Cosmographic analysis from distance indicator and dynamical redshift drift postprint

Authors: Ming-Jian Zhang, Hong Li, Jun-Qing Xia

Date: 2016-08-30T00:00:00+00:00

Abstract

Cosmography is a model-independent description. To ensure data lie within the convergence radius, the y-redshift defined as y=z/(1+z) was introduced. However, discussions regarding the utility of y-redshift and the fundamental cause of the convergence issue are generally absent. In the present paper, we investigate cosmography in both z and y redshifts using supernova and mock redshift drift data. By introducing the bias-variance tradeoff, we reveal that the large squared bias between cosmography and Union2.1 supernova data is the chief culprit of the convergence issue. Moreover, neither higher-order expansions nor the introduction of y-redshift are effective in reconciling this contradiction. Risk minimization suggests that a Taylor expansion up to the second term is a better choice for available supernova data. Forecasts from future supernova data and redshift drift show that redshift drift can provide much tighter constraints on cosmography. We also investigate the effect of the convergence issue on the deceleration parameter and dark energy. This inspires us that dynamical observations including redshift drift can give more detailed information on cosmic evolution.

Full Text

Preamble

Cosmographic analysis from distance indicators and dynamical redshift drift

Ming-Jian Zhang[a], Hong Li[a], and Jun-Qing Xia[b*]

[a]Key Laboratory of Particle Astrophysics, Institute of High Energy Physics, Chinese Academy of Sciences, P.O. Box 918-3, Beijing 100049, China

[b]Department of Astronomy, Beijing Normal University, Beijing 100875, China

Cosmography provides a model-independent description of cosmic evolution, but suffers from a serious convergence issue when confronted with supernova data,

particularly for high redshift $z > 1$. To ensure data fall within the convergence radius, the $y = z/(1 + z)$ redshift parameter was defined. However, discussions about the usefulness of y-redshift and the root cause of this issue are generally absent. In this paper, we study cosmography in both z and y redshift using supernova and mock redshift drift data. By introducing the bias-variance tradeoff, we reveal that the large squared bias between cosmography and Union2.1 supernova data is the "chief culprit" of the convergence issue. Moreover, extending the expansion to higher orders and introducing y-redshift are both ineffective at reconciling this contradiction. Minimizing risk suggests that Taylor expansion up to the second-order term is a better choice for available supernova data. Forecasts from future supernova data and redshift drift show that redshift drift can provide much tighter constraints on cosmography. We also investigate the effect of the convergence issue on the deceleration parameter and dark energy, which inspires us that dynamical observations including redshift drift can provide more detailed information about cosmic evolution.

Introduction

Two teams of type Ia supernova (SNIa) research [?, ?] independently discovered the cosmic accelerating expansion at the end of the last century. This landmark discovery was gradually affirmed by multiple other evidences, such as large-scale structure [?], cosmic microwave background anisotropies [?], and baryon acoustic oscillation (BAO) peaks [?] observations. In general, dynamical cosmological models reserve the right to explain this acceleration. However, such interpretation requires a breakthrough in contemporary physics. The paradigms include exotic dark energy with repulsive gravity, modifications to general relativity, or violation of the cosmological principle. The dark energy model posits that there exists a pervasive new component in the universe, with many variants possible including the cosmological constant [?], phantom [?], and quintom [?]. Conservative modified gravity approaches claim that cosmic acceleration can also be realized in $f(R)$ gravity theory [?] or higher-dimensional theory [?] without exotic components. Even more remarkably, violation of the cosmological principle in the form of the inhomogeneous Lemaître-Tolman-Bondi void model [?, ?, ?] represents another attempt.

Different from the above dynamical templates, kinematics provides a more moderate approach to understanding cosmic acceleration. It only assumes a homogeneous and isotropic universe at large scales. In this framework, the scale factor $a(t)$, a function of cosmic time, becomes essential due to its direct description of how the universe evolves over time. The Hubble parameter $H(z) = \dot{a}/a$ is important for connecting theoretical models with observational variables, where the dot denotes the derivative with respect to cosmic time. The deceleration parameter is well-known for its immediate description of the decelerating (or accelerating) expansion of the universe. Collecting these kinematic parameters, Chiba and Nakamura [?] and Visser [?] created cosmography via the Taylor expansion of the luminosity distance in the redshift parameter. Mathematically, this expansion should be performed near a small quantity. Nevertheless, many observations focus on the high-redshift region. Logically, cosmography may suffer from a critical issue—the convergence problem—a dramatic discrepancy between cosmographic values and observational data, especially for high redshift $z \gtrsim 1$ [?]. To break this disharmony, an improved redshift parametrization $y = z/(1+z)$ was invented. The new redshift $0 < y < 1$ can theoretically ensure the safety of Taylor expansion up to higher redshift. Since the proposal of cosmography, SNIa data have become the first target for testing this notable theory [?, ?]. In combination with supernova compilations, some auxiliary datasets have also been chosen for different analyses [?]. The results confirmed public concerns: fewer series truncations lead to larger uncertainties, and more terms lead to larger error bars. The crisis naturally turns to the question of where the "sweet spot" lies—that is, the most optimized series truncation. In our previous work [?, ?], we identified two nested models using the F-test and found an answer. In the present paper, we are not satisfied with merely determining which order is preferable, but strive to identify the latent fatal factor in the convergence issue. Therefore, a new approach is necessary for our analysis. We also introduce an observation completely different from current distance indicators to understand this problem.

