1}^{+} + c_{-1,1}^{-} \Psi_1^{-} + c_{-1,-2}^{-} \Psi_{-2}^{-} + O(1/m_Q^2),$$

where we give J^P in parentheses on the left-hand side and all constants $c^{\pm}_{k,k'}$ are of order $1/m_Q$. On the right-hand side appear wave functions with negative-energy components of the heavy quark, Ψ^-, together with positive-energy ones, Ψ^+.

After some calculations, we obtain the matrix elements:

$$\langle\Psi^{\pm}_{\ell'}|M|\Psi^{\pm}_{\ell'}\rangle = \int d^3r \left[-v_{k'}(r)y^{(-k')\dagger}_{j'm'}\sigma_n f_n(\vec{r},\vec{p})u_k(r)y^{(k)}_{jm} + u_{k'}(r)y^{(k')\dagger}_{j'm'}\sigma_n f_n(\vec{r},\vec{p})v_k(r)y^{(-k)}_{jm}\right],$$

where σ_i are Pauli matrices, p_i is the momentum operator, and $f_n(\vec{r},\vec{p})$ is defined in Eq. (4).

When estimating $\langle\psi_\ell(0^-)|M|\psi_\ell(0^-)\rangle$, there is no surviving term up to first order in $1/m_Q$. This is because $\langle\Psi^+_{-1}|M|\Psi^+_{-1}\rangle$ vanishes due to the lemma, and cross terms of Ψ^+_{-1} and Ψ^-_{-1} vanish because of orthogonality. Hence, the surviving term starts from order $(1/m_Q)^2$. When estimating $\langle\psi_\ell(1^-)|M|\psi_\ell(1^-)\rangle$ and taking into account the above estimate and $c^{\pm}_{-1,1} \sim 1/m_Q$, there remain cross terms with different k quantum numbers and their conjugates, which are of order $1/m_Q$ and hence suppressed for large m_Q. Similar arguments for other higher states give the same conclusion: the expectation value of the matrix element for any higher state is at most of order $1/m_Q$.

To obtain complete symmetry, we need only neglect the lower-component radial wave function $v_k(r)$, which makes Eq. (11) vanish. Neglecting $v_k(r)$ in Eq. (7), we obtain a nonrelativistic wave function in the heavy quark system, and a simple calculation shows that this is an eigenfunction of $\vec{L}^2$:

$$\vec{L}^2 y^{(k)}_{jm} = k(k+1)y^{(k)}_{jm} = L(L+1)y^{(k)}_{jm},$$

where we have used the formula $\vec{L}\cdot\vec{\sigma}_q \otimes y^{(k)}_{jm} = -(k+1)y^{(k)}_{jm}$ for $k > 0$ and $ky^{(k)}_{jm}$ for $k < 0$, and the fact that $k = L$ or $-(L+1)$. Inclusion of the radial wave function does not change the result because $\vec{L}u_k(r) = 0$. Eq. (12) means that the nonrelativistic approximation of the heavy quark system has rotational symmetry and hence, in this approximation, states with the same L are degenerate.

III. Conclusions and Discussion

In this article, we have pointed out that there exists an approximate degeneracy among heavy-light systems with the same L. This is supported by experimental facts visible in Table I. This approximate symmetry explains why the GI model obtains results similar to those of heavy-light systems that fit experimental data well: the GI model possesses this symmetry from the start, which is only broken by spin-orbit interactions. Numerical results from the GI model, together with those from other models respecting heavy quark symmetry, have been compared with experimental D meson data in Table I, and they all give similar results.

We have analytically shown that expectation values of $[H_0, \vec{L}^2]$ are at most of order $1/m_Q$ for 0^- and 1^- states, and similar arguments yield the same conclusion for other higher states in our model, which respects heavy quark symmetry.

Note that this order of magnitude, $1/m_Q$, is the same as that which breaks degeneracy of a spin doublet in heavy-light systems. We have demonstrated that there is rotational symmetry in the limit $m_Q \to \infty$ and the nonrelativistic limit of heavy-quark symmetry, as shown in Eq. (12).

Simple applications of our idea to other states include baryons QQq like Ξ_{cc}^{+}, multiquark states containing one light quark like $Q\bar{Q}Qq$, and possibly other states where a couple of light quarks can be regarded as a brown muck. A good example is given by the Λ_c spectrum, which yields $\Lambda_c(2286)$ with $L = 0$, $\Lambda_c^{+}(2625)$ with $L = 1$, and $\Lambda_c^{+}(2940)$ with $L = 2$ [?], where a spin multiplet consists of member(s) with the same L. Here L is defined as the angular momentum between the heavy quark c and the two light quarks (ud). One can easily see that gaps between different spin multiplets are nearly equal to $\Lambda_{\rm QCD} \sim 300$ MeV, which coincides with our observation for heavy-light mesons.

Future measurements of higher orbitally and/or radially excited states and their masses by LHCb and the forthcoming Belle II are awaited to test our observation.

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References

[1] K. A. Olive et al. [Particle Data Group Collaboration], Chin. Phys. C 38, 090001 (2014).

[2] S. Godfrey and N. Isgur, Phys. Rev. D 32, 189 (1985).

[3] S. Godfrey and R. Kokoski, Phys. Rev. D 43, 1679 (1991).

[4] T. Matsuki, T. Morii and K. Sudoh, Prog. Theor. Phys. 117, 1077 (2007).

[5] T. Matsuki and T. Morii, Phys. Rev. D 56, 5646 (1997) [hep-ph/9702366].

[6] Q. F. Lü and D. M. Li, Phys. Rev. D 90, no. 5, 054024 (2014).

[7] Y. Sun, X. Liu and T. Matsuki, Phys. Rev. D 88, 094020 (2013).

[8] O. Lakhina and E. S. Swanson, Phys. Lett. B 650, 159 (2007).

[9] D.-M. Li, P.-F. Ji and B. Ma, Eur. Phys. J. C 71, 1582 (2011).

[10] O. Lakhina and E. S. Swanson, Phys. Lett. B 650, 159 (2007).

[11] D.-M. Li, P.-F. Ji and B. Ma, Eur. Phys. J. C 71, 1582 (2011).

[12] J. Zeng, J. W. Van Orden, and W. Roberts, Phys. Rev. D 52, 5229 (1995).

[13] M. Di Pierro and E. Eichten, Phys. Rev. D 64, 114004 (2001).

[14] D. Ebert, R. N. Faustov, and V. O. Galkin, Eur. Phys. J. C 66, 197 (2010).

[15] K. A. Olive et al. [Particle Data Group Collaboration], Chin. Phys. C 38, 090001 (2014).

[16] R. Aaij et al. (LHCb Collaboration), J. High Energy Phys. 1309, 145 (2013).

[17] R. Aaij et al. (LHCb Collaboration), Phys. Rev. D 91, 092002 (2015).

[18] R. Aaij et al. (LHCb Collaboration), Phys. Rev. D 92, 032002 (2015).

[19] P. del Amo Sanchez et al. (BaBar Collaboration), Phys. Rev. D 82, 111101 (2010).

[20] T. Matsuki, T. Morii and K. Sudoh, Phys. Lett. B 659, 593 (2008) doi:10.1016/j.physletb.2007.11.065 [arXiv:0710.0325 [hep-ph]].

[21] T. Matsuki, K. Mawatari, T. Morii and K. Sudoh, hep-ph/0408326.

[22] T. Matsuki and K. Seo, Prog. Theor. Phys. 121, 1141 (2009) [hep-ph/0703158].

Note: Figure translations are in progress. See original paper for figures.