Critical Temperature of Chiral Symmetry Restoration for Quark Matter with a Chiral Chemical Potential postprint

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Abstract

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Full Text

Preamble

Critical Temperature of Chiral Symmetry Restoration for Quark Matter with a Chiral Chemical Potential

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Abstract. In this article we study restoration of chiral symmetry at finite temperature for quark matter with a chiral chemical potential, µ , by means of a nonlocal Nambu-Jona-Lasinio model. This model allows us to introduce in the simplest way possible a Euclidean momentum, p , dependent quark mass function which decays (neglecting logarithms) as $1/p^{2}$ for large p , in agreement with asymptotic behaviour expected in QCD in presence of a nonperturbative quark condensate. We focus on the critical temperature for chiral symmetry restoration in the chiral limit, Tc, versus µ , as well as on the order of the phase transition. We find that Tc increases with µ , and that the transition remains of the second order for the whole range of µ considered.

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1. Introduction

Systems with chirality imbalance, namely with a finite chiral density n = n − n generated by quantum anomalies, have attracted considerable interest in recent years. Gauge field configurations with a finite winding number, Qw, can change fermion chirality according to the Adler-Bell-Jackiw anomaly [1, 2]. In the context of Quantum Chromodynamics (QCD), such nontrivial gauge field configurations with Qw 0 are instantons and sphalerons, the latter being produced copiously at high temperature [3, 4]. The large number of sphaleron transitions in the high-temperature quark-gluon plasma phase of QCD suggested the possibility that net chirality due to quarks interacting with sphalerons, in combination with the magnetic field produced in high-energy collisions, might lead to a charge separation phenomenon named the Chiral Magnetic Effect (CME) [5, 6]. Since the first articles about the CME, interest in the study of media with net chirality has spread from QCD to hydrodynamics and condensed matter systems [7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22].

To describe systems with finite chirality in thermodynamical equilibrium, it is customary to introduce the chiral chemical potential, µ , which is conjugated to n [23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39]. Because of quantum anomalies as well as chirality-changing processes, n is not a strictly conserved quantity, hence the concept of µ might sound ambiguous; however, naming the typical time scale in which chirality-changing processes take place, it can be assumed that µ 0 describes a system in thermodynamical equilibrium with a fixed value of n on a time scale much larger than . For example, in the quark-gluon plasma phase of QCD, chirality-changing processes have been studied in [40] where it was found that 50 ÷ 140 fm/c in the temperature range T 225 ÷ 500 MeV.

An interesting problem in the context of QCD is the study of chiral symmetry restoration at finite temperature and µ 0. Some previous calculations based on chiral models predicted Tc, the critical temperature for chiral symmetry restoration, to decrease with µ [24, 25, 26, 27, 28]. On the other hand, recent lattice data have shown that the critical temperature increases with µ [30, 31], in agreement with results obtained by solving Schwinger-Dyson equations at finite µ [34, 35].

In this article we study chiral symmetry restoration at finite temperature with µ 0 within a Nambu-Jona-Lasinio (NJL) model [41, 42, 43, 44] with a nonlocal interaction kernel [45, 46, 47, 48, 49, 50, 51] built in agreement with the current-current quark interactions in the Instanton Liquid Model of QCD vacuum [52]. We use a nonlocal interaction kernel to mimic the ultraviolet behaviour of the constituent quark mass in QCD [53]. We work in the chiral limit, which allows us to define mathematically the critical temperature as the zero of the second-order Ginzburg-Landau (GL) coefficient.

The main result of our study is that the critical temperature increases with µ for all the nonlocal kernels used in actual calculations. Moreover, we discuss the order of the chiral phase transition at finite µ : we find that although the chiral chemical potential makes the phase transition sharper, it remains of second order in the range of µ studied, that is for µ up to O(1) GeV. Both Tc(µ) and the absence of a critical endpoint are in agreement with the most recent lattice results mentioned above.

The plan of the article is as follows. In Section II we briefly describe the nonlocal NJL model used in our calculation, presenting the several choices we make for the running mass. In Section III we perform a small µ computation of the second GL coefficient in the free energy. In Section IV we relax the small µ approximation and compute the critical temperature for chiral symmetry restoration as a function of µ , as well as determine the order of the phase transition. Finally, in Section V we draw our conclusions.