Different from geometric measurements, Sandage [?] proposed as early as 1962 a dynamical survey to measure the secular variation of $\dot{a}(t)$, namely the redshift drift. Interestingly, this concept is independent of any cosmological model, requiring only a Friedmann-Robertson-Walker universe. Decades later, Loeb [?] proposed several innovative ways to measure this effect, such as the wavelength shift of quasar (QSO) Lyα absorption lines, emission spectra of galaxies, and other powerful radio sources. As expected, a new-generation European Extremely Large Telescope (E-ELT) would monitor this variation via the shift of QSO Lyα absorption lines in the region $z = 2-5$ where other observations are inaccessible [?]. Taking advantage of the E-ELT' s capacity, mock redshift drift data can be generated [?, ?, ?]. Numerous works agree that this future probe could produce excellent constraints on cosmological models, such as holographic dark energy [?], new agegraphic and Ricci dark energy models [?]. Additionally, redshift drift is also useful for breaking degeneracies between model parameters in $f(R)$ modified gravity and $f(T)$ gravity theory [?]. More importantly, the effect of redshift drift could even extend to testing the fundamental Copernican principle [?] and enabling direct measurement of cosmic acceleration [?]. In our previous work, we found that redshift drift can remarkably relax the dynamical coincidence problem [?] and is more stable for exploring the expansion history than distance indicators [?].

For cosmography, various works have reported confusion about the convergence problem. However, the question remains: which element leads to this problem? In the present paper, we focus on identifying the villain of this problem with the help of future mock data. Moreover, we discuss the seriousness of this problem for the deceleration parameter and dark energy equation of state.

This paper is organized as follows: In Section II, we introduce cosmography for supernovae and redshift drift. We present the observations in Section III. In Section IV, we emphatically analyze the convergence problem and its detriment to the deceleration parameter and dark energy. Finally, in Section V we draw conclusions and discussion.

Cosmography

A key advantage of cosmography is its minimal hypothesis of large-scale homogeneity and isotropy, without Friedmann equations. In this framework, introducing the cosmographic parameters is appropriate.

The Hubble parameter $H(t) = \dot{a}/a$ accurately connects cosmological models with observational data. The deceleration parameter $q(t) = -\frac{1}{aH^2}\frac{d^2a}{dt^2}$ directly represents the decelerating or accelerating expansion of the universe. The jerk parameter $j(t) = \frac{1}{aH^3}\frac{d^3a}{dt^3}$ and snap parameter $s(t) = \frac{1}{aH^4}\frac{d^4a}{dt^4}$ are often used as geometrical diagnostics of dark energy models [?, ?]. An important feature to note is that jerk has been a traditional tool for testing the spatially flat cosmological constant dark energy model in which $j(z) = 1$ at all times. The lerk parameter $l(t) = \frac{1}{aH^5}\frac{d^5a}{dt^5}$ is a higher-order parameter for measuring cosmic expansion.

With this preparation, the Hubble parameter can be expressed via the Taylor series [?, ?]:

$$H(z) = H_0\left[1 + (1+q_0)z + \frac{1}{2}(-q_0 + j_0)z^2 + \frac{1}{6}(3q_0 + 3q_0^2 - 4q_0j_0 - 3j_0 - s_0)z^3 + \frac{1}{24}(-12q_0 - 24q_0^2 - 15q_0^3 + 3\right.$$

where the subscript "0" indicates the cosmographic parameters evaluated at the present epoch. According to the differential relations between the Hubble parameter and luminosity distance, the distance modulus of supernovae in cosmography studies can be conveniently expressed in logarithmic form [?, ?]:

$$\mu_{\cos}(z) = \frac{5}{\ln 10}\left[\ln z + C_1 z + C_2 z^2 + C_3 z^3 + C_4 z^4 + \mu_0\right],$$

where

$$C_1 = \frac{1}{2}(1 - q_0),$$

$$C_2 = -\frac{1}{6}(7 - 10q_0 - 9q_0^2 + 4j_0),$$

$$C_3 = \frac{1}{24}(5 - 9q_0 - 16q_0^2 - 10q_0^3 + 200j_0^2 + 36s_0 - 24l_0),$$

$$C_4 = \frac{1}{120}(-469+1004q_0+2654q_0^2+3300q_0^3-1148j_0-2620q_0j_0-1800q_0^2j_0+1575q_0^4+300q_0s_0),$$

and

$$\mu_0 = 42.38 - 5\log_{10} h,$$

with h being the Hubble constant H_0 in units of 100 km s^{-1} Mpc^{-1}. Previous cosmography studies found that supernova distance modulus cannot give robust constraints on the third-order parameter s_0. In the present paper, we extend the corresponding analysis to higher order. To be immune from the convergence problem at high redshift, Cattoën and Visser [?] introduced a y-redshift:

$$y = \frac{z}{1+z}.$$

For the favored y redshift, the distance modulus is

$$\mu_{\cos}(y) = \frac{5}{\ln 10}\left[\ln y + C_1 y + C_2 y^2 + C_3 y^3 + C_4 y^4 + \mu_0\right],$$

with

$$C_1 = \frac{1}{2}(3-q_0),$$

$$C_2 = -\frac{1}{6}(11-q_0+2q_0^2-4j_0),$$

$$C_3 = \frac{1}{24}(17-2q_0+9q_0^2-10q_0^3-j_0+8q_0j_0+s_0),$$

$$C_4 = \frac{1}{120}(971-76q_0+134q_0^2-300q_0^3-68j_0+260q_0j_0-1800q_0^2j_0+1575q_0^4+300q_0s_0).$$

For the y-redshift, when the redshift $0 < z < +\infty$, its corresponding value is $0 < y < 1$, which commonly ensures the expansion falls within the convergence radius. Therefore, it has the responsibility to solve the alarming convergence problem even for observational data in the high-redshift region. However, discussion about its usefulness has been stranded.