2. NJL Model with Momentum Dependent Quark Mass Function

In this section we describe the model used to compute the critical line for chiral symmetry restoration in the T-µ plane. We use a Nambu-Jona-Lasinio (NJL) model [41, 42] (see [43, 44] for reviews) with a nonlocal interaction kernel inspired by the Instanton Liquid picture of the QCD vacuum [47, 45, 46, 48, 49], which has the advantage of introducing in the simplest way possible a Euclidean momentum-dependent quark mass function in agreement with QCD [53, 54]. The action of the model is given by

$$S = \int d^4x(\mathcal{L}_0 + \mathcal{L}_4),$$

where $\mathcal{L}_0$ is the Lagrangian density of free massless quarks with a chiral chemical potential:

$$\mathcal{L}_0 = \bar{\psi} i \partial_\mu \gamma^\mu + \mu_5 \bar{\psi} \gamma^0 \gamma^5 \psi$$

with ψ being a quark field with Dirac, color, and flavor indices. In this equation µ is the chiral chemical potential, and its conjugated quantity is the chiral charge density, $n_5 \equiv n_R - n_L$: a finite µ induces a chiral density in the system, and in general the relation between n and µ has to be computed numerically within some model [25, 26].

In Eq.(1), $\mathcal{L}_4$ corresponds to the interaction term that in compact form can be written as

$$\mathcal{L}_4 = G[j_s(x) j_s(x) + j_{ps}(x) j_{ps}(x)],$$

where

$$j_s(x) = \int d^4y d^4z F^\star(y-x) F(z-x) \bar{\psi}(y) \psi(z),$$

$$j_{ps}(x) = \int d^4y d^4z F^\star(y-x) F(z-x) \bar{\psi}(y) i\gamma_5 \tau \psi(z),$$

correspond to scalar and pseudoscalar currents respectively. The interaction term in Eq. (3) is formally equivalent to a local NJL interaction,

$$\mathcal{L}_4 = G[(\bar{\Psi}(x)\Psi(x))^2 + (\bar{\Psi}(x) i\gamma_5 \tau \Psi(x))^2]$$

written in terms of the dressed quark fields

$$\Psi(x) \equiv \int d^4y F(y-x) \psi(y).$$

In this study we assume chiral symmetry is spontaneously broken by the nonvanishing expectation value of the dressed quark field operator $\sigma \equiv G\langle \bar{\Psi}(x)\Psi(x)\rangle \neq 0$, hence neglecting pion condensates. Working at finite temperature T, the action in momentum space and in finite quantization volume V can be written as

$$S = -\beta V \int \frac{d^4p}{(2\pi)^4} \bar{\psi}(p)[\gamma^\mu p_\mu - M(p) + \mu_5 \gamma^0 \gamma^5] \psi(p),$$

with $\beta = 1/T$. In the above equation we have introduced the quark mass function $M(p) \equiv -2\sigma C(p)$, with $C(p) \equiv F^2(p)$ and $F(p)$ corresponding to the Fourier transform of the form factor F in Eq. (7). The above equation is formally in agreement with nonlocal models in which the nonlocality comes from an effective one-gluon exchange picture [55, 56, 50, 51, 58, 57].

In QCD it has long been known that quarks develop a constituent quark mass function in the presence of spontaneous chiral symmetry breaking [53], which for large values of Euclidean momentum p_E and in the chiral limit behaves like [58]

$$M_{QCD}(p_E^2) = -\frac{4\pi^2 d_m}{[\log(\mu^2/\Lambda_{QCD}^2)]^{1-d_m}}[\log(p_E^2/\Lambda_{QCD}^2)]^{-d_m}\langle\bar{\psi}\psi\rangle(\mu),$$

where µ is the renormalization point and $\langle\bar{\psi}\psi\rangle(\mu)$ is the chiral condensate at the scale µ used as a parameter in Operator Product Expansion; $d_m = 12/(33-2N_f)$ is the anomalous dimension of the current mass which for $N_f = 2$ gives $d_m = 12/27$. The NJL model with a local interaction kernel misses the asymptotic behaviour of the quark mass function, which instead we will show to be important in the context of matter with $n_5 \neq 0$.

In particular we will assume several specific functional forms for M(p) in Eq. (10), which besides log terms are consistent with the ultraviolet behaviour of the QCD quantity in Eq. (11). Equation (11) is valid only in the ultraviolet limit, therefore we need to rely on some ansatz to describe the quark mass function at small Euclidean momentum. We assume the form factor

$$C(p_E) = \theta(\Lambda^2 - p_E^2)\left(\frac{\Lambda^2}{p_E^2}\right)^\gamma + \theta(p_E^2 - \Lambda^2)\left(\frac{\log\Lambda^2/\Lambda_{QCD}^2}{\log p_E^2/\Lambda_{QCD}^2}\right)^\gamma,$$

where p_E^2 is the Euclidean squared momentum. For the exponent   in Eq.(12) we consider three cases: = 0 for simplicity;   = 1, inspired by the large momentum behaviour of the form factor introduced in [55, 56]; and finally   = 1 - d_m where d_m corresponds to the anomalous mass dimension, inspired by the quark mass function derived by Politzer [53].