In our cosmographic study, we adopt a different probe from geometrical measurement: the dynamical redshift drift. More detailed descriptions follow.

In an expanding universe, we observe at time t_0 a signal emitted by a source at $t_{\rm em}$. The source' s redshift can be represented through the cosmic scale factor:

$$z(t_0) = \frac{a(t_0)}{a(t_{\rm em})} - 1.$$

Over the observer' s time interval Δt_0, the source' s redshift becomes

$$z(t_0 + \Delta t_0) = \frac{a(t_0 + \Delta t_0)}{a(t_{\rm em} + \Delta t_{\rm em})} - 1.$$

In terms of the Hubble parameter $H(z) = \dot{a}(t_{\rm em})/a(t_{\rm em})$, we simplify Eq. (15) to:

$$\dot{z} = (1+z)H_0 - H(z).$$

What we should highlight is its independence from any prior and dark energy model. For this unique advantage, many analyses have demonstrated that redshift drift is not only able to provide much stronger constraints on dynamical cosmological models [**?**, **?**], but also to solve some crucial cosmological problems [**?**, **?**], and even allows us to test the Copernican principle [**?**]. Observationally, it is convenient to probe the spectroscopic velocity drift:

$$\dot{v} \equiv \frac{\Delta v}{\Delta t_0} = \frac{c\dot{z}}{1+z},$$

which is of order several cm s^{-1} yr^{-1}. The signal naturally accumulates with increased observational time.

Taylor expansion tells us that the redshift drift should be:

$$\dot{z} = \left.\frac{dz}{dt_0}\right|_{t_0} \Delta t_0 + \frac{1}{2}\left.\frac{d^2 z}{dt_0^2}\right|_{t_0} \Delta t_0^2 + \cdots.$$

Using the Taylor series of the Hubble parameter in Eq. (6), we can put Eq. (18) into practice:

$$\dot{z}(z) = cH_0 \left[-q_0 z + (2q_0 + q_0^2 - j_0)z^2 + (-6q_0 - 6q_0^2 - 3q_0^3 + 4q_0 j_0 + 6j_0 + s_0)z^3 + (24q_0 + 36q_0^2 + 36q_0^3 + 15q_0^4 \cdots\right.$$

where $\Delta t_{\rm em}$ is the time interval for the source to emit another signal, satisfying $\Delta t_{\rm em} = \Delta t_0/(1+z)$. As a consequence, the observed redshift variation of the source is:

$$\dot{z} = \frac{a(t_0 + \Delta t_0)}{a(t_{\rm em} + \Delta t_{\rm em})} - \frac{a(t_0)}{a(t_{\rm em})}.$$

For the y-redshift, this simplifies to:

$$\dot{z}(y) = cH_0 \Big[-q_0 y + \frac{1}{2}(2q_0 + q_0^2 - j_0)y^2 + \frac{1}{6}(-6q_0 - 6q_0^2 - 3q_0^3 + 4q_0 j_0 + 6j_0 + s_0)y^3 + \frac{1}{24}(24q_0 + 36q_0^2 + 36q_0^3 -$$

Taking the first-order approximation to Eq. (14), the physical interpretation of redshift drift can be exposed:

$$\dot{z} = \frac{\dot{a}(t_0) - \dot{a}(t_{\rm em})}{a(t_{\rm em})}.$$

Obviously, we should note that the secular redshift drift monitors a variation of $\dot{a}$ during the evolution of the universe. For distance measurements, information content is commonly extracted via the integral of the Hubble parameter, approximately the integral of $\dot{a}$. Compared with distance measurements, Hubble parameter measurement may be a more effective probe of cosmic expansion information. However, acquisition of the Hubble parameter in observational cosmology must be accomplished indirectly from the differential ages of galaxies [?, ?, ?], from BAO peaks in the galaxy power spectrum [?, ?], or from BAO peaks using the Lyα forest of QSOs [?].

Observational Constraints and Approach

In this section, we report the relevant data used in our calculations. The current data we employ are the canonical distance modulus from the Union2.1 compilation. To test whether future SNIa observations can alleviate or resolve the important convergence problem, we also produce mock data from the Wide-Field InfraRed Survey Telescope-Astrophysics Focused Telescope Assets (WFIRST-AFTA) [?]. More importantly, the dynamical redshift drift is also forecasted to provide a brand-new inspection of cosmography. Parameters can be estimated through a Markov chain Monte Carlo method by modifying the publicly available code COSMOMC [?]. As introduced in Section II, cosmography is independent of dynamical models. Therefore, we fix the background variables and treat the cosmographic parameters as free parameters in our calculations.

Current Supernovae

One important reason why supernova data are widely used is their extremely plentiful resource. In this paper, we use the latest supernova Union2.1 compilation of 580 datasets from the Hubble Space Telescope Supernova Cosmology Project [?]. The data are typically presented as tabulated distance modulus with errors. In this catalog, the redshift spans $0 < z < 1.414$, with about 95% of samples in the low-redshift region $z < 1$.

To eliminate the nuisance parameter μ_0, an effective approach is generally performed by marginalizing over it in the likelihood [?, ?, ?]. A new form of the χ^2 statistics independent of μ_0 is eventually reduced to:

$$\tilde{\chi}^2(z,p) = A - \frac{B^2}{C},$$

where

$$A(p) = \sum_i \frac{[\mu_{\rm obs}(z_i) - \mu_{\rm cos}(z_i; \mu_0 = 0, p)]^2}{\sigma_i^2(z_i)},$$

$$B(p) = \sum_i \frac{\mu_{\rm obs}(z_i) - \mu_{\rm cos}(z_i; \mu_0 = 0, p)}{\sigma_i^2(z_i)},$$

$$C = \sum_i \frac{1}{\sigma_i^2(z_i)}.$$

The vector p indicates the parameters different from H_0 embedded in the cosmographic models. In our calculation, we include the covariance matrix with systematic errors.

Future Supernovae

In cosmology, forecasting the constraints from future observations on cosmological models is quite useful for theoretical research and equipment design. Estimating the uncertainty of observational variables is a core matter. In previous cosmography studies, one typically generates several mock datasets from a conceptual telescope or satellite [?], or extrapolates from current observational data [?]. To be more reliable, in the present paper we plan to use a realistic program. WFIRST-2.4 not only holds tremendous potential for key scientific programs but also enables a survey with more supernovae in a more uniform redshift distribution. One of its science drivers is to measure the cosmic expansion history. According to the updated report by the Science Definition Team [?], we obtain 2725 SNIa over the region $0.1 < z < 1.7$ with a bin size $\Delta z = 0.1$.

Redshift Drift

As suggested by Loeb [?], the redshift drift probe can be realized via the wavelength shift of QSO Lyα absorption lines, emission spectra of galaxies, and other radio techniques. The ground-based largest optical/near-infrared telescope E-ELT will preferentially provide continuous monitoring from the Lyα forest in the spectra of high-redshift QSOs [?]. These spectra are not only immune from noise due to peculiar motions relative to the Hubble flow but also contain a large number of lines in a single spectrum [?]. According to the capability of E-ELT, the uncertainties of velocity drift can be modeled as [?, ?]:

$$\sigma_{\Delta v} = 1.35 \frac{2370}{\mathrm{S/N}} \sqrt{\frac{30}{N_{\mathrm{QSO}}}} \left(\frac{1 + z_{\mathrm{QSO}}}{5} \right)^q \mathrm{cm/s},$$

with $q = -1.7$ for $2 < z < 4$, or $q = -0.9$ for $z > 4$, where the signal-to-noise ratio S/N is assumed to be 3000, the number of QSOs $N_{\mathrm{QSO}} = 30$, and z_{QSO} is the redshift in the range $2 < z < 5$.

Following previous works [?, ?, ?, ?], we obtain mock data assumed to be uniformly distributed among the redshift bins: $z_{\mathrm{QSO}} = [2.0, 2.8, 3.5, 4.2, 5.0]$ under the fiducial model from the best-fit values using Union2.1 samples. Unless specifically declared, the observational time is set to ten years.

Constraints Analysis

In this section, we perform related analyses from three perspectives. We first discuss the key convergence problem: what causes it and what contribution future observations can make toward solving it. In the following two sections, we focus on the cosmological deceleration parameter and dark energy in cosmography.

Convergence Problem

Using the Union2.1 compilation, we obtain the cosmographic parameters up to fourth order. We show the corresponding results in Figs. 1, 2, 3 and Table I.

For z-redshift, the current deceleration parameter gradually decreases with increasing series truncation. For fourth order, its best-fit value is $q_0 = -0.59$, which equals the derived estimation in a fiducial ΛCDM model. Most constraints on the parameter q_0 are less than zero, satisfying the discovery of recent accelerating expansion. However, we should note that q_0 for first order is positive, indicating a slowing down of acceleration. This novel phenomenon has attracted much attention [?, ?]. Their parameterized analysis found that cosmic acceleration may have already peaked, and expansion may be slowing down. Compared with other cosmographic parameters, q_0 is the best-constrained. Its errors for different orders are comparatively small. For example, $q_0 = -0.59 \pm 0.37$ for fourth order. For l_0, it is constrained as $l_0 = 130.45 \pm 176.22$. This is precisely what previous work pointed out—higher order brings non-physical constraints. Thus, the convergence problem naturally focuses on which series truncation favors the data best.

Traditionally, to display this problem, previous authors would compare a fiducial model (e.g., ΛCDM) with cosmographic models. We believe that residuals between observational data and cosmographic models can make the problem more apparent. In Fig. 3 [Figure 3: see original paper], we find that Taylor series with first order and fourth order are worse than the others. In particular, the situation becomes worse in the intermediate region $0.4 < z < 1$, regardless of order.

To mathematically reconcile this issue, Cattoën and Visser [?] introduced the y-redshift, with $0 < y < 1$. As expected, the new redshift parameter thus ensures that observations in the high-redshift region fall within the convergence radius. We also fit the supernova data to cosmography in y-redshift. Comparison between Fig. 1 and Fig. 2 shows that degeneracies among parameters do not change. Conversely, errors of the corresponding parameters are magnified much more for y-redshift. For fourth order, the parameters become quite nonphysical, e.g., $l_0 = 2361.94 \pm 3150.97$. In fact, the usefulness of y-redshift should receive more attention, though it is usually absent in previous literature.