We also consider smooth form factors. In particular we consider a Yukawa-type form factor [50, 51]:

$$C(p_E) = \frac{\Lambda^2}{p_E^2 + \Lambda^2};$$

then we consider a form factor inspired by the Instanton Liquid Model (ILM) of the QCD vacuum [52]:

$$C(p_E) = \frac{4}{\pi^2 d^2 |p_E|^2}[I_0(x)K_0(x) - I_1(x)K_1(x)],$$

where d corresponds to the typical instanton size $d \approx 0.36$ fm and $x = |p_E|d/2$. Finally we consider a nonlocal kernel used in nonlocal NJL model studies [55, 56]:

$$C(p_E) = \theta(\Lambda^2 - p_E^2)e^{-p_E^2 d^2/2} + \theta(p_E^2 - \Lambda^2)\frac{\Lambda^2}{p_E^2}\left(\frac{\log \Lambda^2/\Lambda_{QCD}^2}{\log p_E^2/\Lambda_{QCD}^2}\right)e^{-\Lambda^2 d^2/2},$$

where d corresponds to the instanton size used in Eq. (14) and $\Lambda = O(1)$ GeV. Equation (15) offers a smooth version of the form factor in Eq. (12) with = 1.

In this article we use imaginary time formalism to deal with finite temperature calculations, thus we introduce $p_0 = ip_4 = i\omega_n$ with $\omega_n = \pi T(2n+1)$ being the fermionic Matsubara frequency. The thermodynamic potential per unit volume in the mean field approximation is then given by

$$\Omega = -\frac{N_c N_f T}{V}\sum_n \int \frac{d^3p}{(2\pi)^3}\mathrm{Tr}\log \beta S^{-1}(\omega_n, p),$$

where the inverse fermion propagator is given by

$$S^{-1}(\omega_n, p) = i\omega_n\gamma^0 - \gamma \cdot p - M(\omega_n, p) + \mu_5\gamma^0\gamma^5.$$

A straightforward evaluation of the determinant of S^{-1} gives

$$\Omega = -\frac{N_c N_f T}{V}\sum_n \int \frac{d^3p}{(2\pi)^3}\log \beta^4(\omega_n^2 + E_+^2)(\omega_n^2 + E_-^2),$$

where we have defined

$$E_\pm = \sqrt{(p \pm \mu_5)^2 + M(\omega_n, p)^2}.$$

In this article we will use Eq. (18) to compute the critical line for chiral symmetry restoration with µ 0, by a Ginzburg-Landau (GL) expansion in powers of the condensate .

3. Small µ Analysis

In this article we are interested in the relation between µ and the critical temperature for a second-order phase transition within the non-local model specified by the potential in Eq. (18). Close to the second-order phase transition we make a Ginzburg-Landau (GL) expansion of the thermodynamic potential:

$$\Omega - \Omega_0 = \alpha_2\sigma^2 + \alpha_4\sigma^4 + O(\sigma^6),$$

where the coefficient depends on temperature and the critical temperature is defined as the solution of the equation (Tc) = 0; Ω corresponds to the free energy in the phase with = 0, which is just an irrelevant constant and has been subtracted.

In this section we focus on µ T, hence we expand

$$\alpha_2 = \alpha_{2,0} + \mu_5^2\alpha_{2,2}.$$

The above equation allows us to compute, to lowest order in µ /T, the shift of the critical temperature due to µ :

$$\delta T_c = -\frac{\alpha_{2,2}(T_c^0)}{a}\mu_5^2,$$

where T_c^0 corresponds to the critical temperature at µ = 0 and $a \equiv d\alpha_{2,0}/dT$ at $T = T_c^0$. The quantity a depends on the specific model used but is positive by definition because , is negative for T < Tc and positive for T > Tc; thus the sign of Tc is determined only by the sign of , . A straightforward computation starting from Eq. (18) shows that

$$\alpha_{2,2} = -\frac{4N_cN_fT}{V}\sum_n\int\frac{d^3p}{(2\pi)^3}C^2(\omega_n,p)\frac{2(3p^2-\omega_n^2)}{(p^2+\omega_n^2)^3},$$

where C is the nonlocal interaction kernel. Once , is known, the critical temperature versus µ can be computed as

$$T_c(\mu_5) = T_c^0\left(1 - \frac{\alpha_{2,2}(T_c^0)}{aT_c^0}\mu_5^2\right).$$