We have been expecting that the villain in the cosmographic divergence problem should be identified. In our previous work [?], we introduced the F-test, which provides exactly a criterion for comparison, identifying which of two alternatives fits better. In such a test, one should assume the correctness of one model (the one with fewer parameters is preferable). Then we can assess the probability for the alternative model by fitting the data as well. If this probability is high, then no statistical benefit comes from the extra degrees of freedom associated with the new model. Hence, the smaller the probability, the more significant the data fitting of the second model against the first one will be. Using the F-test, we found that higher-order terms do not fit the SNIa data significantly better. Cosmographic expansion up to the jerk term j_0 is better for luminosity distance. For further analysis of this issue, we recommend the bias-variance tradeoff [?]:

$$\text{risk} = \text{bias}^2 + \text{variance} = \frac{1}{N}\sum_{i=1}^{N}[\mu_{\text{cos}}(z_i) - \mu_{\text{obs}}(z_i)]^2 + \sigma^2(\mu_{\text{cos}}(z_i)),$$

where $\mu_{\text{cos}}(z_i)$ is the reconstructed cosmographic distance modulus for different series truncations, $\mu_{\text{obs}}(z_i)$ is the observed data, and $\sigma(\mu_{\text{cos}}(z_i))$ is the uncertainty of reconstruction. Obviously, the bias-variance tradeoff can reveal more detailed information, being influenced not only by observational precision but also by its accuracy. Theoretically, few parameters lead to an estimator with large bias but small variance. By contrast, more parameters lead to an estimator with smaller bias but larger variance. Minimizing risk corresponds to a balance between bias and variance. In cosmology, this promising approach has been widely utilized to find an effective way of obtaining information about the dark energy equation of state $w(z)$ [?, ?].

In Fig. 4 [Figure 4: see original paper], we plot this tradeoff for current supernova observational data. The line graphs show that the bias squared for different orders is more than forty. More importantly, it does not markedly decrease with increasing series truncation. That is, Taylor expansion with higher order does not improve the fit. The disharmony between cosmography and supernova data is not reconciled. This is not what we expect. Considering the variance, it is small for lower order. However, for third order, the variance starts to increase markedly. This is because even though current supernova data can constrain the parameter s_0, its error is still large. For fourth order, the variance is amazingly large, up to 2540.

To explicitly validate our concerns, we also plot the bias-variance tradeoff for y-redshift in the right panel. The bias squared in this case is not optimistic compared with z-redshift. It is also more than forty, even fifty. From this perspective, the introduction of y-redshift does not improve the trouble. Comparing the panels, we can steadily deduce that Taylor expansion up to the j_0 term, where the risk is minimum, is a better choice for SNIa data. This consequence is consistent with our previous work using the F-test [**?**]. Using results from our previous work, we also calculate the bias squared for the Union2 compilation containing 557 samples. For second order it is 46.64, which is slightly larger than 42.38 from the Union2.1 sample. Obviously, improvement in supernova observation does not markedly reduce the bias squared. There is no doubt that bias turns out to be the leading killer in the convergence problem. Therefore, how to reduce the large bias should be the most important matter.

We simulate mock SNIa data using WFIRST and redshift drift using E-ELT. We show the results in Table I. With improved observational precision, future observations can give tighter constraints on cosmography. For example, compared with $\sigma_{l_0} = 176.22$ from the Union2.1 sample, the upcoming WFIRST improves the parameter l_0 to $\sigma_{l_0} = 25.66$. Undoubtedly, WFIRST improves the precision by one order of magnitude. For redshift drift, as expected, it gives much more robust constraints on the parameters, e.g., $\sigma_{l_0} = 3.08$. In Fig. 5 [Figure 5: see original paper], we compare the variances for three different observations. In our reconstruction, we use general error propagation including the covariance matrix. We confirm that future WFIRST can dramatically narrow the constraints, especially for higher-order Taylor series. For secular redshift drift, it provides an unimaginable constraint on higher-order terms. This is partly because of the precise error of redshift drift from E-ELT. The more important reason lies in the fact that redshift drift is a dynamical probe that carries more detailed information about cosmic expansion.

One point we should explain is that the variance for third order from redshift drift appears slightly larger than that from WFIRST. In fact, the errors of corresponding parameters given by redshift drift are much smaller. The larger variance is due to degeneracies between parameters contained in the covariance matrix. Returning to the main point, variances from the Union2.1 compilation are in fact already considerably small from the analysis in Fig. 4. What makes cosmography troublesome is the bias. It cannot be suppressed even when we extend the Taylor series. This may be a "congenital disease" of distance measurement. Owing to its model independence and direct measurement of expansion, redshift drift is more likely to give lower bias, thus reconciling the convergence problem.

Deceleration Parameter

The deceleration parameter $q(z)$ is an important factor for describing cosmic expansion history. Especially, its positive (negative) sign immediately indicates decelerating (accelerating) expansion. Current observations commonly favor the transition from deceleration to acceleration at redshift $z < 1$. However, the deceleration parameter is not an observable quantity. Therefore, multiform parameterized $q(z)$ have been proposed. Different parameterizations commonly give different results when confronted with observational data. Nevertheless, cosmography could theoretically serve as a relatively fair ruler for describing cosmic expansion due to its model independence. Unfortunately, cosmography suffers from a serious convergence problem. Next, we investigate its effect on the deceleration parameter.

Using the COSMOMC constraints, we can reconstruct the deceleration parameter via the differential relation:

$$q(z) = \frac{d\ln H(z)}{d\ln(1+z)} - 1.$$

In Fig. 6 [Figure 6: see original paper], we show the cosmographic distance modulus, Hubble parameter, and deceleration parameter for different orders. For the distance modulus, they apparently all fit the observational data well and are almost indistinguishable. However, we should not be fooled by this illusion. We cannot forget that $H(z)$ is the first derivative of luminosity distance, while $q(z)$ is its second derivative. In the middle panel, we find that conflict concealed in the distance is now disclosed by the Hubble parameter. The harm of the order problem has affected the reconstruction. At low redshift, $H(z)$ for different orders are consistent. However, their differences gradually enlarge for $z > 0.4$. For the deceleration parameter, the series truncation issue is highlighted. No cosmographic $q(z)$ looks the same as others. Compared with $q(z)$ from the fiducial ΛCDM model, cosmographic ones in fourth order at low redshift are consistent with it. Just as the residual analysis in Fig. 3 shows, data in the region $z > 0.4$ are not good. The bottom panel shows that cosmographic $q(z)$ in fourth order can be distinguished from the ΛCDM model at $z > 0.4$. Therefore, if supernova data at high z can be improved, the dynamical ΛCDM model may act as a prolocutor of cosmography. We also note that $q(z)$ in fourth order is negative. This is because $H(z)$ in this order decreases at high redshift. Therefore, reasonable determination of series truncation is so important.

In fact, Fig. 6 implies that dynamical measurement can give more stable estimation of cosmic expansion history. In our previous work [**?**], we confirmed that inclusion of $H(z)$ data can lead to strong constraints on cosmographic parameters. Distance measurements are unable to discriminate cosmography from several cosmological models, while $H(z)$ data are successful at a high level. Recently, Neben and Turner [**?**] revisited cosmography up to second order using mock data. They showed that distance indicators cannot directly measure q_0

with both accuracy and precision. However, redshift drift may be able to do so. Therefore, it is reasonable for us to anticipate that dynamical redshift drift could provide a much more stable evaluation of cosmic expansion history.

Dark Energy Equation of State

Because the hypothesis in cosmography is so pure that it does not depend on any dynamical model, it is often considered a tool for selecting among competing models. Many believe that cosmography is a selection criterion to discriminate which model works fairly well over others. On one hand, if one fixes the cosmological observables in terms of cosmography, the degeneracy problem among dark energy models can in principle be alleviated. On the other hand, once a cosmography is fixed, it is possible to reconstruct the dynamical cosmological model in terms of their relations. For example, with two cosmographic parameters (q_0, j_0), we can derive new dynamical parameters for a dark energy model with constant equation of state (EoS):

$$\Omega_m(q_0, j_0) = \frac{2(j_0 - q_0 - 2q_0^2)}{1 + 2j_0 - 6q_0},$$

$$w_0(q_0, j_0) = \frac{-3 + 6q_0}{1 + 2j_0 - 6q_0}.$$

If the values on the left side of the equation differ from those obtained by other methods, it may indicate the validity of a different theory.

Whether for constant EoS or varying EoS dark energy models, it cannot be hidden that corresponding results strongly depend on the considered dark energy model. We should elaborate on the influence of series truncation issues on dark energy. To extract an EoS that is model-independent to the greatest extent possible, our study is made in the normal cosmological model:

$$H^2(z) = H_0^2\left[\Omega_m(1+z)^3 + (1-\Omega_m)\exp\left(3\int_0^z \frac{1+w(z')}{1+z'}dz'\right)\right].$$

In our analysis, we do not impose any specific form of dark energy, but use the generic $w(z)$. From Eq. (30), we obtain a very ordinary dark energy:

$$1 + w(z) = \frac{[H^2(z) - H_0^2\Omega_m(1+z)^3]'(1+z)}{3[H^2(z) - H_0^2\Omega_m(1+z)^3]},$$

where the prime denotes the derivative with respect to redshift z. For Eq. (31), we note that the denominator may be zero when $H(z)^2 = H_0^2\Omega_m(1+z)^3$ is satisfied. For convenience, we fix the matter density $\Omega_m = 0.3$ for a flat curvature space in the calculation. In Fig. 7 [Figure 7: see original paper], we plot the EoS for different series truncations. We find that they vary intensely for different orders. At low redshift, most are close to $w = -1$, implying that the cosmological constant model is consistent with cosmography. At high redshift, dark energy becomes more complicated to understand. Especially for second and fourth orders, the deceleration parameter exhibits singularities. We verify that the Hubble parameter for second and fourth orders shown in Fig. 6 indeed has intersection points with the factor $\Omega_m(1+z)^3$. We also change the matter density Ω_m from 0.2 to 0.35, demonstrating that the corresponding conclusions are consistent. Therefore, on one hand, one should be careful when treating cosmography from supernovae as a selection criterion to discriminate dark energy models. On the other hand, from Eq. (31), we can see that redshift drift can provide a more impartial estimation of the EoS, due to its superiority over distance indicators for measuring the Hubble parameter.

Conclusion and Discussion

Cosmography describes cosmic evolution via Taylor expansion. However, when fitting supernova data, a dramatic discrepancy arises—the convergence issue. To reconcile this problem, a $y = z/(1+z)$ redshift was defined to ensure observational data fall within the convergence radius $0 < y < 1$, especially for redshift $z > 1$. However, discussions about the usefulness of y-redshift and the key reason for the convergence issue are generally absent. In the present paper, we try to catch the villain with the help of dynamical redshift drift.