In Figure 1 we plot the coefficient , computed from Eq. (23) for the several form factors described in Section 2. For all models of p -dependent quark mass functions we find that , < 0, and because of Eq. (24) this implies µ tends to increase the critical temperature for chiral symmetry restoration within the model at hand. This differs from what is obtained within local models, in which the critical temperature has been found to decrease with µ , with the exception of [23] where renormalization was used to treat the divergent vacuum term.

4. The Critical Line within Ginzburg-Landau Expansion

In this section we compute the critical line for chiral symmetry restoration as a function of the chiral chemical potential within a Ginzburg-Landau (GL) expansion of the thermodynamic potential in Eq. (18). We do not rely on a small µ expansion as done in the previous section, and we limit ourselves to computing numerically the relevant GL coefficients from which we extract both the critical temperature and the order of the phase transition.

Close to a second-order phase transition we can write Eq. (18) as

$$\Omega - \Omega_0 = \alpha_2 \sigma^2 + \alpha_4 \sigma^4 + O(\sigma^6),$$

where we have subtracted the thermodynamic potential at = 0, namely Ω ; and can be computed by taking derivatives of Ω with respect to at = 0. We find

$$\alpha_2 = -\frac{N_c N_f T}{V} \sum_n \int \frac{d^3p}{(2\pi)^3} C^2(\omega_n, p) \frac{16(p^2 + \omega_n^2 + \mu_5^2)}{[\omega_n^2 + (p-\mu_5)^2][\omega_n^2 + (p+\mu_5)^2]},$$

and

$$\alpha_4 = -\frac{N_c N_f T}{V} \sum_n \int \frac{d^3p}{(2\pi)^3} C^4(\omega_n, p) \frac{5\mu_5^4 + 2\mu_5^2(3p^2 - \omega_n^2) + (p^2+\omega_n^2)^2 - 384[p^4 + 2p^2(\omega_n^2 - \mu_5^2) + (\omega_n^2 + \mu_5^2)^2}{[\omega_n^2 + (p-\mu_5)^2]^2[\omega_n^2 + (p+\mu_5)^2]^2}$$

The nontrivial solution of the gap equation, Ω/ = 0, for T Tc is given by

$$\sigma^2(T, \mu_5) = -\frac{\alpha_2(T, \mu_5)}{\alpha_4(T, \mu_5)},$$

and the critical temperature is defined by the condition (Tc, µ) = 0.

In the upper panel of Fig. 2 we plot the coefficient in units of the parameter Λ² as a function of temperature for three different values of µ . For each value of µ the critical temperature Tc corresponds to (Tc) = 0. We show data for the nonlocal model with mass function given by Eqs. (12) and (10) with = 0 and Λ = 900 MeV; for other models we obtain qualitatively the same results. We notice that, in agreement with the small µ analysis shown in the previous section, increasing µ results in an increasing critical temperature. We also notice that the slope of at T = Tc increases with µ . Together with the behaviour of discussed below, this is a signature of the phase transition becoming sharper with µ .

In the middle panel of Fig. 2 we plot the coefficient versus temperature for the same values of µ shown in the upper panel. We notice that for any value of µ the coefficient decreases in magnitude, but it is always positive at the critical temperature, meaning the phase transition is second order. We also notice that the magnitude of at T = Tc(µ) decreases compared to the case µ = 0, implying that the phase transition becomes sharper with increasing µ . In fact, because of Eq. (28) we can write the solution of the gap equation for T Tc as

$$\sigma^2 = -\frac{1}{\alpha_4(T_c)} \left.\frac{d\alpha_2}{dT}\right|_{T=T_c} (T - T_c) + O[(T - T_c)^2],$$

so the slope of the condensate at the critical temperature is given by

$$\left.\frac{d\sigma^2}{dT}\right|_{T=T_c} = -\frac{1}{\alpha_4(T_c)} \left.\frac{d\alpha_2}{dT}\right|_{T=T_c},$$

which becomes larger as we increase µ because decreases and the slope of at the critical temperature increases with µ . Our conclusion is that within the range of µ explored in our study, we have firm evidence that the phase transition becomes sharper as µ is increased, but there is no critical endpoint because does not change sign at the critical temperature. This is in agreement with lattice simulations [30, 31] but disagrees with previous model studies that used a different regularization scheme [24, 26, 25, 27], showing how the existence of the critical point in the phase diagram is very sensitive to the regularization prescription, as already noted in [28].

Finally, in the lower panel of Fig. 2 we plot the coefficient at T = Tc(µ) for several models. We notice that although the numerical value of strongly depends on the model, we find it is always positive when computed at Tc.

In Fig. 3 we plot the critical temperature versus µ for the nonlocal models described in the text, obtained from the zero of the coefficient . In the left panel we collect results for the sharp models. Circles correspond to the mass function given by Eqs. (12) and (10) with = 0 for two different values of Λ; diamonds correspond to the same mass function with = 1 - d . In the left panel we also show results for two local NJL models. In particular, we denote by squares the results for a local NJL model with a 4-dimensional sharp cutoff on the vacuum term but no cutoff on the thermal part of the free energy; empty squares correspond to a model dubbed Λ-NJL, in which there is a 4D sharp cutoff both on the vacuum and on the thermal contribution to the gap equation. In both cases Λ = 900 MeV.

In the right panel of Fig. 3 we plot the critical temperature for smooth form factors. Data with triangles pointing upwards correspond to a Yukawa-like form factor in Eq. (13) with two values of Λ. Data denoted by triangles pointing downwards correspond to the Instanton Liquid Model (ILM) form factor in Eq. (14). Stars correspond to the nonlocal form factor in Eq. (15). In both panels, temperature and chemical potential are measured in units of the critical temperature at µ = 0. In each calculation we have fixed the value of the parameter Λ in the form factor, then tuned the NJL coupling constant G to obtain Tc = 170 MeV at µ = 0 for every model.

The results in Fig. 3 show that for all nonlocal models studied in this article, the critical temperature increases with µ , at least for µ Λ where Λ is the scale at which we match the nonperturbative mass function with its perturbative counterpart, Λ O(1) GeV. We remark that our interaction kernels lack a backreaction of µ , hence our results should be taken with caution for µ = O(1 GeV), while they are reliable for smaller values of µ . For the case of local models, we find that the Λ-NJL model still predicts Tc increases with µ , at least up to values of µ of order Λ. This agrees with the previous analysis of [28] where a Λ-NJL with a 3-dimensional cutoff was considered. For the NJL model result in Fig. 3 we find that Tc increases with µ for small values, then decreases. We mention that in this case the critical line bends downward for values of µ considerably smaller than the ultraviolet cutoff, hence the change of slope cannot be directly related to the cutoff; we have checked instead that this change of slope is due to taking into account all modes in the gap equation at finite temperature rather than cutting them at p = Λ, in agreement with [28].

A detailed comparison with lattice data [30, 31] is premature because those data have not been obtained in the chiral limit; moreover some lattice data correspond to Nc = 2 QCD while here we consider Nc = 3. However, we can at least compare the magnitude of the increase of the critical temperature obtained within the nonlocal models and within lattice simulations. In Fig. 3 we show lattice results for Tc(µ) for Nc = 2 adapted from Ref. [31], in which the critical temperature at µ = 0 is Tc = 195.8 ± 0.4 MeV. We find that among the models considered here, those with Gaussian and ILM form factors, respectively Eqs. (15) and (14), better reproduce the magnitude of the variation of the critical temperature with µ .

5. Conclusions

In this article we have presented a model study of the critical temperature of chiral symmetry restoration, Tc, as a function of chiral chemical potential, µ . We used a nonlocal NJL model with several Euclidean interaction kernels chosen to mimic the constituent quark mass of QCD in the ultraviolet.

We performed a Ginzburg-Landau expansion of the thermodynamic potential in the vicinity of the second-order critical line, focusing on the second-order coefficient , which allows us to determine how the critical temperature runs with µ . The results for Tc(µ) are collected in Fig. 3 for the different models. We found that within all nonlocal models used in our study, Tc always increases with µ at least for µ Λ, where Λ is the scale at which we match the nonperturbative mass function with its perturbative counterpart, Λ O(1) GeV. We remark that our interaction kernels lack a backreaction from µ , hence our results should be taken with caution for µ = O(1 GeV), while they are reliable for smaller values.