Our study is performed in both z and y redshift. The bias-variance tradeoff is an effective approach in previous cosmology studies due to its dual dependence on observational precision and accuracy. By introducing this method, we reveal that the large squared bias between cosmography and current supernova data seriously manifests the convergence issue. We find that it does not significantly decrease when we extend the expansion to higher series truncation. It also does not significantly decrease when we introduce y-redshift. Obviously, extending to higher order and introducing y-redshift are both ineffective at solving the convergence problem. Conversely, the corresponding variances sharply increase with extension to higher order, especially for the l_0 term. This is precisely the non-physical result often mentioned. Minimizing risk suggests that expansion up to the j_0 term is a better choice for available supernova data, consistent with our previous work using the F-test [**?**]. Forecasts from mock SNIa and redshift drift data show that they can certainly improve constraints significantly. Moreover, redshift drift can give much tighter constraints on cosmography. This is because canonical redshift drift is a dynamical probe, providing direct measurement of cosmic expansion history, while distance measurement is geometric. In fact, this phenomenon has attracted attention. Maor et al. [**?**] studied the luminosity distance for determining the EoS of the universe and proved that luminosity distance depends on w through a multiple integral relation that smears out much information.

We also investigate the effect of the convergence issue on the deceleration parameter and dark energy. We find that conflict is concealed in distance, but disclosed by the Hubble parameter, and highlighted by the deceleration parameter. In our previous work [?], we also confirmed that inclusion of $H(z)$ data can lead to strong constraints on cosmographic parameters. Reconstruction of the EoS indicates that one should be careful when treating cosmography from supernovae as a selection criterion to discriminate dark energy models. Therefore, it is reasonable to believe that dynamical measurements such as redshift drift can provide more detailed information about cosmic expansion history.

Although many works have pointed out the serious convergence issue, the "chief culprit" remains unknown. Our study identifies that the squared bias between cosmography and supernova data is very dangerous. For comparison, we also calculate the bias squared of the Union2 compilation containing 557 samples using our previous work [?]. For second order it is 46.64, slightly larger than 42.38 from the Union2.1 sample. We worry that improvement in supernova observation does not markedly reduce the bias. Redshift drift as a dynamical measurement has very good potential to overcome this puzzle. Moreover, it can be realized via multiple wavebands and methods. Recently, a tentative measurement has been made with the Green Bank Telescope for low redshift [?]. Tests with the German Vacuum Tower Telescope also demonstrate that Laser Frequency Combs have an advantage in long-term calibration precision and accuracy for realizing the redshift drift experiment [?]. It is reasonable to expect that redshift drift will play an increasingly important role in studying cosmic expansion history. In our future work, we would also like to further explore which elements influence the bias and how to reduce the bias for supernova observation, to fully mine the value of supernova data.

Acknowledgments

We thank Wei Zheng and Si-Yu Li for their help with the calculations. M.-J. Zhang is funded by the China Postdoctoral Science Foundation under grant No. 2015M581173. H. Li is supported in part by NSFC under grant Nos. 11033005 and 11322325 and by the 973 program under grant No. 2010CB83300. J.-Q. Xia is supported by the National Youth Thousand Talents Program and the National Science Foundation of China under grant No. 11422323. The research is also supported by the Strategic Priority Research Program "The Emergence of Cosmological Structures" of the Chinese Academy of Sciences, grant No. XDB09000000.

References

[1] A. Riess et al., Astron. J 116, 1009 (1998). [2] S. Perlmutter, G. Aldering, G. Goldhaber, et al., Astrophys. J. 517, 565 (1999). [3] M. Tegmark, M. Strauss, M. Blanton, et al., Phys. Rev. D 69, 103501 (2004). [4] D. Spergel et al., Astrophys. J. Suppl 148, 170 (2003). [5] D. J. Eisenstein, I. Zehavi, D. W. Hogg, et al., The Astrophysical Journal 633, 560 (2005). [6] S. M. Carroll, W. H. Press, and