We then checked the order of the phase transition by computing the coefficient of the GL effective potential: we found that although µ makes the transition sharper because the magnitude of decreases with µ at Tc, the coefficient never vanishes as would happen at a critical endpoint. Our conclusion is that there is no trace of a critical endpoint in the phase diagram, at least within the range of µ explored in this article, namely for µ approximately up to 1 GeV.

We also studied how a sharp but covariant 4-dimensional cutoff scheme affects Tc(µ) within the local NJL model: we found that although for a small range of µ the critical temperature increases, then for µ quite smaller than the cutoff it decreases, as happens in local chiral model calculations with a 3-dimensional cutoff. However, removing by hand the contribution of thermal excitations with p > Λ in the gap equation, we find Tc(µ) increases also for larger values of µ , in agreement with [28].

Finally, we mention that during the final stage of preparation of this manuscript, Ref. [59] appeared in which the same problem was studied and an increasing Tc versus µ was found, in agreement with the results presented here.

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Appendix A. Analysis of Modes Contributing to ,

In this appendix we present a short numerical analysis of the coefficient , defined in Eq. (23), which illuminates the difference between nonlocal and local models concerning Tc(µ) for small values of µ . For simplicity, we focus on the form factor given by Eq.(12) with = 0, which allows easier manipulations and clearer mode separation.

A direct computation of , gives

$$\alpha_{2,2} = I_1 + I_2 + J_1 + J_2,$$

where we have split contributions depending on the momentum region of quarks and on temperature:

- Modes with $p_E^2 \leq \Lambda^2$ at T = 0:

$$\frac{4N_c N_f}{V} I_1 = -a_2 \int_{m_0^2}^{\Lambda^2} dp_E^2 \frac{1}{p_E^2}$$

- Modes with $p_E^2 > \Lambda^2$ at T = 0:

$$\frac{4N_cN_f}{V} I_2 = -a_1 \int_{\Lambda^2}^{\infty} dp_E^2 \frac{1}{(p_E^2)^2}$$

- Modes with $p_E^2 \leq \Lambda^2$ at T > 0:

$$\frac{4N_cN_f}{V} J_1 = -\frac{a_2}{2\pi^2} |F(\beta\Lambda)|$$

- Modes with $p_E^2 > \Lambda^2$ at T > 0:

$$\frac{4N_cN_f}{V} J_2 = -\frac{a_1}{2\pi^2} |G(\beta\Lambda)|$$

In the above equations $a_1 \approx 0.25$ and $a_2 \approx 0.938$. Moreover, we have introduced an infrared cutoff m_0 which appears in intermediate steps when contributions are split; this fictitious cutoff disappears when the sum is performed, as is clear from Eqs.(A.4) and (A.2). In the left panel of Fig. A1 we plot the functions F, G, and their sum to understand the role of the various terms in Eq.(A.1). The modes in Eq.(A.3) come from the high-momentum part of the Dirac sea; they are not usually considered in local model calculations because their contribution is divergent and simply subtracted. We notice this contribution to , is negative, thus helping to keep the critical temperature at finite μ higher than at μ = 0.

We now compare results of the 4D nonlocal model with those of local models. We introduce a Λ-NJL model in which we remove all modes with $p_E^2 > \Lambda^2$; the coefficient is thus given by

$$\alpha_{2,2}^{\Lambda-NJL} = -\frac{4N_cN_f}{V}[a_2 \log \beta\Lambda - |F(\beta\Lambda)|].$$

We also introduce a local NJL model in which we remove ultraviolet modes $p_E^2 > \Lambda^2$ only at T = 0, and integrate over all momenta at finite temperature:

$$\alpha_{2,2}^{NJL} = -\frac{4N_cN_f}{V}[a_2 \log \beta\Lambda - |F(\infty)|],$$

where $F(\infty) \equiv \lim_{x\to\infty} F(x)$.

In the right panel of Fig. A1 we plot the coefficient , for the local NJL model (green dot-dashed line), the local Λ-NJL model (maroon dashed line), and the nonlocal model with mass function given by Eqs. (12) and (10) with = 0. For local models there exists a window of Λ in which , > 0; on the other hand, for the nonlocal model considered here we find , < 0 for any value of Λ. The fact that , can be positive in local models is due to the absence of the vacuum term in Eq. (A.3) which would give a negative contribution to , . The range of Λ in which , > 0 is quite wide for the NJL model, being shrunk in the case of Λ-NJL. The results in Fig. A1 suggest that in the local NJL model the slope of Tc(μ) at small μ depends on the ratio Tc/Λ; this is true also in the Λ-NJL model, but in this case the range of Tc/Λ with positive , is much smaller.

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