E. L. Turner, Annu. Rev. Astron. Astrophys. 30, 499 (1992). [7] R. Caldwell, M. Kamionkowski, and N. Weinberg, Phys. Rev. Lett. 91, 71301 (2003). [8] B. Feng, X. Wang, and X. Zhang, Phys. Lett., B 607, 35 (2005). [9] J. D. Barrow and S. Cotsakis, Phys. Lett. B 214, 515 (1988). [10] G. Dvali, G. Gabadadze, and M. Porrati, Phys. Lett. B 485, 208 (2000). [11] G. Lemaître, Annales de la Societe Scietifique de Bruxelles 53, 51 (1933). [12] R. C. Tolman, Proceedings of the national academy of sciences of the United States of America 20, 169 (1934). [13] H. Bondi, Mon. Not. R. Astron. Soc. 107, 410 (1947). [14] T. Chiba and T. Nakamura, Progress of theoretical physics 100, 1077 (1998). [15] M. Visser, Classical and Quantum Gravity 21, 2603 (2004). [16] C. Cattoën and M. Visser, Classical and Quantum Gravity 24, 5985 (2007). [17] C. Cattoën and M. Visser, Phys. Rev. D 78, 063501 (2008). [18] S. Capozziello, R. Lazkoz, and V. Salzano, Physical Review D 84, 124061 (2011). [19] J.-Q. Xia, V. Vitagliano, S. Liberati, and M. Viel, Phys. Rev. D 85, 043520 (2012). [20] V. Vitagliano, J.-Q. Xia, S. Liberati, and M. Viel, Journal of Cosmology and Astroparticle Physics 2010, 005 (2010). [21] A. Sandage, Astrophys. J. 136, 319 (1962). [22] A. Loeb, Astrophys. J. Letters 499, L111 (1998). [23] L. Pasquini, S. Cristiani, H. Dekker, et al., The Messenger 122, 10 (2005). [24] J. Liske, A. Grazian, E. Vanzella, et al., Mon. Not. R. Astron. Soc. 386, 1192 (2008). [25] J. Liske, A. Grazian, E. Vanzella, et al., The Messenger 133, 10 (2008). [26] A. Balbi and C. Quercellini, Mon. Not. R. Astron. Soc. 382, 1623 (2007). [27] H. Zhang, W. Zhong, Z.-H. Zhu, and S. He, Phys. Rev. D 76, 123508 (2007). [28] J. Zhang, L. Zhang, and X. Zhang, Phys. Lett. B 691, 11 (2010). [29] Z. Li, K. Liao, P. Wu, H. Yu, and Z.-H. Zhu, Phys. Rev. D 88, 023003 (2013). [30] J.-P. Uzan, C. Clarkson, and G. F. Ellis, Physical Review Letters 100, 191303 (2008). [31] H.-R. Yu, T.-J. Zhang, and U.-L. Pen, Physical Review Letters 113, 041303 (2014). [32] M.-J. Zhang and W.-B. Liu, The European Physical Journal C 74, 1 (2014). [33] M.-J. Zhang, J.-Z. Qi, and W.-B. Liu, International Journal of Theoretical Physics 54, 2456 (2014). [34] V. Sahni, T. D. Saini, A. A. Starobinsky, and U. Alam, Journal of Experimental and Theoretical Physics Letters 77, 201 (2003). [35] U. Alam, V. Sahni, T. D. Saini, and A. Starobinsky, Monthly Notices of the Royal Astronomical Society 344, 1057 (2003). [36] R. Jimenez and A. Loeb, The Astrophysical Journal 573, 37 (2008). [37] J. Simon, L. Verde, and R. Jimenez, Phys. Rev. D 71, 123001 (2005). [38] D. Stern, R. Jimenez, L. Verde, M. Kamionkowski, and S. A. Stanford, Journal of Cosmology and Astroparticle Physics 2010, 008 (2010). [39] E. Gaztanaga, A. Cabre, and L. Hui, Monthly Notices of the Royal Astronomical Society 399, 1663 (2009). [40] M. Moresco, A. Cimatti, R. Jimenez, et al., Journal of Cosmology and Astroparticle Physics 08, 006 (2012). [41] N. G. Busca, T. Delubac, J. Rich, et al., Astronomy and Astrophysics 552, A96 (2013), 1211.2616. [42] M. Martinelli, S. Pandolfi, C. Martins, and P. Vielzeuf, Phys. Rev. D 86, 123001 (2012). [43] B. Moraes and D. Polarski, Phys. Rev. D 84, 104003 (2011). [44] A. Lewis and S. Bridle, Physical Review D 66, 103511 (2002). [45] N. Suzuki, D. Rubin, C. Lidman, et al., Astrophys. J. 746, 85 (2012). [46] E. D. Pietro and J.-F. Claeskens, Mon. Not. R. Astron. Soc. 341, 1299 (2003). [47] S. Nesseris and L. Perivolaropoulos, Phys. Rev. D 72, 123519 (2005). [48] L. Perivolaropoulos, Phys. Rev. D 71, 063503 (2005). [49] B. Bochner, D. Pappas, and M. Dong, Astrophys. J. 814, 7 (2015), 1308.6050. [50] D. Spergel, N. Gehrels, C. Baltay, D. Bennett, J. Breckinridge, M. Donahue, A. Dressler, B. Gaudi, T. Greene, O. Guyon, et al., arXiv:1503.03757 (2015). [51] L. Pasquini, S. Cristiani, H. Dekker, et al., Scientific Requirements for Extremely Large Telescopes 232, 193 (2006). [52] A. Shafieloo, V. Sahni, and A. A. Starobinsky, Phys. Rev. D 80, 101301 (2009). [53] V. H. Cárdenas, C. Bernal, and A. Bonilla, Mon. Not. R. Astron. Soc. 433, 3534 (2013). [54] L. Wasserman, All of Nonparametric Statistics (Springer Science & Business Media, 2006). [55] D. Huterer and G. Starkman, Phys. Rev. Lett. 90, 031301 (2002). [56] W. Zheng, S.-Y. Li, H. Li, J.-Q. Xia, M. Li, and T. Lu, Journal of Cosmology and Astroparticle Physics 2014, 030 (2014). [57] A. R. Neben and M. S. Turner, Astrophys. J. 769, 133 (2013). [58] I. Maor, R. Brustein, and P. J. Steinhardt, Physical Review Letters 86, 6 (2001). [59] J. Darling, Astrophys. J. Letters 761, L26 (2012). [60] T. Steinmetz, T. Wilken, C. Araujo-Hauck, R. Holzwarth, T. W. Hänsch, L. Pasquini, A. Manescau, S. D' Odorico, M. T. Murphy, T. Kentischer, et al., Science 321, 1335 (2008). [61] http://wfirst.gsfc.nasa.gov/

Note: Figure translations are in progress. See original paper for figures.